International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2013): 4.438

A Mathematical Model of Vehicle Route Cost Estimation Problem

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Abstract: ***The present model deals with the uncertainty in the transportation problems. The proposed algorithm is used to solve a special class of transportation problem in which uncertainty is present in the items to be transported by different types of vehicles. The transportation cost involves the vehicles types and the revenue lost due the unsatisfied demand of destinations. The uncertainty shows with the help of fuzzy number representation.***

Keywords: Fuzzy Set, Trapezoidal fuzzy number, Greenberg's Algorithm, Parametric programming and Integer programming.

1. Introduction

The transportation problem model has wide practical applications, not only in transportation problems put also in the case of production, management, planning etc. In practical situations every vehicle has variation in capacity which can be depend on route or types of item be transported. These inaccuracies can be due to environment conditions or uncertainty present in transportation routes.

The generalized classical transportation problem was first emerged by Ferguson & Dantzig [1] in a paper discussing to the allocation of aircraft routes. Other researchers have also structured the transportation problems (TP) into classical set theory. In the classical TP with integer demands and supply, the solution can be determined with Simplex method or by Vogel approximation etc. But to tackle the uncertainty in nature the fuzzy set theoretic approach helps to remove such types of uncertainty at a limit. This has been developed and introduce by Zadeh [6].

2. Preliminary

"Zadeh [6]" first introduced the fuzzy set for dealing with vagueness type of uncertainty. A fuzzy set Ã define on the universe X which is characterized by a membership function such that $\mu_{\tilde{A}} : X \to [0,1]$. Then a fuzzy set Ã on X is a set of ordered pairs :

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$$

where $\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership function or degree of compatibility or degree of truth that maps to the membership space [0, 1]. When this space contains only two points 0 & 1 then Ã is said to be non fuzzy or crisp set and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a non fuzzy set. The family of all fuzzy sets in X is denoted *F (X)*.

Def 2.1: The ***support*** of fuzzy set Ã is the crisp set of all $x \in$ X such that $\mu_{\tilde{A}}(x) \geq 0$ & denoted by S (Ã).

Def 2.2: The set of elements that belong to the fuzzy set Ã at least to the degree α is called the ***α-level set***

$$\tilde{A}^{\alpha} = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$$

Def. 2.3: **Generalized Fuzzy Number:-**

A generalized fuzzy number Ã, conventionally represented by Ã = (a, b; β, γ) or Ã = (left normalized point, right normalized point ; left spread, right spread) and is a normalized convex fuzzy subset on the real line R if -

(i) S (Ã) is a closed & bounded interval i.e. [a- β, b + γ]

(ii) $\mu_{\tilde{A}}(x)$ is an upper semi continuous function.

(iii) a - β ≤ a ≤ b ≤ a + γ

(iv) The membership function is of the following form.

$$\mu_{\tilde{A}}(x) = \begin{cases} f(x) & \text{for} \quad x \in [a - \beta, a] \\ 1 & \text{for} \quad x \in [a, b] \\ g(x) & \text{for} \quad x \in [b, b + \gamma] \end{cases}$$

where f (x) and g (x) are monotone increasing and decreasing functions respectively.

Def. 2.4: **LR -Type Fuzzy Numbers:-** A generalized Fuzzy Numbers Ã = (a, b; β, γ) is said to be LR type if there exists two reference functions, known as shape function L & R such that

$$\mu_{\tilde{A}}(x) = \begin{cases} L(\frac{a-x}{\beta}) & \text{for} \quad x \in [a - \beta, a] \\ 1 & \text{for} \quad x \in [a, b] \qquad \textit{where } \beta \geq 0, \gamma \geq 0 \\ R(\frac{x-b}{\gamma}) & \text{for} \quad x \in [b, b + \gamma] \end{cases}$$

If the left and right spread functions f(x) & g(x) are linear then the LR-type fuzzy number is said to be linear Trapezoidal fuzzy number given in fig 1.

Some special cases are possible in which the type of one or both of the functions L and R may have no significance i.e.

