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# Magnetohydrodynamic Rayleigh Problem in a Porous Plate

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***Abstract:*** ***This paper presents the study of magnetohydrodynamic flow version of the Rayleigh problem with Hall effect and Rotation in the presence of an inclined magnetic field and porous plate. Exact solution of the governing equation is obtained by Laplace transform technique for the MHD flow of incompressible, electrically conducting, viscous fluid past a uniformly accelerated and insulated infinite plate. The effects of the Hall parameter*** $N$, ***Hartmann number*** $M$, ***Angle of inclination*** $\theta$, ***Porosity parameter*** $K_p$ ***and the Rotation parameter*** $K^2$ ***on the velocity components*** $u$ ***and*** $v$ ***are shown graphically. It is concluded that the axial and transverse velocity components*** $u$ ***and*** $v$ ***increases with the increase in Hall Parameter, Hartmann number, Ekmann number,*** $y$ ***and*** $t$ ***with respect to the increase of Porosity parameter.***

**Keywords:** MHD flow, uniformly accelerated plate, Hall effect, inclined magnetic field, porous plate.

## 1. Introduction

Many engineering problems are susceptible to MHD analysis. The study of MHD flow problems has achieved remarkable interest due to its application in MHD generators, MHD pumps and MHD flow meters etc. The study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes and ionized gases .Geophysics encounters MHD phenomena in interaction on conducting fluids and magnetic fields. The rotating flow of an electrically conducting fluid in presence of magnetic field has got its importance in Geophysical problems. The study of rotating flow problems is also important in the solar physics dealing with the sunspot development, the solar cycle and the structure of rotating magnetic stars. The general theory of rotating fluids has received growing interest during last decade because of its application in cosmic and geophysical science. MHD in the present form is due to pioneer contribution of several notable authors like Alfven[1], Cowling[2]. The MHD Stoke's or Rayleigh problem was first solved by Rossow [3] without taking into account the induced magnetic field. With the induced magnetic field, it was solved by Nanda and Sundaram [4],Chang and Yen[5] and Roscizewski [6]. Steady state channel flows of ionized gases were studied by Sato [7]. The effect of Hall current on MHD Rayleigh's problem in ionized gas where studied by Mohanty [8]. Schlicting [9] has studied the unsteady flow due to an impulsive motion of an infinite plate in a fluid of an infinite extent. MHD flow past a uniformly accelerated plate under a transverse magnetic field was studied by Gupta [10]. Magnetohydrodynamic Rayleigh problem with Hall effect was studied by Haytham Sulieman [11]. Effect of Hall current and rotation on unsteady MHD couette flow in the presence of an inclined Magnetic field was studied by Seth, Nandkeolyar and Ansari [12]. Hall Effect on transient MHD flow past a vertical plate was analysed by Ahmed and Das [13]. In this study we have considered the Magnetohydrodynamic Rayleigh Problem with Hall Effect and Rotation over a porous plate in the presence of an Inclined Magnetic Field

## 2. Formulation of the Problem

Consider the flow of an incompressible electrically conducting, viscous fluid past an infinite and insulated porous flat plate occupying the plane y = 0. Initially the fluid and the plate rotate in unison with a uniform angular velocity Ω about the y - axis normal to the plane. The $x$-axis is taken in the direction of the motion of the plate and $z$ – axis lying on the plate normal to both $x$ and y – axis. Relative to the rotating fluid, the plate is impulsively started from rest and set into motion with uniform acceleration in its own plane along the $x$ - axis. A uniform magnetic field $H_0$ is applied in a direction which makes an angle $\theta$ with the positive direction of $y$ - axis in the $xy-$ plane.

Here the velocity vector $\bar{q}$=(u,0,v), magnetic induction $\bar{H}$=($H_0 \sin\theta$, $H_0 \cos\theta$, 0), Electro static field $\bar{E}$=($E_x$, 0, $E_z$), Uniform angular velocity $\mathbf{\Omega}$=(0,$\Omega_y$,0) (2.1)

Governing equations are:

$$\nabla.\bar{q}=0 \tag{2.2}$$

$$\frac{\partial \bar{q}}{\partial t} + (\bar{q}.\nabla)\bar{\mathrm{q}} + 2\mathbf{\Omega}\times \bar{\mathrm{q}}$$

$$= -\frac{1}{\rho}\nabla \mathrm{P} + \gamma\nabla^2\bar{\mathrm{q}} + \frac{1}{\rho}\bar{\mathrm{J}}\times\bar{\mathrm{H}} - \frac{\gamma}{k}\,\bar{\mathrm{q}} \tag{2.3}$$

$$\nabla \times H = \mu J \tag{2.4}$$

$$\nabla \times E = -\frac{\partial H}{\partial t} \tag{2.5}$$

$$\nabla.H = 0 \tag{2.6}$$

$$\frac{J}{\sigma} = (E + \bar{q}\times H) - \frac{J\times H}{n.e} \tag{2.7}$$

where $\sigma$ is the electrical conductivity.

