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An Approach for Solving Transportation Problem Using Modified Kruskal's Algorithm

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Abstract: ***This paper presents a new approach for finding a minimum feasible solution for transportation problem with different types (balanced and unbalanced). The approach is based mainly on using graph theory in general and Kruskal's algorithm for finding Minimum Spanning Tree(MST) in finding out the first minimum cost between sources and demands. All the edges between sources and demands are sorted in an ascending order according to the weights(costs of unit delivery between sources and demands) in an array. Starting from the first element of the array which represents the absolute minimum cost, then delete either the source vertex with all its outgoing edges if this source is satisfied or deleting the targeted demand with all its incoming edges if this demand is satisfied or even both. Different examples are considered in this paper to study the correctness and the scalability of the proposed approach. The examples cover both balanced and unbalanced transportation models. As Kruskal's algorithm works with O(E log V) time complexity, where E represents the number of edges and V is the number of vertices, however, the proposed approach tends to reduce the number of vertices and the edges after each iteration and hence it will converge faster.***

Keywords: Graph, Algorithms, Transportation Problem, Kruskal's algorithm, MST.

1. Introduction

The Transportation Problem was one of the original applications of linear programming models. The story goes like this. A firm produces goods at m different supply centers, labeled as S_1, S_2, …S_m. The demand for the good is spread out at n different demand centers labeled D_1,D_2,…D_n. The problem of the firm is to get goods from supply centers to demand centers at minimum cost[1]-[4]. Assume that the cost of shipping one unit from supply center i to demand center j is C_{ij} and that shipping cost is linear.

Suppose that there are m sources and n destinations.
Let (a_i) be the number of supply units available at source i , $(i = 1, 2, 3, \ldots, m)$.
Let (b_j) be the number of demand units required at destination j , $(j = 1, 2, 3, \ldots, n)$.
Let (c_{ij}) represents the unit transportation cost for transporting the units from source i to destination j.
Let (x_{ij}) is the number of units shipped from source i to destination j.

The objective is to determine the number of units to be transported from source i to destination j so that the total transportation cost is minimum. Figure 1 represents a typical transportation problem with m supplies and n demands [4],[6].

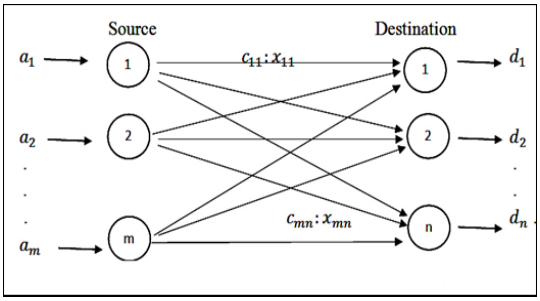

Figure 1: Network Representation of the Transportation Problem

and it can be represented by table 1.

Table 1: Sources and Demands of the Typical Transportation Problem

To / From	D_1	D_2	…	D_n	Supply
S_1	C_{11} x_{11}	C_{12} x_{12}	…	C_{1n}	A_1
S_2	C_{21} x_{21}	C_{22} x_{22}	…	C_{2n}	A_2
. . .	. . .	. . .	…	. . .	. . .
S_m	C_{m1} x_{m1}	C_{m2} x_{m2}	…	C_{mn}	A_m
B_j	B_1	B_2	…	B_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Minimize $$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \quad (1)$$

Subject to $$\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1, 2, 3, \ldots, m \quad (2)$$

and $$\sum_{i=1}^{m} x_{ij} = b_j, \; j = 1, 2, 3, \ldots, n \quad (3)$$

where $$x_{ij} \geq 0 \quad (4)$$

The two sets of constraints will be consistent i.e.; the system will be in balance if :

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad (5)$$

The resulting formulation in equations 1-5 is called a balanced transportation model. In case where total supply is different from total demand, this type of transportation model is called unbalanced. [1],[2],[5],[6].
For unbalanced $a_{ij} < b_{ij}$ or $a_{ij} > b_{ij}$

Different methods and techniques were used to solve the transportation problem which can be specified as two-phase

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solution. The first solution is used to find the minimum feasible solution, where different techniques and algorithms are used such as North-West Corner, Lowest Cost Entry Method and Vogel Approximation Method. In addition there are two more methods to find out the optimum solution, these include Modified Distribution (MODI) and Stepping Stone Method [3]-[12]. The proposed method depends on graph theory and Kruskal's algorithm for finding MST.

1.1 Kruskal's Algorithm

Kruskal's algorithm is used to find out the minimum Spanning Tree in a network model [1],[3].

```
KRUSKAL(G):
1 A = Ø
2 foreach v ∈ G.V:
3    MAKE-SET(v)
4 foreach (u, v) ordered by
   weight(u, v), increasing:
5    if FIND-SET(u) ≠ FIND-SET(v):
6       A = A U {(u, v)}
7       UNION(u, v)
8 return A
```

Where E is the number of edges in the graph and V is the number of vertices.

