

Intuitionistic Fuzzy Regular Generalized Semi Continuous Mappings

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Abstract: In this paper we introduce the concept intuitionistic fuzzy regular generalized semi continuous mappings and investigate some of their properties.

Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy topology, Intuitionistic fuzzy topological space, Intuitionistic fuzzy regular generalized semi closed set, Intuitionistic fuzzy regular generalized semi continuous mappings.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [13] in 1965. Later, Chang [4] proposed fuzzy topology in 1967. Atanassov [3] introduced the idea of intuitionistic fuzzy sets using the notion of fuzzy sets. In 1997 Coker [5] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy regular semi continuous mappings and obtain some theorems.

2. Preliminaries

Definition 2.1:[3] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the function $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [3] Let A and B be two IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

- $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
- $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
- $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. The IFS $0 \sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1 \sim = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition 2.3: [5] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFS in X satisfying the following axioms:

- $0 \sim, 1 \sim \in \tau$
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X . The complement A^c of an IFOS A in (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X .

Definition 2.4: [5] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$$

$$\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$$

Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$

Definition 2.5: [8] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

- intuitionistic fuzzy semi closed set (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$
- intuitionistic fuzzy α closed set (IF α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$
- intuitionistic fuzzy regular closed set (IFRCS in short) if $A = \text{cl}(\text{int}(A))$
- intuitionistic fuzzy pre closed set (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$

The respective complements of the above IFCSs are the respective IFOSs.

Definition 2.6: [1] An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy regular generalized semiclosed set (IFRGSCS in short) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X .

Definition 2.7: [2] An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy regular generalized semiopen set (IFRGSOS in short) if $\text{sint}(A) \supseteq U$ whenever $A \supseteq U$ and U is an IFRCS in X .

Note that the complement A^c of an IFRGSCS A in an IFTS (X, τ) is an IFRGSOS in (X, τ) .

Corollary 2.8: [5] Let $A, A_i (i \in J)$ be intuitionistic fuzzy sets in X and $B, B_j (j \in K)$ be intuitionistic fuzzy sets in Y and $f : X \rightarrow Y$ be a function. Then

- a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$
- b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
- c) $A \subseteq f^{-1}(f(A))$ [If f is injective , then $A=f^{-1}(f(A))$]
- d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective , then $B=f(f^{-1}(B))$]
- e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
- f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
- g) $f^{-1}(0\sim) = 0\sim$
- h) $f^{-1}(1\sim) = 1\sim$
- i) $f^{-1}(B^c) = (f^{-1}(B))^c$

Definition 2.9: [12] Two IFSs A and B are said to be q -coincident ($A \text{ }_q \text{ } B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.10: [12] Two IFSs A and B are said to be not q -coincident ($A \text{ }^c \text{ } B$ in short) if and only if $A \subseteq B^c$.

Definition 2.11: [6] An intuitionistic fuzzy point (IFP in short), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An IFP $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.12: [11] Let $p_{(\alpha, \beta)}$ be an IFP of an IFTS (X, τ) . An IFS A of X is called an intuitionistic fuzzy neighbourhood of $p_{(\alpha, \beta)}$ if there exists an IFOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

Definition 2.13: [7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy continuous (IF continuous in short) mapping if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$.

Definition 2.14: [8] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an

- (i) intuitionistic fuzzy semi continuous (IFS continuous in short) mapping if $f^{-1}(B) \in \text{IFSO}(X)$ for every $B \in \sigma$
- (ii) intuitionistic fuzzy α continuous (IF α continuous in short) mapping if $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$ for every $B \in \sigma$
- (iii) intuitionistic fuzzy pre continuous (IFP continuous in short) mapping if $f^{-1}(B) \in \text{IFPO}(X)$ for every $B \in \sigma$

Definition 2.15: [2] If every IFRGSCS in (X, τ) is an IFSCS in (X, τ) , then the space can be called as an intuitionistic fuzzy regular semi $T_{1/2}$ space (IF $_{rs}T_{1/2}$ in short).

Definition 2.16: [2] An IFTS (X, τ) is said to be an intuitionistic fuzzy regular semi $T_{1/2}^*$ space (IF $_{rs}T_{1/2}^*$ space in short) if every IFRGSCS is an IFCS in (X, τ) .