- $a = -\infty : \mu_{\tilde{A}}(x) = 1$ *for* $x \leq a$
- $b = +\infty : \mu_{\tilde{A}}(x) = 1$ *for* $x \geq b$
- $\beta = 0 : \mu_{\tilde{A}}(x) = 0$ *for* $x \leq a$
- $\gamma = 0 : \mu_{\tilde{A}}(x) = 1$ *for* $x \geq b$

Def. 2.5: **Trapezoidal Fuzzy Number**: An fuzzy number $\tilde{A}$ is represented as $\tilde{A}=[a, b, c, d]$ is known as trapezoidal fuzzy number and it is the special case of LR-type fuzzy number. In which left and right spread of the functions are linear in nature its membership function is shown as in fig 1.

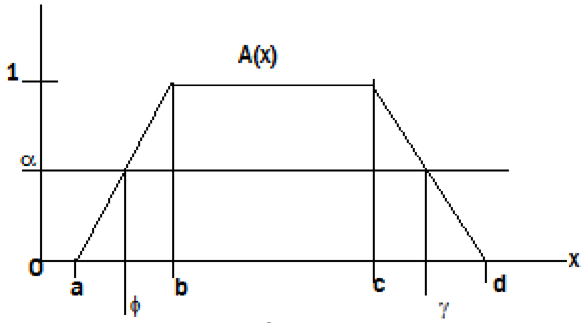

Fig-1
Trapezoidal Fuzzy Number

3. Problem description

The parameters of each transportation problem are unit cost demand and supply values and these type of problems can be operated by integer transportation problem. But in practical situation the above parameters are not known exactly and may have variation, these type of situation can be removed by considering the parameter in the form of Trapezoidal fuzzy numbers. Stefan Chanas & Dorota Kuchta [5] developed an algorithm to obtain such type of problems using parametric techniques in which uncertainty is present in supply and demand. Similarly Omer M. Saad [2] has given another algorithm for solving parametric transportation problem with variation in quantity in transported items. The proposed algorithm is to determine the solution of generalized transported problem having variation in transported items capacity of each type of vehicle. The problem is defined as follows-

Table 3.1

Source→ Destination↓	**S_1**	**S_2**	**S_3**	**S_4**	**…**	**S_m**	**Demand**
D_1	C_{11} $\tilde N_{ij}$	C_{21} $\tilde N_{ij}$	C_{31} $\tilde N_{ij}$	…	…	C_{m1} $\tilde N_{ij}$	d_1
D_2	C_{12} $\tilde N_{ij}$	C_{22} $\tilde N_{ij}$	…	…	…	…	d_2
D_3	C_{13} $\tilde N_{ij}$	…	…	…	…	…	d_3
…	…	…	…	…	…	…	
…	…	…	…	…	…	…	
D_n	C_{1n} $\tilde N_{ij}$	C_{2n} $\tilde N_{ij}$	C_{3n} $\tilde N_{ij}$	…	…	C_{mn} $\tilde N_{ij}$	d_n
Availability	a_1	a_2	a_3	a_4		a_m	

where the parameters are defined as-

C_{ij} denotes transported cost from source jth to ith destination in one cycle

$\tilde N_{ij}$ denotes transported quantity of items from source jth to ith destination in one cycle and in form of fuzzy number

d_j demand of jth destination

a_i availability of ith source

x_{ij} be the assigned transported quantity from jth source to ith destination

k_j loss due to the unsatisfied demand per item

p_j unsatisfied demand quantity of items

The above transportation problem can be formulated as follows.

$$(1)\ Minimize\ Z = \sum_{i=1}^{m} . \sum_{j=1}^{n} C_{ij} \frac{x_{ij}}{\tilde N_{ij}} + \sum_{j=1}^{n} k_j p_j$$

$$subject\ to \quad \sum_{i=1}^{m} x_{ij} \le d_j$$

$$\sum_{i=1}^{n} x_{ij} \le a_i$$

$$p_j = d_j - \sum_{i=1}^{n} \frac{x_{ij}}{\tilde N_{ij}}$$

where C_{ij}, a_j, d_j, k_j are positive integers and $\tilde N_{ij}$ are Trapezoidal continuous fuzzy numbers on the real line, whose membership function is defined as.

$$\tilde N_{ij} = [n_{ij}^1, n_{ij}^2, n_{ij}^3, n_{ij}^4]$$

1. It is continuous mapping from R to the closed interval [0, 1].
2. $\tilde N_{ij} = 0\ on\ (-\infty, n_{ij}^1]$.
3. $\tilde N_{ij}\ is\ strictly\ increasing\ on\ [n_{ij}^1, n_{ij}^2]$
4. $\tilde N_{ij} = 1\ on\ [n_{ij}^2, n_{ij}^3]$
5. $\tilde N_{ij}\ is\ strictly\ decreasing\ on\ [n_{ij}^3, n_{ij}^4]$
6. $\tilde N_{ij} = 0\ on\ [n_{ij}^4, \infty)$

Now we introduce the concept of α-level set or α-cut of the fuzzy numbers as follows.