2. Proposed Algorithm

The proposed algorithm consists of the following steps:
Step1: Convert the problem into graph and corresponding table.
Step2:Create a sorted array in ascending order containing the unit costs as the edges between the sources and demands.
Step3:Pick the first element in the array(minimum cost) and the related source and demand vertices.
Step 4:Delete all the rows in the array that are related to the source or the demand depending on which one is satisfied first and rearrange the array.
Step 5: Repeat from step 3 until the array is empty.

2.1 Proposed Algorithm Pseudo Code

The proposed algorithm represents a modification of Kruskal's algorithm for finding Minimum Spanning Tree (MST). The Algorithm Pseudo code

```
KRUSKAL(G):
1 A = Ø
2 foreach (u, v) ordered by
    Weight(u, v), increasing:
3       A = A U {(u, v)}
4       If Vcapacity is covered
5              Delete all rows
          corresponding to V
6       Endif
7       If Ddemand is covered
8              Delete all rows
            corresponding to D
9       Endif
10     UNION(u, v)
11  return A
```

In all cases, the output of the algorithm will be either a tree or unconnected graph depending on the verification of the term (m+n-1).

3. Testing Examples

Four different examples were taken to study the correctness and effectiveness of the proposed algorithm, these include both balanced and unbalanced transportation models.

Example 1: Balanced model with number of sources is greater than the demands.

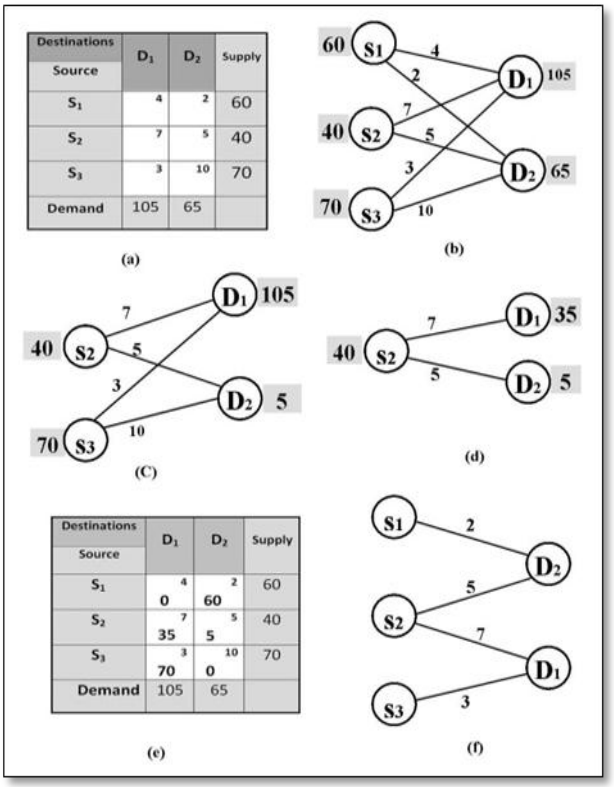

Figure 2: Graph Representation for example 1. (b)-(f) Solution Steps

And the results can be shown in Table 2.

Table 2: Table Representation of the Proposed Algorithm for Example 1

N	Edge	cost	Kruskal step1 Cost=2*60		Kruskal step2 Cost=2*60+3*70		Kruskal step3 Cost=2*60+3*70 +5*2		Kruskal step4 Cost=2*60+3*70+ 5*2+7*35	
1	S_1D_2	2	X		X		X		X	
2	S_3D_1	3	S_3D_1	3	X		X		X	
3	S_1D_1	4	X		X		X		X	
4	S_2D_2	5	S_2D_2	5	S_2D_2	5	X		X	
5	S_2D_1	7	S_2D_1	7	S_2D_1	7	S_2D_1	7	X	
6	S_3D_2	10	S_3D_2	10	X		X		X	

Example 2: A balanced transportation model with number of demands is greater than the sources.