Definition 2.17: [2] An IFTS (X, τ) is said to be an intuitionistic fuzzy regular generalized semi $T_{1/2}$ (IF $_{rgs}T_{1/2}$ in short) space if every IFRGSCS in X is an IFGSCS in X .

Definition 2.18: [6] An IFTS (X, τ) is said to be an intuitionistic fuzzy $T_{1/2}$ (IFT $_{1/2}$ in short) space if every intuitionistic fuzzy generalized closed set (IFGCS in short) in X is an IFCS in X .

3. Intuitionistic Fuzzy Regular Generalized Semi Continuous Mappings

In this section we introduce the notion of intuitionistic fuzzy regular generalized semi continuous mappings and study some of their properties.

Definition 3.1: A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy regular generalized semi continuous (IFRGS continuous in short) mapping if $f^{-1}(B)$ is an IFRGSCS in (X, τ) for every IFCS B of (Y, σ) .

Example 3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.6), (0.3, 0.4) \rangle$ where $\mu_a=0.5$, $\mu_b=0.6$, $\nu_a=0.3$, $\nu_b=0.4$ and $G_2 = \langle x, (0.2, 0.3), (0.7, 0.6) \rangle$ where $\mu_a=0.2$, $\mu_b=0.3$, $\nu_a=0.7$, $\nu_b=0.6$ and $G_3 = \langle y, (0.6, 0.6), (0.3, 0.4) \rangle$ where $\mu_u=0.6$, $\mu_v=0.6$, $\nu_u=0.3$, $\nu_v=0.4$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.3, 0.4), (0.6, 0.6) \rangle$ is an IFCS in Y . Then $f^{-1}(G_3^c) = \langle x, (0.3, 0.4), (0.6, 0.6) \rangle$ where $\mu_a=0.3$, $\mu_b=0.4$, $\nu_a=0.6$, $\nu_b=0.6$ is an IFS in X . Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X . Now $\text{scl}(f^{-1}(G_3^c)) = (f^{-1}(G_3^c)) \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X .

Theorem 3.3: Every IF continuous mapping is an IFRGS continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF continuous mapping. Let A be an IFCS in Y . Since f is IF continuous mapping, $f^{-1}(A)$ is an IFCS in X . Since every IFCS is an IFRGSCS [1], $f^{-1}(A)$ is an IFRGSCS in X . Hence f is an IFRGS continuous mapping.

Example 3.4: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.3), (0.4, 0.2) \rangle$, $G_2 = \langle x, (0.1, 0.2), (0.9, 0.8) \rangle$ and $G_3 = \langle y, (0.5, 0.4), (0.4, 0.2) \rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.4, 0.2), (0.5, 0.4) \rangle$ is an IFCS in Y . Then $f^{-1}(G_3^c) = \langle x, (0.4, 0.2), (0.5, 0.4) \rangle$ is an IFS in X . Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X . Now $\text{scl}(f^{-1}(G_3^c)) = (f^{-1}(G_3^c)) \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an IFCS in X , since $\text{cl}(f^{-1}(G_3^c)) = G_1^c \neq f^{-1}(G_3^c)$. Therefore f is an IFRGS continuous mapping but not an IF continuous mapping.

Theorem 3.5: Every IFS continuous mapping is an IFRGS continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFS continuous mapping. Let A be an IFCS in Y . Then by hypothesis $f^{-1}(A)$ is an IFSCS in X . Since every IFSCS is an IFRGSCS [1], $f^{-1}(A)$ is an IFRGSCS in X . Hence f is an IFRGS continuous mapping.

Example 3.6: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.2, 0.5), (0.7, 0.4) \rangle$, $G_2 = \langle x, (0.2, 0.4), (0.7, 0.6) \rangle$ and $G_3 = \langle y, (0.8, 0.6), (0.2, 0.4) \rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.2, 0.4), (0.8, 0.6) \rangle$ is an IFCS in Y . Then $f^{-1}(G_3^c) = \langle x, (0.2, 0.4), (0.8, 0.6) \rangle$ is an IFS in X . Then $f^{-1}(G_3^c) \subseteq G_1$ where

G_1 is an IFROS in X . Now $\text{scl}(f^{-1}(G_3^c)) = G_2 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an IFSCS in X , since $\text{int}(\text{cl}(f^{-1}(G_3^c))) = G_2 \not\subseteq f^{-1}(G_3^c)$. Therefore f is an IFRGS continuous mapping but not an IFS continuous mapping.

