

# Formal Deductive Proof in Geometry and its Implication in Teaching Process

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**Abstract:** *Mathematical researchers and cognitive psychology focused on several principal reasons based on difficulties of student in deductive proof solving geometry problem. Literatures about geometry problems widely reveal two reasons knowing the difficulties. First, a larger number don't possess the required content knowledge. Second, properties of deductive proof different from most of others mathematical problems. Since, this problem is non-algorithmic, content knowledge is very important, but it is not enough. Thus, almost all problems were not understood by students, therefore they required greater instructional support during the solving process.*

**Keywords:** Geometry, Multiple Regression Analysis, Non Algorithmic.

## 1. Introduction

In 1970s and 1980s there was an intensive discussion among the mathematical teachers in United State of America to discuss whether the mathematical proof must be included as a part of higher school curriculum. Mathematical teachers assume that mathematical proof in school had developed in a subject focus to the formal aspects, but get a less attention on the mathematical comprehension. (Hanna 1983). This opinion is developed up to National Council of Teachers of Mathematics, NCTM (1989) issue a statement that (1) deductive proof did not taught in school because heuristic method is useful for student in developing of reasoning skill and justification rather than proof deductively, and (2) teaching of proof in school only to the student who will continue their study to the college.

The last opinion about the necessity of mathematic poof introduced in school is recommended by NCTM (2000) in which proof is a part of mathematic curriculum in all of level. Section of "Reasoning and proof" in NCTM's document stated that student must:

- To know to reasoning and proof as mathematical fundamental aspects.
- To make conjecture and check the trueness of the conjecture.
- To develop and evaluate the argument and mathematical proof.
- To choose and use various reasoning and proofing method.

Recommendation of NCTM (2000) indicates that mathematical proof is one of considered factor in mathematical teaching in school. A few or more the experiences of student in arrange a proof in higher school will influence the proof capability when they study in first year in college as said by Moore (1994) that one of reason why the college student get difficulties in proof is their limited experiences in construct a proof in geometry proof in school. Therefore, based on a study of Sabri (2003) on a mathematical proof concept of the college student as teacher candidate it is suggested that the higher school curriculum must prepare the student In mathematical proof teaching.

This indicates that mathematical proof concept of junior high school student is very poor.

In addition, the reasoning aspects in mathematical proof are not developed optimally, so the increasing of mathematical thinking capability of senior high school is late. The reasoning aspect is conjecture aspect (to make assumption about the main idea in proof), analysis aspect (to analyze the available facts), connection aspect (to make a correlation between facts and required conclusion), synthesis aspect (to make a synthesis by manipulation of facts to achieve a conclusion), even evaluation (to evaluate the rule of conclusion drawing based on the available facts and critical proof strategy) and mathematical communication aspects (to express idea and proof process verbally or non-verbally).

The difficultness of senior high school student in proof has influence to the topic of mathematical teaching with complexes proof topic such as geometry, etc. geometry is special subject that help the development of deductive that proof formally in general educational level. Problem solving in formal deductive proof is assumed as poof of achievement of mathematical competency. Therefore, the formal deductive proof will be fair indicator/measurement instrument in which the student of junioOr high school must have reasoning competency is not only in geometry but also to the other subject in curriculum of junior high school.

Nowadays, it is a few of research on the development of formal deductive proof problem to the student of junior high school. In this level, the student learns especially how to solve any geometrical problems through deductive proof. This will increase the thinking logic of student really rather than the other lessons on the level. Formal deductive proof in curriculum will familiarize the student to solve problem systematically.

But in fact, according to the early observation on how the capability of higher school student in solve geometrical problem through formal deductive proof it concluded that student in this observation is in junior high school indicates a poor performance in geometrical problem solving by formal deductive proof. This is suitable to result obtained by a National Examinations and Testing Service (NETS) indicates that the student has a lower interest in solve the

geometrical problem by formal deductive proof and only a few of sample achieve a satisfied score (NETS, 2003 : 14).

Based on aforementioned description, a researcher will take effort to develop a teaching model in handle the difficultness of junior high school student and to increase their thinking way in order to increase their competency that developed optimally and increase the interest of student in using formal deductive proof for geometrical problem solving so the formal deductive proof will be effective in teaching process to help student solve the geometrical problem.