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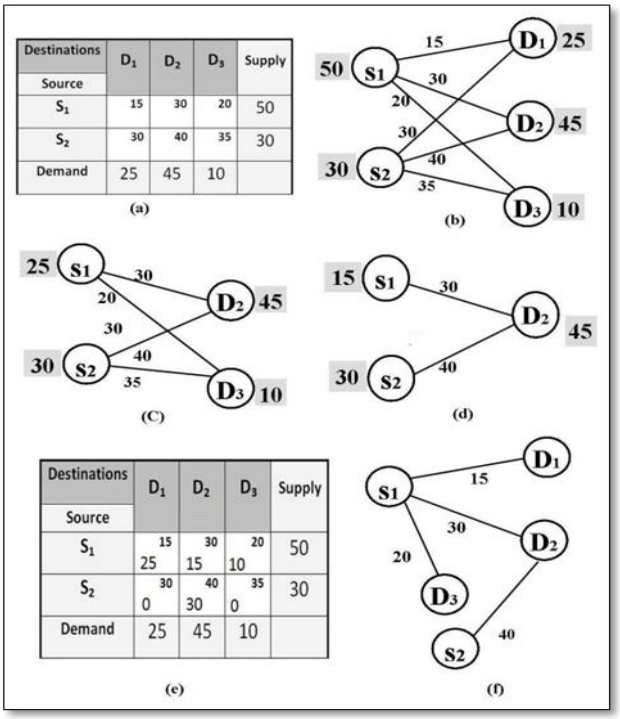

Figure 3: Graph Representation for Example 2. (b)-(f) Solution Steps

Table 3: Table Representation of the Proposed Algorithm for Example 2

N	Edge	cost	Kruskal step1 Cost = 15*25 = 375		Kruskal step2 Cost=375+20*10 = 575		Kruskal step3 Cost = 575+30*15 =1025		Kruskal step4 Cost = 1025+40*30 =2225	
1	S_1D_1	15	X		X		X		X	
2	S_1D_3	20	S_1D_3	20	X		X		X	
3	S_1D_2	30	S_1D_2	30	S_1D_2	30	X		X	
4	S_2D_1	30	X		X		X		X	
5	S_2D_3	35	S_2D_3	35	X		X		X	
6	S_2D_2	40	S_2D_2	40	S_2D_2	40	S_2D_2	40	X	

Example 3: Unbalanced model with number of sources is greater than the number of demand.

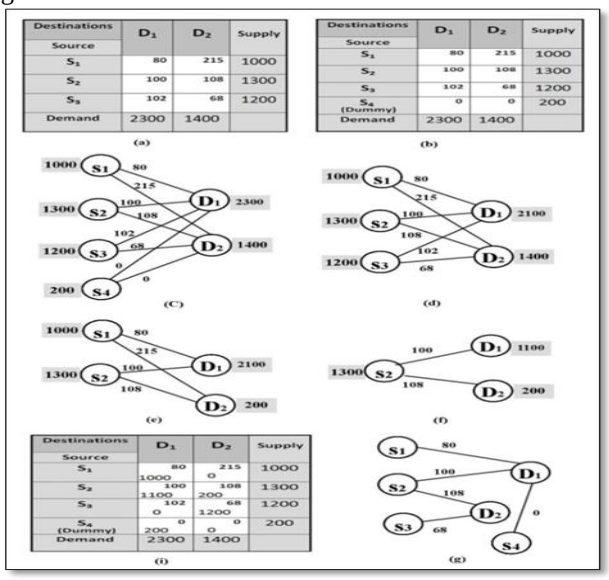

Figure 4: Graph Representation for Example 3. (b)-(g) Solution Steps

Table 4: Table Representation of the Proposed Algorithm for Example 3

N	Edge	cost	Kruskal step1 Cost = 0*200 =0		Kruskal step2 Cost=0+68*1200 =81600		Kruskal step3 Cost = 81600+80*1000 =161600		Kruskal step4 Cost = 161600+100*1100 =271600		Kruskal step5 Cost = 271600+108*200 = 293200	
1	S_4D_1	0	X		X		X		X		X	
2	S_4D_2	0	X		X		X		X		X	
3	S_3D_2	68	S_3D_2	68	X		X		X		X	
4	S_1D_1	80	S_1D_1	80	S_1D_1	80	X		X		X	
5	S_2D_1	100	S_2D_1	100	S_2D_1	100	S_2D_1	100	X		X	
6	S_3D_1	102	S_3D_1	102	X		X		X		X	
7	S_2D_2	108	S_2D_2	108	S_2D_2	108	S_2D_2	108	S_2D_2	108	X	
8	S_1D_2	215	S_1D_2	215	S_1D_2	215	X		X		X	

Example 4: Unbalanced model with number of sources is less than the number of demand.