Def 3.1: The α-level set of fuzzy number is defined as the interval set for which the membership value exceeds the level value α∈[0, 1].

$$(N_{ij})_\propto = \{n_{ij} : \tilde N_{ij} \ \ge \propto, i = 1, \dots m, j = 1, \dots n\}$$

$$(N_{ij})_\propto = [\underline{(N_{ij})_\propto}, \overline{(N_{ij})_\propto}]$$

$\tilde N_{ij}$ is the fuzzy number then $1/\tilde N_{ij}$ is also a fuzzy number let it is denoted by ${}^c\tilde N_{ij}$,which can be defined as-

$${}^c\tilde N_{ij} = \cup_{\propto\in[0,1]} \left(\frac{1}{\tilde N_{ij}}\right)_\propto$$

Where ${}_\propto(1/\tilde N_{ij}) = \propto . (N_{ij})_\propto$ and hence

$${}^c(N_{ij})_\propto = [1/\overline{(N_{ij})_\propto}, 1/\underline{(N_{ij})_\propto}] \equiv [{}^c\underline{(N_{ij})}_\propto, {}^c\overline{(N_{ij})}_\propto]$$

For the solution of the above fuzzy integer linear programming problem we start with α-cut problem that is non-fuzzy integer problem and that problem can be solved by simplex integer programming methods. That can be formulated as-

$$(2)\ Minimize\ Z = \sum_{i=1}^{m} . \sum_{j=1}^{n} C_{ij} \tilde n_{ij} x_{ij} + \sum_{j=1}^{n} k_j p_j$$

$$subject\ to \quad \sum_{i=1}^{m} x_{ij} \le d_j$$

$$\sum_{i=1}^{n} x_{ij} \le a_i$$

$$\sum_{i=1}^{n} \tilde n_{ij} x_{ij} + p_j = d_j$$

$$\tilde n_{ij} \in {}^c(N_{ij})_\propto$$

C_{ij}, a_j, d_j, $k_j \ge 0$ for all $i = 1, \dots m; j = 1, \dots n$.

Now the problem can be rewritten in the following form-

$$(3)\ Minimize\ Z = \sum_{i=1}^{m} . \sum_{j=1}^{n} C_{ij} \tilde n_{ij} x_{ij} + \sum_{j=1}^{n} k_j p_j$$

$$subject\ to \quad \sum_{i=1}^{m} x_{ij} \le d_j$$

$$\sum_{i=1}^{n} x_{ij} \le a_i$$

$$\sum_{i=1}^{n} \tilde n_{ij} x_{ij} = r_j\ ; \quad x_{ij} \ge 0$$

Where $r_j = d_j - p_j$ and $\tilde n_{ij} \in {}^c(N_{ij})_\propto$

C_{ij}, a_j, d_j, $k_j \ge 0$ for all $i = 1, \dots m; j = 1, \dots n$.

Now using Greenberg's algorithm[] to solve the problem. The algorithm proceeds in the following ways:

On solving the problem considering as parametric linear problem let we reach at k^{th} iteration, the vectors variable X is expressed in terms of a vector of nonnegative integer parameters Z. Hence the above problem can be reconsidered as follows:

$$(4)\ Minimize\ Z^k = L^k + C^k y^k$$
$$subject\ to \qquad X = T^k + Q^k y^k$$
$$S = P^k + R^k y^k$$

Where all integer constants $L^k \geq 0$; $C^k \geq 0$ and S stands for all slack variables. Initially the objective function is equal to L^k and $y^k = 0\ whenever\ T^k \geq 0\ , P^k \geq 0$ proceeds to provide the optimal solution in finite number of steps.

4. Algorithm

In the following we describe an algorithm to solve the Fuzzy Transportation problem. The capacity of each type of vehicle is fuzzy in nature therefore represented by fuzzy number in the above types of problems. The proposed solution algorithm can be summarized in the following steps:

Step-1: First of all formulate the transportation problem in the form of fuzzy linear programming problem as (1).