Formal deductive proof is one of teaching approach that enable the growth and development of proof capability in geometric subject. This approach is one of teaching approach that begin by the problem serve to the student of junior high school and they required to elaborate each information and available fact. Through this approach, the problem must introduce the student of junior high school in understand the mathematical objects and a correlation between the objects of mathematic.

The teacher motivate the junior high school student to do the transactive reasoning such as criticize, describe, clarify, justify and elaborate a suggested idea, either initiated by student or teacher. To enable participate in transactive discussion, the initial mathematic capability of student has an important role, so an idea will be developed step by step to build a comprehensive mathematic concept.

Based on aforementioned description, researcher do a research entitled: Formal deductive proof in geometry and its implication in teaching process.

## 2. Method of Research

### Steps of research

#### Step – I

- To predict Geometrical problem indicator in Junior High School (PJG)
- Hypothesis formulation
- Content of geometric knowledge (KPG)
- General Problem Solving (PMU)
- Mathematic reasoning competency (KPM)

#### Step – II

- Result of Step 1 + Literature on deductive thinking theory
- To analyze the result and problem in geometrical problem
- Instructional report / as basic design
- Result of Report and Design Planning

The result of research in step 1 plus a wide literature review about consideration of thinking theory in formal deductive for the proof of geometrical problem, to report teaching consideration and requirement of instructional/teaching provides basic for design of an environment based teaching.

The method applied in this research is quantitative method. This research is a descriptive – quantitative research. Its

focus is how the influence of formal deductive proof in geometric and its implication in mathematic teaching. This research was conducted at SMP Negeri 1 Medan. The population is all of students of SMP Negeri 1 Medan. And the sample is 166 students of junior high school that take as randomly sample of five classes.

In this research it will describe formal deductive proof in geometric that classified into three partitions, i.e. geometrical problem solving proof (PJG), content of geometrical knowledge IKPG), and General Problem Solving (PMU),, mathematic reasoning competency (KPM) of junior high school student (SMP) at SMP Negeri 1 Medan that formulated in multi linear as follows :

$$Y = a_0 + a_1X_1 + a_2X_2 + a_3X_3$$

Where :

Y : Mathematic reasoning competency (KPM)

X1 : Geometrical problem solving proof IPJG)

X2 : Content of Geometrical Knowledge IKPG)

X3 : General problem solving (PMU)

a0 : Constanta

a1, a2, a3, : Coefficients

Multi linear regression analysis is used to measure the influence between more than one independent variables and the dependent variable.

## 3. Result and Discussion

### 3.1 General description of test result of student

A student assigned to the geometrical topic mathematical test in SMP that consist of items of geometrical problem solving proof (PJG), Content of geometrical knowledge (KPG), and general problem solving (PMU). The test item written on geometrical problem solving proof (PJG) and general problem solving (PMU) is suitable to the indicators of problem solving. The content of geometrical knowledge (PKG) is assumed as common method. The score of mathematic reasoning competency (KPM) is calculated from the school appraisal. Statistic problem solving software (SPSS) is used to do the multi linear regression analysis of all the score.

The multi linear analysis is used to study the influence between two or more independent variables in which in this research the independent variables are geometrical problem solving proof (PJG), content of geometrical knowledge (KPG) and general problem solving (PMU). One of dependent variable is mathematic reasoning competency (KPM) score. The results of its analysis is as follows :

Entered / Removed Variables

| Model | Entered Variable               | Removed Variable | Method  |
|-------|--------------------------------|------------------|---------|
| 1     | PMU(X1)<br>KPG(X2)<br>PJG (X3) |                  | Entered |

The above table indicates the applied regression method. In multi linear regression analysis, there are three independent variables, i.e. geometrical problem solving proof (PJG),

Content of geometrical knowledge (KPG), and general problem solving (PMU). Based on the above table indicates that all of submitted independent variable will included in this research or there is not removed variable. While the applied method is entered method that enable pout all of independent variable for analysis simultaneously.