Here $J$ is the current density, t is the time, ρ is density, γ is kinematic viscosity, $e$ is electric charge, $m$ is mass of an electron, $n$ is the electron number density, τ is the mean collision time , $k$ is the permeability of the fluid and μ is magnetic permeability.

The initial and boundary conditions are

$t \le 0: u = 0, v = 0$ for $y \ge 0$,


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$t > 0: u = U_0, v = 0$ for $y = 0$,

$u \to 0: v = 0$ as $y \to \infty$ (2.8)

Now introducing the non-dimensional quantities

$$y^* = \frac{U_0 . y}{\gamma}, u^* = \frac{u}{U_0}, v^* = \frac{v}{U_0}, t^* = \frac{U_0^2 t}{\gamma} \quad (2.9)$$

The equation of motion (2.3) in component term becomes

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{\sigma H_0^2 \cos^2\theta \gamma}{\rho U_0^2 (1+\omega^2 \tau^2 \cos^2\theta)} (u + \omega\tau v \cos\theta) - \frac{2\gamma}{U_0^2} v\Omega_y - \frac{\gamma^2}{kU_0^2} u \quad (2.10)$$

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial y^2} + \frac{\sigma H_0^2 \cos^2\theta \gamma}{\rho U_0^2 (1+\omega^2 \tau^2 \cos^2\theta)} (\omega\tau u \cos\theta - v) + \frac{2\gamma}{U_0^2} u\Omega_y - \frac{\gamma^2}{kU_0^2} v \quad (2.11)$$

Now let $M^2 = \frac{\sigma H_0^2 \gamma}{\rho U_0^2}$ is the Hartman number, $N = \omega\tau$ is the Hall Parameter, $K_p = \frac{\gamma^2}{kU_0^2}$ is the porosity parameter and $K^2 = \frac{\gamma \Omega_y}{U_0^2}$ is the Rotation parameter i.e., the reciprocal of Ekmann number. The initial and boundary conditions are

$u(0, y) = v(0, y) = 0;$

$u(t, 0) = 1, v(t, 0) = 0$

$u(t, y)$ and $v(t, y) \to 0$ as $y \to \infty$ (2.12)

Now multiplying both sides of equation (2.10) and (2.11) by $e^{-st}$ and integrating from 0 to $\infty$ with respect to $t$ we get

$$\frac{d^2\hat{u}}{dy^2} - \left(\frac{M^2\cos^2\theta}{1+N^2\cos^2\theta} + s + K_p\right)\hat{u} = \left(\frac{NM^2\cos^3\theta}{1+N^2\cos^2\theta} + 2K^2\right)\hat{v} \quad (2.13)$$

$$\frac{d^2\hat{v}}{dy^2} - \left(\frac{M^2\cos^2\theta}{1+N^2\cos^2\theta} + s + K_p\right)\hat{v} = -\left(\frac{NM^2\cos^3\theta}{1+N^2\cos^2\theta} + 2K^2\right)\hat{u} \quad (2.14)$$

where $\hat{u}(s, y) = L\{u(t, y)\} = \int_0^\infty u(t, y) e^{-st}\, dt$, $\hat{v}(s, y) = L\{v(t, y)\} = \int_0^\infty v(t, y) e^{-st}\, dt$

By introducing the complex function $\hat{q} = \hat{u} + i\hat{v}$, then equation (2.10) and (2.11) can be combined into the single equation

$$\frac{d^2\hat{q}}{dy^2} - \left(\frac{M^2\cos^2\theta}{1+N^2\cos^2\theta} + s + K_p\right)\hat{q} = -i\left(\frac{NM^2\cos^3\theta}{1+N^2\cos^2\theta} + 2K^2\right)\hat{q} \quad (2.15)$$

## 3. Analytical Solution

By introducing the complex function $\hat{q} = \hat{u} + i\hat{v}$, then equation (2.13) and (2.14) becomes

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} - \left[\left(\frac{M^2\cos^2\theta}{1+N^2\cos^2\theta} + K_p\right)(1 - iN\cos\theta) - 2iK^2\right] q \quad (3.1)$$