Theorem 3.7: Every IFG continuous mapping is an IFRGS continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFS continuous mapping. Let A be an IFCS in Y . Then by hypothesis $f^{-1}(A)$ is an IFGCS in X . Since every IFGCS is an IFRGSCS [1], $f^{-1}(A)$ is an IFRGSCS in X . Hence f is an IFRGS continuous mapping.

Example 3.8: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.4, 0.5), (0.6, 0.5) \rangle$, $G_2 = \langle x, (0.2, 0.5), (0.8, 0.5) \rangle$ and $G_3 = \langle y, (0.7, 0.8), (0.2, 0.2) \rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.2, 0.2), (0.7, 0.8) \rangle$ is an IFCS in Y . Then $f^{-1}(G_3^c) = \langle x, (0.2, 0.2), (0.7, 0.8) \rangle$ is an IFS in X . Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X . Now $\text{scl}(f^{-1}(G_3^c)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an IFGCS in X , since $\text{cl}(\text{int}(\text{cl}(f^{-1}(G_3^c)))) = G_1^c \not\subseteq f^{-1}(G_3^c)$. Therefore f is an IFRGS continuous mapping but not an IFG continuous mapping.

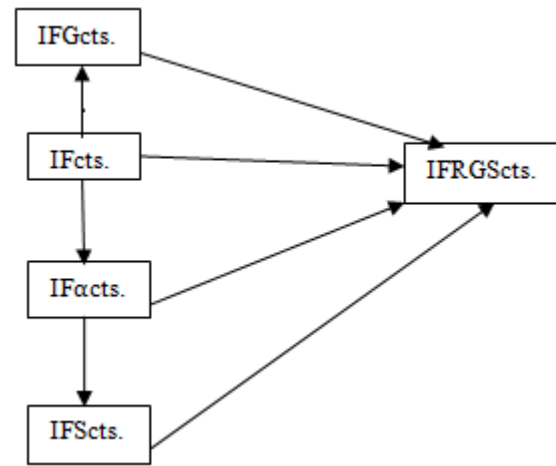
Theorem 3.9: Every $\text{IF}\alpha$ continuous mapping is an IFRGS continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an $\text{IF}\alpha$ continuous mapping. Let A be an IFCS in Y . Then by hypothesis $f^{-1}(A)$ is an $\text{IF}\alpha$ CS in X . Since every $\text{IF}\alpha$ CS is an IFRGSCS [1], $f^{-1}(A)$ is an IFRGSCS in X . Hence f is an IFRGS continuous mapping.

Example 3.10: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.4, 0.5), (0.6, 0.5) \rangle$, $G_2 = \langle x, (0.3, 0.4), (0.7, 0.6) \rangle$ and $G_3 = \langle y, (0.6, 0.6), (0.4, 0.3) \rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.4, 0.3), (0.6, 0.6) \rangle$ is an IFCS in Y . Then $f^{-1}(G_3^c) = \langle x, (0.4, 0.3), (0.6, 0.6) \rangle$ is an IFS in X . Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X . Now $\text{scl}(f^{-1}(G_3^c)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an $\text{IF}\alpha$ CS in X , since $\text{cl}(\text{int}(\text{cl}(f^{-1}(G_3^c)))) = G_1^c \not\subseteq f^{-1}(G_3^c)$. Therefore f is an IFRGS continuous mapping but not an $\text{IF}\alpha$ continuous mapping.

The relation between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram 'cts' means continuous.

In the above diagram the reverse implications are not true in general.



Theorem 3.11: A mapping $f : X \rightarrow Y$ is an IFRGS continuous mapping if and only if the inverse image of each IFOS in Y is an IFRGSOS in X .

Proof: Necessity: Let A be an IFOS in Y . This implies A^c is an IFCS in Y . Since f is an IFRGS continuous mapping, $f^{-1}(A^c)$ is an IFRGSCS in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRGSOS in X .