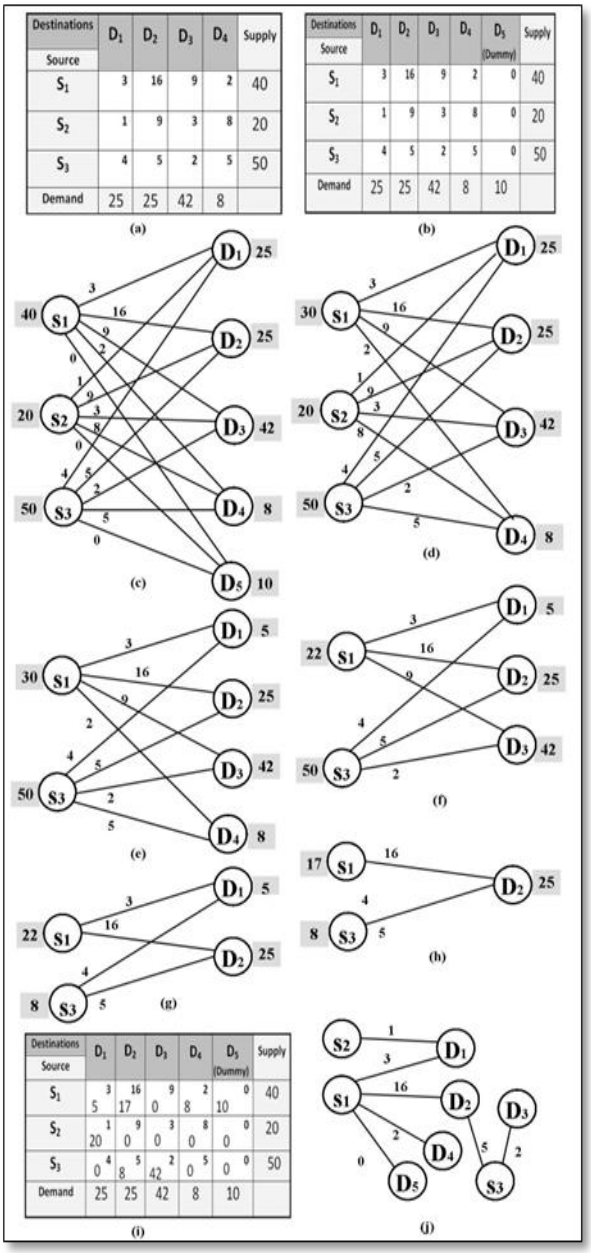

Figure 5: Graph Representation for Example 4. (b)-(j) Solution Steps

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Table 5: Table Representation of the Proposed Algorithm for Example 4.

N	Edge	cost	Kruskal step1 Cost = 0*10 = 0		Kruskal step2 Cost=0+1*20 = 20		Kruskal step3 Cost = 20+2*8 =36		Kruskal step4 Cost = 36+2*42 =120		Kruskal step5 Cost = 120+3*5 =135		Kruskal step 6 Cost = 135+5*8 =175		Kruskal step 7 Cost = 175+16*17 =447	
1	S_1D_5	0	X		X		X		X		X		X		X	
2	S_2D_5	0	X		X		X		X		X		X		X	
3	S_3D_5	0	X		X		X		X		X		X		X	
4	S_2D_1	1	S_2D_1	1	X		X		X		X		X		X	
5	S_1D_4	2	S_1D_4	2	S_1D_4	2	X		X		X		X		X	
6	S_3D_3	2	S_3D_3	2	S_3D_3	2	S_3D_3	2	X		X		X		X	
7	S_1D_1	3	S_1D_1	3	S_1D_1	3	S_1D_1	3	S_1D_1	3	X		X		X	
8	S_2D_3	3	S_2D_3	3	X		X		X		X		X		X	
9	S_3D_1	4	S_3D_1	4	S_3D_1	4	S_3D_1	4	S_3D_1	4	X		X		X	
10	S_3D_2	5	S_3D_2	5	S_3D_2	5	S_3D_2	5	S_3D_2	5	S_3D_2	5	X		X	
11	S_3D_4	5	S_3D_4	5	S_3D_4	5	X		X		X		X		X	
12	S_2D_4	8	S_2D_4	8	X		X		X		X		X		X	
13	S_1D_3	9	S_1D_3	9	S_1D_3	9	S_1D_3	9	X		X		X		X	
14	S_2D_2	9	S_2D_2	9	X		X		X		X		X		X	
15	S_1D_2	16	S_1D_2	16	S_1D_2	16	S_1D_2	16	S_1D_2	16	S_1D_2	16	S_1D_2	16	X	

4. Conclusion

The proposed algorithm for finding the minimum feasible solution for the transportation problem is based mainly on using Kruskal's algorithm for finding out the MST with some modification. Four different examples representing different transportation problem cases were taken into account to study the correctness and expandability of the proposed algorithm. From the results given in figures 2-5 and tables 2-5,it is clear that the proposed algorithm can be used in different transportation models and gives faster convergence criteria since it is based mainly on the reduction of the number of vertices and edges after each iteration since the time complexity of such algorithms is highly related to E(number of edges) and V (number of vertices)

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