The initial and boundary conditions take the form $q(0, y) = 0, q(t, 0) = 1,$

$$q(t, y) \to 0 \text{ as } y \to \infty \quad (3.2)$$

Using the abbreviation

$$\alpha = -\left[\left(\frac{M^2\cos^2\theta}{1+N^2\cos^2\theta} + K_p\right)(1 - iN\cos\theta) - 2iK^2\right]$$

Equation (3.1) can be written as

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} + \alpha q \quad (3.3)$$

$$\text{Let } \phi(t, y) = e^{-\alpha t} q(t, y) \quad (3.4)$$

$$\text{From (3.3) we get } \frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} \quad (3.5)$$

From equations (3.2) and (3.4) we conclude that $\phi(0, y) = 0, \phi(t, 0) = e^{-\alpha t},$

$$\phi(t, y) \to 0 \text{ as } y \to \infty \quad (3.6)$$

To solve (3.5) subject to the initial and boundary conditions (3.6) we apply the Laplace transform method and obtain the solution as

$$\begin{aligned} q(t, y) = e^{at} \cos bt\ erfc\left(\frac{y}{2\sqrt{t}}\right) \\ - \int_0^t e^{a\tau}\ erfc\left(\frac{y}{2\sqrt{\tau}}\right)[a\cos b\tau \\ - b\sin b\tau] d\tau \\ + i\left[e^{at}\sin bt\ erfc\left(\frac{y}{2\sqrt{t}}\right)\right] \\ - \int_0^t e^{a\tau}\ erfc\left(\frac{y}{2\sqrt{\tau}}\right)[a\sin b\tau \\ + b\cos b\tau] d\tau \end{aligned}$$

where $\alpha = a + ib$ with $a = -\frac{M^2\cos^2\theta}{1+N^2\cos^2\theta} + K_p$, $b = \frac{NM^2\cos^3\theta}{1+N^2\cos^2\theta} + 2K^2$ and

$erfc(x) = 1 - \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2}\, du$

In order to get a clear understanding of the flow fluid we have carried out numerical calculations of equation (2.15). The boundary value problem can be stated as

$$\frac{d^2\hat{q}}{dy^2} - \omega\hat{q} = 0 \quad (3.7)$$

$\hat{q}(0, s) = \frac{1}{s}, \hat{q}(\infty, s) = 0$

Where

$$\omega = \left(\frac{M^2\cos^2\theta}{1+N^2\cos^2\theta} + s + K_p\right) - i\left(\frac{NM^2\cos^3\theta}{1+N^2\cos^2\theta} + 2K^2\right) \quad (3.8)$$

To ensure that the Laplace transforms are well-defined, it should be assumed that $s > 0$. This implies

$Re(\omega) = \frac{M^2\cos^2\theta}{1+N^2\cos^2\theta} + s > 0$. Hence there exists $\eta$ in the complex number such that $\eta^2 = \omega$ with $Re(\eta) < 0$. Furthermore

$$\hat{q}(y, s) = \frac{e^{\eta y}}{s} \quad (3.9)$$

satisfy the boundary value problem (3.7) and (3.8). For $y = 0$ we have

$$\hat{q}(0, s) = \frac{1}{s} = \int_0^\infty 1.e^{-st}\, dt$$

$$= \int_0^\infty (1 + 0i).e^{-st}\, dt$$

Thus $u(0, t) \equiv 1$ and $v(0, t) \equiv 0$ for all $t$.

Recall that the inverse Laplace transform is

$q(y, t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \hat{q}(y, s) e^{st} ds$

Where $\gamma > 0$ is chosen so that all the singularities of $\hat{q}(y, s)$ are to the left of $\gamma$. The above integral is over the vertical line z=$\gamma$ in the complex plane. Since $\hat{q}(y, s) = \frac{e^{\eta y}}{s}$, we can choose $\gamma$ to be any positive number. In the calculations below we choose $\gamma$=0.25.

We will define $q$ strictly as a function of $t$ using Mathematic's NIntegrate command. We will approximate the integral above by integrating from 0.25 – 500i to 0.25 + 500i.