Sufficiency: Let A be an IFCS in Y . This implies A^c is an IFOS in Y . By hypothesis, $f^{-1}(A^c)$ is an IFRGSOS in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, where $(f^{-1}(A))^c$ is an IFRGSOS in X . This implies $f^{-1}(A)$ is an IFRGSCS in X . Hence f is an IFRGS continuous mapping.

Theorem 3.12: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping and let $f^{-1}(A)$ be an IFRCS in X for every IFCS A in Y . Then f is an IFRGS continuous mapping.

Proof: Let A be an IFCS in Y . Then $f^{-1}(A)$ is an IFRCS in X , by hypothesis. Since every IFRCS is an IFRGSCS [1], $f^{-1}(A)$ is an IFRGSCS in X . Hence f is an IFRGS continuous mapping.

Theorem 3.13: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFRGS continuous mapping, then

- (i) f is an IF continuous mapping if X is an $\text{IF}_{rs}T_{1/2}^*$ space
- (ii) f is an IFGS continuous mapping if X is an $\text{IF}_{rgs}T_{1/2}$ space
- (iii) f is an IFS continuous mapping if X is an $\text{IF}_{rs}T_{1/2}$ space

Proof: (i) Let A be an IFCS in Y . Then $f^{-1}(A)$ is an IFRGSCS in X , by hypothesis. Since X is an $\text{IF}_{rs}T_{1/2}^*$ space, $f^{-1}(A)$ is an IFCS in X . Hence f is an IF continuous mapping.

(ii) Let A be an IFCS in Y . Then $f^{-1}(A)$ is an IFRGSCS in X , by hypothesis. Since X is an $\text{IF}_{rgs}T_{1/2}$ space, $f^{-1}(A)$ is an IFGSCS in X . Hence f is an IFGS continuous mapping.

(iii) Let A be an IFCS in Y . Then $f^{-1}(A)$ is an IFRGSCS in X , by hypothesis. Since X is an $\text{IF}_{rs}T_{1/2}$ space, $f^{-1}(A)$ is an IFSCS in X . Hence f is an IFS continuous mapping.

Theorem 3.14: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFRGS continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be an IF continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is an IFRGS continuous mapping.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IFCS in Y , by hypothesis. Since f is an IFRGS continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFRGSCS in X . Hence $g \circ f$ is an IFRGS continuous mapping.

Theorem 3.15: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFRGS continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ is an IFG continuous mapping and Y is an $IFT_{1/2}$ space, then $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is an IFRGS continuous mapping.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IFGCS in Y , by hypothesis. Since Y is an $IFT_{1/2}$ space, $g^{-1}(A)$ is an IFCS in Y . Therefore $f^{-1}(g^{-1}(A))$ is an IFRGSCS in X , by hypothesis. Hence $g \circ f$ is an IFRGS continuous mapping.

Theorem 3.16: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFRGS continuous mapping, then for each IFP $p_{(\alpha, \beta)}$ of X and each $A \in \sigma$ such that $f(p_{(\alpha, \beta)}) \in A$, there exists an IFRGSOS B of X such that $p_{(\alpha, \beta)} \in B$ and $f(B) \subseteq A$.

Proof: Let $p_{(\alpha, \beta)}$ be an IFP of X and $A \in \sigma$ such that $f(p_{(\alpha, \beta)}) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis B is an IFRGSOS in X such that $p_{(\alpha, \beta)} \in B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Theorem 3.17: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFRGS continuous mapping, then for each IFP $p_{(\alpha, \beta)}$ of X and each $A \in \sigma$ such that $f(p_{(\alpha, \beta)}) \in A$, there exists an IFRGSOS B of X such that $p_{(\alpha, \beta)} \in B$ and $f(B) \subseteq A$.