**Table 1:** The Coefficient of Multi Linear equation Coefficients<sup>a</sup>

| Model        | Unstandardized Coefficient |           | Standardized Coefficient | t      | Sig  |
|--------------|----------------------------|-----------|--------------------------|--------|------|
|              | B                          | Std error | B                        |        |      |
| 1 (Constant) | 40.779                     | 5.491     |                          | 7.428  | .000 |
| PJG (X1)     | 1.253                      | .098      | .670                     | 12.837 | .000 |
| KPG (X2)     | .542                       | .084      | .325                     | 6.471  | .000 |
| PMU (X3)     | .110                       | .081      | .070                     | 1.357  | .177 |

a. Dependent Variable : KPM

On coefficient table, there is a constant value of linear equation will used in this research. Based on the coefficient table, the multi linear regression equation is as follows:

$$Y = 40.779 + 1.253X_1 + 0.542 X_2 + 0.110 X_3$$

Based on the aforementioned multi linear regression equation, the coefficient of geometric teaching regression by formal deductive proof on mathematic reasoning competency test are as follows :

a. Regression coefficient X1 (Geometrical problem solving (PJG)).

The value of regression coefficient of the data is 1.253. this indicates that each the increasing of 1 unit of geometrical problem solving (PJG) it cause the increasing of mathematic reasoning competency (KPM) for 1.253 (in assumption of X2 and X3 is constant)

b. Regression coefficient X2 (Content of Geometrical Knowledge (KPG))

The value of regression coefficient of the data is 0.542. This indicates that each the increasing of content of geometrical knowledge (KPG) for 1 unit, it cause the increasing the mathematic reasoning competency (KPM) for 0.542 (in assumption X1 and X3 is constant)

c. Regression coefficient X3 (General problem solving (PMU))

The value of regression coefficient of data is 0.110. This indicates that each increasing of general problem solving (PMU) for 1 unit mathematic reasoning competency (KPM) for 0.110 (in assumption X1 and X2 is constant)

**Table 2:** The Value of contribution of Variable Model Summary<sup>b</sup>

| Model | R                 | R Square | Adjusted R Square | Std. Error of the Estimate |
|-------|-------------------|----------|-------------------|----------------------------|
| 1     | .793 <sup>a</sup> | .692     | .6622             | 7.223                      |

a. Predictors : (Constant), PMU (X1), KPG (X2), PJG (X3)

b. Dependent variable : KPM

Based on the aforementioned table, it indicates the contribution of variable based on determination coefficient. The R square value or R<sup>2</sup> (Determination coefficient) of result of regression with R square variable is 0.629, and the result indicates that the influence of geometrical problem solving proof (PJG), content of geometrical knowledge

(KPG) and general problem solving (PMU) has R square for 62.9% while its remain is 37.1% (100% - 62.9%) as a value that influenced by other variables.

### 3.2 Hypothesis Test

#### a. F-test

**Table 3:** Table of G-test ANOVA<sup>b</sup>

| Model        | Sum of squares | df  | Mean Square | F      | Sig               |
|--------------|----------------|-----|-------------|--------|-------------------|
| 1 regression | 14329.575      | 33  | 4776.525    | 91.543 | .000 <sup>a</sup> |
| Residual     | 8452.786       | 162 | 52.178      |        |                   |
| Total        | 22782.361      | 165 |             |        |                   |

a. Predictor (Constant), PMU(X1), KPG (X2), PJG (X3)

b. Dependent variable : KPM

Ha : The influence of geometric problem solving proof (PJG), content of geometrical knowledge (KPG) and general problem solving (PMU) simultaneously influence the dependent variable of mathematic reasoning competency (KPM)

Ho : The influence of geometric problem solving proof (PJG), content of geometrical knowledge (KPG) and general problem solving (PMU) simultaneously did not influence the dependent variable of mathematic reasoning competency (KPM)

The result of F test for coefficient of regression equation based on F-calculated is 91.543 with significant value is 0.000.. For value of F-table that find by excel in formula = finv (significant level, df1, df2), the value of F-table is 2.66. Based on the result it concluded that F-calculated > F-table or 91.543 > 2.66 and significant value is 0.000 < 0.05, so the decision is to reject Ha. Therefore : Ha : All of variables of influence of geometric problem solving proof (PJG), content of geometrical knowledge (KPG) and general problem solving (PMU) simultaneously influence the dependent variable of mathematic reasoning competency (KPM)

#### b. T-test

T-test is used to study whether the variable of influence of geometric problem solving proof (PJG), content of geometrical knowledge (KPG) and general problem solving (PMU) influence the dependent variable of mathematic reasoning competency (KPM). In order to proof it, the t-test applies the significant value of r95% and degree of freedom df3.