The effect of the Hall parameter $N$, the Hartmann number $M$, the angle of inclination $\theta$, the porosity parameter $K_p$ and


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the rotation parameter $K^2$ in the velocity components $u$ and $v$ are illustrated in the following figures

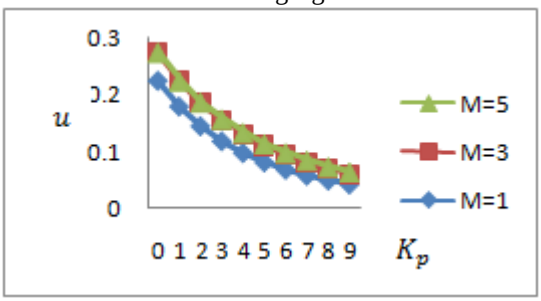



**Figure 1:** Effect of Hartmann number in $u$ with $K_p$ variation and with N=1, y=1, t=0.5, θ =30, $K^2$=1

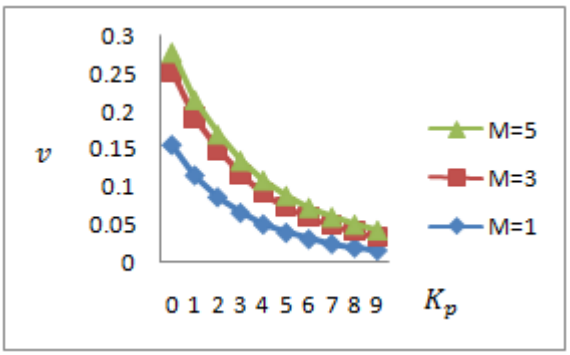



**Figure 2:** Effect of Hartmann number in $v$ with $K_p$ variation and with N=1, y=1, t=0.5, θ =30, $K^2$=1

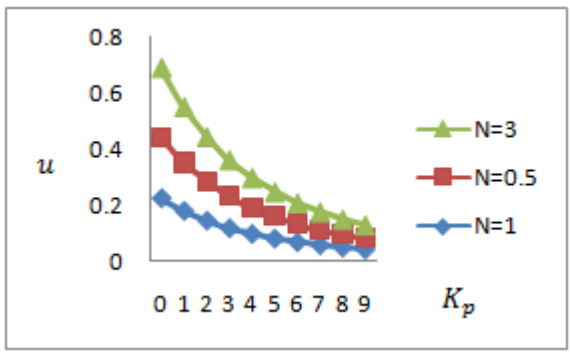



**Figure 3:** Effect of Hall Parameter in $u$ with $K_p$ variation and with M=1, y=1, t=0.5, θ =30, $K^2$=1

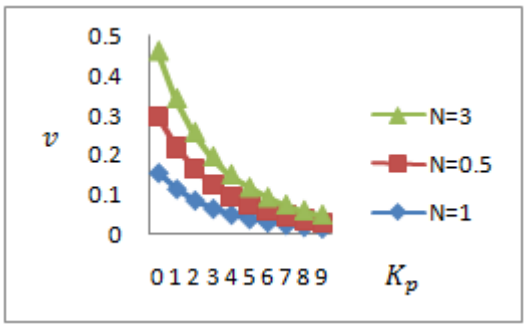



**Figure 4:** Effect of Hall Parameter in $v$ with $K_p$ variation and with M=1, y=1, t=0.5, θ =30, $K^2$=1

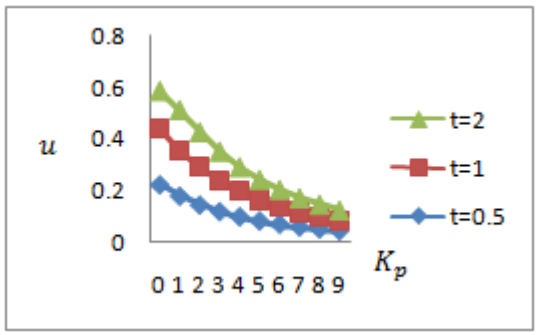



**Figure 5:** Effect of t in $u$ with $K_p$ variation and with M=1, y=1, N=1, θ =30, $K^2$=1

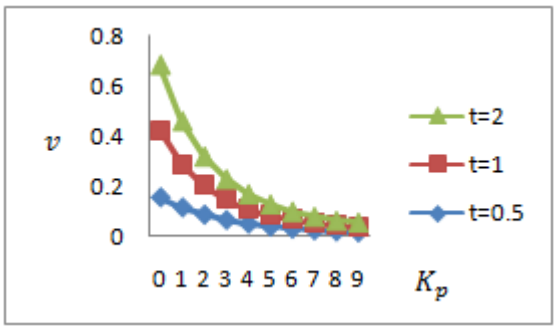



**Figure 6:** Effect of t in $v$ with $K_p$ variation and with M=1, y=1, N=1, θ =30, $K^2$=1

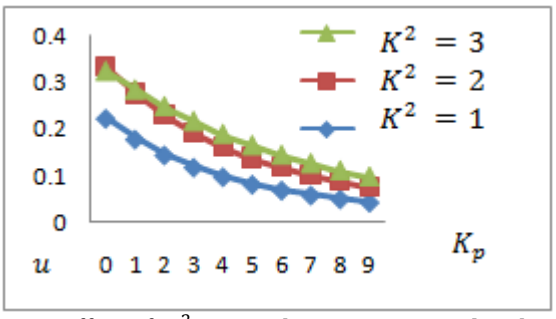