Proof: Let $p_{(\alpha, \beta)}$ be an IFP of X and $A \in \sigma$ such that $f(p_{(\alpha, \beta)}) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis B is an IFRGSOS in X such that $p_{(\alpha, \beta)} \in B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Theorem 3.18: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if X and Y are $IF_{rs}T_{1/2}$ spaces:

- (i) f is an IFRGS continuous mapping
- (ii) $f^{-1}(B)$ is an IFRGSOS in X for each IFOS B in Y
- (iii) for every IFP $p_{(\alpha, \beta)}$ in X and for every IFOS B in Y such that $f(p_{(\alpha, \beta)}) \in B$, there exists an IFRGSOS A in X such that $p_{(\alpha, \beta)} \in A$ and $f(A) \subseteq B$.

Proof: (i) $\Rightarrow$ (ii) is obvious from the Theorem 3.11.
(ii) $\Rightarrow$ (iii) Let B be any IFOS in Y and let $p_{(\alpha, \beta)} \in X$. Given $f(p_{(\alpha, \beta)}) \in B$. By hypothesis $f^{-1}(B)$ is an IFRGSOS in X . Take $A = f^{-1}(B)$. Now $p_{(\alpha, \beta)} \in f^{-1}(f(p_{(\alpha, \beta)}))$. Therefore $f^{-1}(f(p_{(\alpha, \beta)})) \subseteq f^{-1}(B) = A$. This implies $p_{(\alpha, \beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.
(iii) $\Rightarrow$ (i) Let A be an IFCS in Y . Then its complement, $A^c = B$ (say) is an IFOS in Y . Let $p_{(\alpha, \beta)} \in X$ and $f(p_{(\alpha, \beta)}) \in B$. Then there exists an IFRGSOS, say $C = f^{-1}(B)$ in X such that $p_{(\alpha, \beta)} \in C$ and $f(C) \subseteq B$. Therefore $f^{-1}(B)$ is an IFRGSOS in X . That is $f^{-1}(A^c)$ is an IFRGSOS in X and hence $f^{-1}(A)$ is an IFRGSCS in X as $f^{-1}(A^c) = (f^{-1}(A))^c$. Thus f is an IFRGS continuous mapping.

Theorem 3.19: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if X is an $IF_{rs}T_{1/2}^*$ space:

- (i) f is an IFRGS continuous mapping

- (ii) for each IFP $p_{(\alpha, \beta)}$ in X and for every IFN A of $f(p_{(\alpha, \beta)})$, there exists an IFRGSOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq f^{-1}(A)$
- (iii) for each IFP $p_{(\alpha, \beta)}$ in X and for every IFN A of $f(p_{(\alpha, \beta)})$, there exists an IFRGSOS B in X such that $p_{(\alpha, \beta)} \in B$ and $f(B) \subseteq A$

Proof: (i) $\Rightarrow$ (ii) Let $p_{(\alpha, \beta)} \in X$ and let A be an IFN of $f(p_{(\alpha, \beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha, \beta)}) \in C \subseteq A$. Since f is an IFRGS continuous mapping, $f^{-1}(C) = B$ (say), is an IFRGSOS in X and $p_{(\alpha, \beta)} \in f^{-1}(f(p_{(\alpha, \beta)})) \in f^{-1}(C) \subseteq f^{-1}(A)$. Therefore $p_{(\alpha, \beta)} \in B \subseteq f^{-1}(A)$.

(ii) $\Rightarrow$ (iii) Let $p_{(\alpha, \beta)} \in X$ and let A be an IFN of $f(p_{(\alpha, \beta)})$. Then there exists an IFRGSOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq f^{-1}(A)$, by hypothesis. Therefore $p_{(\alpha, \beta)} \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(iii) $\Rightarrow$ (i) Let B be an IFOS in Y and let $p_{(\alpha, \beta)} \in f^{-1}(B)$. Then $f(p_{(\alpha, \beta)}) \in B$. Therefore B is an IFN of $f(p_{(\alpha, \beta)})$. Then by hypothesis there exists an IFRGSOS A in X such that $p_{(\alpha, \beta)} \in A$ and $f(A) \subseteq B \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(B)$ is an IFRGSOS in X [2]. Hence f is an IFRGS continuous mapping.