Step-2: Apply α-cut to the Trapezoidal fuzzy parameter $\tilde{N}_{ij}$.

Step-3: Set the certain degree $\alpha=\alpha^*$ into [0, 1] we can initiate taking α=0.

Step-4: Determine the α-cut interval and proceed to determine its inverse interval.

Step-5: Choose $\tilde{n}_{ij} \in {}^c\ (N_{ij})_\propto$ for that α^*.

Step-6: Use the Greenberg Algorithm to solve the problem as explained at (3) – (4) and find the integer solution of the problem.

Step-7: Set $\alpha=(\alpha^* + \varepsilon)$ into [0, 1].

Step-8: Repeat again the process Step-1 to 7 until the interval [0, 1] is fully exhausted then, stops.

Step-9: Compare all the solution for different value of α^* and find the optimum value of the objective function.

5. Example

Let us take a numerical example to explain the proposed model. Let we have Three destination and four warehouses. The vehicle capacity for each route is given in the form of Trapezoidal fuzzy number. The problem is defined as follows-

Destination → Source↓	**D_1**	**D_2**	**D_3**	**Available**
S_1	**$C_{11}=3$** $\tilde{N}_{11} = [2\ 4\ 5\ 7]$	**$C_{11}=6$** $\tilde{N}_{11} = [0\ 3\ 4\ 7]$	**$C_{11}=1$** $\tilde{N}_{11} = [3\ 4\ 5\ 7]$	**$S_1=170$**
S_2	**$C_{12}=6$** $\tilde{N}_{12} = [1\ 5\ 7\ 10]$	**$C_{12}=5$** $\tilde{N}_{12} = [0\ 3\ 5\ 6]$	**$C_{12}=3$** $\tilde{N}_{12} = [1\ 6\ 7\ 9]$	**$S_2=60$**
S_3	**$C_{13}=3$** $\tilde{N}_{13} = [1\ 2\ 5\ 7]$	**$C_{13}=8$** $\tilde{N}_{13} = [1\ 3\ 4\ 6]$	**$C_{13}=10$** $\tilde{N}_{13} = [0\ 2\ 3\ 5]$	**$S_3=35$**
S_4	**$C_{14}=4$** $\tilde{N}_{14} = [2\ 4\ 5\ 7]$	**$C_{14}=15$** $\tilde{N}_{14} = [0\ 3\ 4\ 7]$	**$C_{14}=6$** $\tilde{N}_{14} = [3\ 4\ 5\ 7]$	**$S_4=60$**
DEMAND	**140**	**175**	**10**	**320**

The unsatisfied demand penalty of each destination is-

$D_1=5$, $D_2=6$, $D_3=4$;

The calculation of the example is conduct by the help of MATLAB and by doing all iteration work we find that for α=1, we find the Integer solution of the given problem As follows-

$$X = \begin{bmatrix} 50 & 110 & 8 \\ 6 & 54 & 0 \\ 32 & 0 & 2 \\ 50 & 8 & 0 \end{bmatrix}$$

Numbers of round of each vehicle from source to destinations are as follows-

$$X = \begin{bmatrix} S\backslash D & D_1 & D_2 & D_3 \\ S_1 & 10 & 27 & 2 \\ S_2 & 1 & 11 & 0 \\ S_3 & 6 & 0 & 0 \\ S_4 & 10 & 2 & 0 \end{bmatrix}$$

Hence the Transportation Cost is Rs. 343

The Unsatisfied demands of the destination are-

$D_1=2$, $D_2=2$, $D_3=2$;

Hence the Penalty cost of each destinations-

$D_1=10$, $D_2=12$, $D_3=8$;

Hence the Total Cost Bear by the destinations is equal to-

Total Cost = Transportation cost + Penalty Cost

= 343+20=363.

6. Conclusion

In this paper Trapezoidal fuzzy set concept has been used with Greenberg's Algorithm to evaluate the Integer solution of the parametric form of the Transportation Linear Programming problem. The model can be used by managers to optimize the transportation cost items of own company. The problem is the modified version of Osman and Saad. In the future the other properties Fuzzy sets and other parameters can be included as fuzzy sets such as demand and supply.

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