**Table 4:** Results of T-test Coefficients<sup>a</sup>

| Model        | Unstandardized Coefficient |           | Standardized Coefficient | t      | Sig  |
|--------------|----------------------------|-----------|--------------------------|--------|------|
|              | B                          | Std error | B                        |        |      |
| 1 (Constant) | 40.779                     | 5.491     |                          | 7.426  | .000 |
| PJG (X1)     | 1.253                      | .098      | .670                     | 12.837 | .000 |
| KPG (X2)     | .542                       | .084      | .325                     | 6.471  | .000 |
| PMU (X3)     | .110                       | .081      | .070                     | 1.357  | .127 |

#### a. Dependent Variable : KPM

##### *T-test for variable of Geometrical Problem Solving Proof (PJG)*

Based on hypothesis test using t-test, the t value for coefficient of geometrical problem solving proof (PJG) is 12.837 with significant value for 0.000. The score of t-table for significant testing is get by the value of alpha for 5% and df 164 and t-table is 2.26 and decision is to accept  $H_0$  and reject  $H_a$  if t-calculated < t-table and significant value > 5% and reject  $H_0$  and accept  $H_a$  if t-calculated > t-table and significant value < 5%.

Based on the aforementioned calculation, it concluded that the value of t-calculated > t-table or 12.837 > 2.26 with significant value for 0.000 or < 5%. It means that there is significant influence between geometric problem solving proof (PJG) as variable X1 to mathematic reasoning competency (KPM) as variable Y.

*t-test for variable of content of geometrical knowledge (KPG) based on hypothesis test using t-test, the t value for coefficient of content of geometrical knowledge (KGP) is 6.471 with significant value is 0.000. the t-table for significance testing it obtain the alpha value for 5% and df 164, so the t-table is 2.26 with decision is to accept  $H_0$  and reject  $H_a$  if t-calculated < t-table and significant value > 5% and reject  $H_0$  and accept  $H_a$  if t-calculated > t-table and significant value < 5%.*

Based on the aforementioned calculation, it concluded that value of t-calculated > t-table or 6.471 > 2.26 with significant value for 0.000.

#### 4. Conclusion

From the results of experiments that have been performed and evaluation described in the test tried before it can be the following conclusions:

1. It shows that in deductive meaning, a pattern, theorem, or corollary about the sum of triangle angles is  $180^\circ$ , has been proofed by using the last theory and mathematic pattern.
2. Formal deductive proof has instructional requirements. It must have elements of proper knowledge regarding content knowledge, skills in the process of solving including representational progress solving and strategic planning.
3. The learning environment is also a prerequisite to facilitate the process of formal deductive proof in mathematical problem that can assist and facilitate the student when he faces problems of mathematics.
4. Geometry is a science that deals with the problem real life.

#### References

- [1] Anderson, J. R. (1995). Learning and memory: An integrated approach. New York: John Wiley & Sons.
- [2] Brodie, Karin. (2010). Teaching Mathematical Reasoning in Secondary School Classrooms (Mathematics Teacher Education. New York : Springer