Theorem 3.20: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if X is an $IF_{rs}T_{1/2}$ spaces:

- (i) f is an IFRGS continuous mapping
- (ii) $f^{-1}(B)$ is an IFRGSOS in X for every IFOS B in Y
- (iii) $f^{-1}(\text{int}(B)) \subseteq \text{cl}(\text{int}(f^{-1}(B)))$ for every IFS B in Y

Proof: (i) $\Rightarrow$ (ii) is obviously true by Theorem 3.11
(ii) $\Rightarrow$ (iii) Let B be any IFS in Y . Then $\text{int}(B)$ is an IFOS in Y . Then $f^{-1}(\text{int}(B))$ is an IFRGSOS in X . Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(\text{int}(B))$ is an IFSOS in X . Then $f^{-1}(\text{int}(B)) \subseteq \text{cl}(\text{int}(f^{-1}(\text{int}(B)))) \subseteq \text{cl}(\text{int}(f^{-1}(B)))$ as $\text{int}(B) \subseteq B$.
(iii) $\Rightarrow$ (i) Let B be an IFOS in Y . Then $\text{int}(B) = B$. By hypothesis $f^{-1}(B) \subseteq \text{cl}(\text{int}(f^{-1}(B)))$. This implies $f^{-1}(B)$ is an IFSOS in X . Therefore it is an IFRGSOS in X [2] and hence f is an IFRGS continuous mapping.

Theorem 3.21: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if X is an $IF_{rs}T_{1/2}$ spaces:

- (i) f is an IFRGS continuous mapping
- (ii) $f^{-1}(B)$ is an IFRGSCS in X for every IFCS B in Y
- (iii) $\text{int}(\text{cl}(f^{-1}(A))) \subseteq f^{-1}(\text{cl}(A))$ for every IFS A in Y

Proof: (i) $\Rightarrow$ (ii) is obvious from the Definition 3.1
(ii) $\Rightarrow$ (iii) Let A be an IFS in Y . Then $\text{cl}(A)$ is an IFCS in Y . By hypothesis, $f^{-1}(\text{cl}(A))$ is an IFRGSCS in X . Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(\text{cl}(A))$ is an IFSCS. Then $\text{int}(\text{cl}(f^{-1}(A))) \subseteq \text{int}(\text{cl}(f^{-1}(\text{cl}(A)))) \subseteq f^{-1}(\text{cl}(A))$. Thus $\text{int}(\text{cl}(f^{-1}(A))) \subseteq f^{-1}(\text{cl}(A))$.
(iii) $\Rightarrow$ (i) Let A be an IFCS in Y . By hypothesis $\text{int}(\text{cl}(f^{-1}(A))) \subseteq f^{-1}(\text{cl}(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFSCS in X and hence it is an IFRGSCS [1]. Thus f is an IFRGS continuous mapping.

Definition 3.22: Let (X, τ) be an IFTS. The regular generalized semi closure ($\text{rgscl}(A)$ in short) for any IFS A is defined as follows.

$\text{rgscl}(A) = \cap \{K / K \text{ is an IFRGSCS in } X \text{ and } A \subseteq K\}$
If A is an IFRGSCS, then $\text{rgscl}(A) = A$.

Theorem 3.25: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFRGS continuous mapping. Then the following conditions hold:

- (i) $f(\text{rgscl}(A)) \subseteq \text{cl}(f(A))$, for every IFS A in X .
- (ii) $\text{rgscl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$, for every IFS B in Y .

Proof: (i) Let A be any IFS A in Y . Since $\text{cl}(f(A))$ is an IFCS in Y and f is an IFRGS continuous mapping, then $f^{-1}(\text{cl}(f(A)))$ is an IFRGSCS in X . That is $\text{rgscl}(A) \subseteq f^{-1}(\text{cl}(f(A)))$. Therefore $f(\text{rgscl}(A)) \subseteq f(f^{-1}(\text{cl}(f(A)))) \subseteq \text{cl}(f(A))$, for every IFS A in X .

(ii) Replacing A by $f^{-1}(B)$ in (i), we get $f(\text{rgscl}(f^{-1}(B))) \subseteq \text{cl}(f(f^{-1}(B))) \subseteq \text{cl}(B)$. Hence $\text{rgscl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$, for every IFS B in Y .

Conclusion

Thus we have analyzed intuitionistic fuzzy regular generalized semi continuous mapping and obtained some interesting theorems.

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