**Figure 7:** Effect of $K^2$ in $u$ with $K_p$ variation and with M=1, y=1, N=1, θ =30, t=0.5

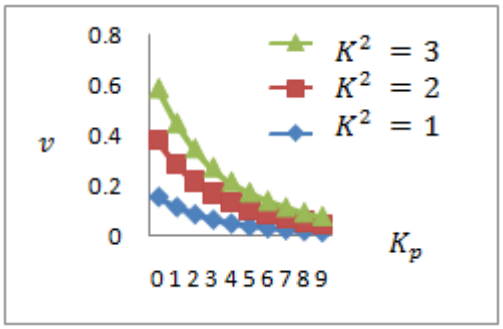



**Figure 8:** Effect of $K^2$ in $v$ with $K_p$ variation and with M=1, y=1, N=1, θ =30, t=0.5


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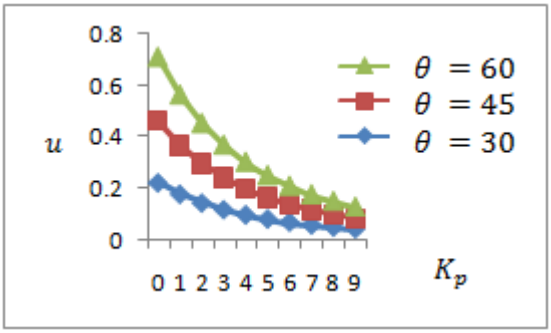



**Figure 9:** Effect of $\theta$ in $u$ with $K_p$ variation and with M=1, y=1, N=1, $K^2$=1, t=0.5

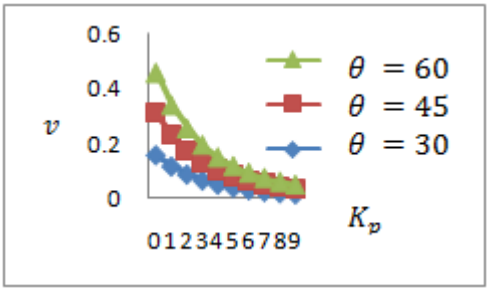



**Figure 10:** Effect of $\theta$ in $v$ with $K_p$ variation and with M=1, y=1, N=1, $K^2$=1, t=0.5

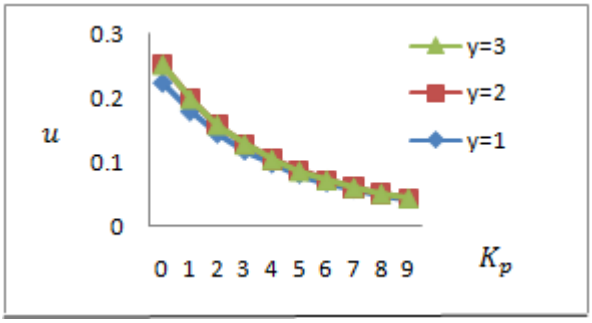



**Figure 11:** Effect of $y$ in $u$ with $K_p$ variation and with M=1, $\theta$=30, N=1, $K^2$=1, t=0.5

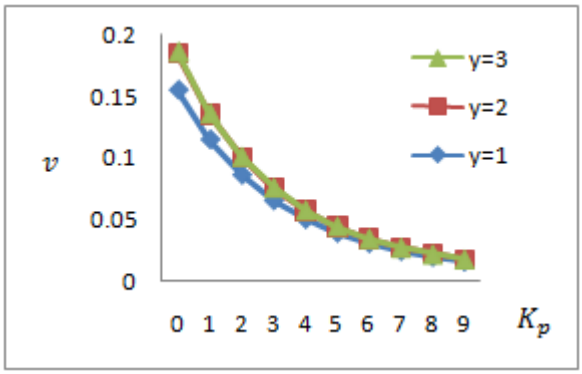



**Figure 12:** Effect of $y$ in $v$ with $K_p$ variation and with M=1, $\theta$=30, N=1, $K^2$=1, t=0.5

## 4. Conclusion

From the above figures 1 to 12, we conclude that the axial and transverse velocity components $u$ and $v$ increases with the increase in Hall Parameter, Hartmann number, Ekmann number, y and $t$ with respect to the increase of Porosity parameter.

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