- [3] Copi, I.M.(1978). Introduction to Logic. New York: Macmillan.
- [4] Dahlan, J.A. (2004). Meningkatkan Kemampuan Penalarandan Pemahaman-Matematis Siswa SLTP melalui Pendekatan Pembelajaran Open-Ended. Desertasi PPSUPI Bandung: Not published.
- [5] Depdiknas. 2002. Pendidikan berorientasi Kecakapan Hidup (Life Skill) Melalui Pendekatan Broad-Based Education (Draft). Jakarta: Departemen Pendidikan Nasional.
- [6] Ekanayake, M.B., Brown, C., Chinnappan, M., (2003). Development of a Web-Based Learning Tool to Enhance Formal Deductive Thinking in Geometry. Conf. Proc. 26th Annual Conference of the Mathematics Education Research Group of Australia, 26(1), 302 - 308.
- [7] Hanna, G. (1983). Rigorous Proof in Mathematics Education, OISE Press, Toronto.
- [8] Hoyle, C. dan Kuchemann, D. (2002). Students' understanding of logical implication. Educational Studies in Mathematics 51, 193-223
- [9] Hudoyo, H. 1988. Mengajar Belajar Matematika. Jakarta: Departemen Pendidikan dan Kebudayaan
- [10] Jacobs, H.R. (1982). Mathematics, A Human Endeavor. San Fransisco: W.H. Freeman and Company.
- [11] Jonassen, D. H. 2000. Toward a Design Theory of Problem Solving. Educational Technology Research and Development 48 (4): 63-85. New York: Springer.
- [12] Jujun S, Suriasumantri. Filsafat Ilmu, Sebuah Pengantar Populer. Pustaka Sinar Harapan, Jakarta, 2001
- [13] Keraf, G. (1982). Argumen dan Narasi. Komposisi Lanjutan III. Jakarta: Gra-media.
- [14] Kirkley, J (2003). Principle for Teaching Problem Solving. Indiana University : Plato Learning
- [15] Klieme, E., Hoinze, A., (2002). Informal Prerequisites for Informal Proofs. ZDM 2002. 34(1), 9-16.
- [16] Koedinger, K. R., & Anderson, J. R. (1993). Reifying implicit planning in geometry: Guidelines for Model-based intelligent tutoring systems design. In S. P. Lajoie (Ed.), Computers as cognitive tools (pp. 15-45). NJ: Lawrence Erlbaum.
- [17] Kusnandi. 2008. Pembelajaran Matematikadengan Strategi Abduktif-Deduktif untuk Menumbuhkembangkan Kemampuan Membuktikan Pada mahasiswa (Disertasi). Bandung: Universitas Pendidikan Indonesia.
- [18] Moore, R.C. (1994). Making the transition to Formal Proof. Educational Studies in Mathematics, 27: 249-266.
- [19] Maduna B. E. (2003). Formal Deductive Proof in Geometry: Implications for Instructional
- [20] Practices. International conference on Sri Lanka Studies. University of Wollongong: Australia.
- [21] National Council of Teachers of Mathematics (NCTM). (2000). Principles and Standards for School Mathematics. Reston, VA: NCTM.
- [22] National Council of Teachers of Mathematics (NCTM). (1989). Curriculum and evaluation standard for school mathematics. Virginia: the national Council of teachers of mathematics, inc.
- [23] National Examinations and Testing Service, (2003), The Report on Performance at GCE (OL), National Examinations and Testing Service, Sri Lanka.

- [24] Pedemonte, B. (2003). What kind of proof can be constructed following an abductive argumentation. Proceeding of the third Conference of the European Society for Research in Mathematics Education.
- [25] Polya, G (1981). Mathematical Discovery on Understanding, Learning and Teaching Problem Solving}. New York: John Wiley & Sons.
- [26] Posamentier, A.S. danStepelman, J. (1986).Teaching secondary mathematics Teachniques and enrichment units. New jersey : Merrill Prentice
- [27] Reiss, K.& Renkl, A. (2002). Lerning to prove: The idea of heuristic exam-ples}. ZDM, 34(1): 29-35.
- [28] Reiss, K., Klieme, E. Hoinze, A., (2001). Prerequisites for the Understanding of Proofs in the GeometryClassroom. Proceedings of the 25th International Conference for the Psychology of Mathematics Education, Volume 4, pp. 97-104.
- [29] Ruseffendi, E.T. (1988). *Pengantar kepada Membantu Guru Mengembangkan Kompetensinya dalam Pengajaran Matematika untuk Meningkatkan CBSA*. Bandung: Tarsto.
- [30] Sabri (2003). Prospective Secondary School Teachers' Conceptions of Mathematical Proof in Indonesia} . Thesis: Curtin University of Technology.
- [31] Schoenfeld, A. H. (1985). Mathematical problem solving. Berkeley, CA: Academic Press.
- [32] Soedjadi, R. 2000. *Kiat Pendidikan Matematika di Indonesia*. Jakarta: Direktorat Pendidikan Tinggi Departemen Pendidikan Nasional.
- [33] Soekardijo, R.G. (1988). *LogikaDasar, Tradisonil, Simbolik dan Induktif*. Jakarta: Gramedia.