

3-Total Super Product Cordial Labeling For Some Graphs

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Abstract: In this paper we investigate a new labeling called 3-total super product cordial labeling. Suppose $G = (V(G), E(G))$ be a graph with vertex set $V(G)$ and edge set $E(G)$. A vertex labeling $f: V(G) \rightarrow \{0, 1, 2\}$. For each edge uv assign the label $(f(u) * f(v)) \bmod 3$. The map f is called a 3-total super product cordial labeling if $|f(i) - f(j)| \leq 1$ for $i, j \in \{0, 1, 2\}$ where $f(x)$ denotes the total number of vertices and edges labeled with $x = \{0, 1, 2\}$ and for each edge uv , $|f(u) - f(v)| \leq 1$. Any graph which satisfies 3-total super product cordial labeling is called 3-total super product cordial graphs. Here we prove some graphs like path; cycle and complete bipartite graph K_1, n are 3-total super product cordial graphs.

Keywords: 3-total super product cordial labeling, 3-total super product cordial graphs

1. Introduction

All graphs in this paper are finite, undirected and simple. For all other terminology and notations, we follow Harray [2]. Let $G(V, E)$ be a graph where the symbols $V(G)$ and $E(G)$ denotes the vertex set and edge set. If the vertices or edges or both of the graph are assigned values subject to certain conditions it is known as graph labeling. A dynamic survey of graph labeling is regularly updated by Gallian [3] and it is published in Electronic Journal of Combinatorics. Cordial graphs were first introduced by Cahit[1] as a weaker version of both graceful graphs and harmonious graphs. The concept of product cordial labeling of a graphs was introduced by Sundaram et. al[4].

Definition 1.1.: Let G be a graph. Let f be a map from $V(G)$ to $\{0, 1, 2\}$. For each edge uv assign the label $[f(u) * f(v)] \bmod 3$. Then the map f is called 3-total product cordial labeling of G , if $|f(i) - f(j)| \leq 1$; $i, j \in \{0, 1, 2\}$ where $f(x)$ denotes the total number of vertices and edges labeled with $x = \{0, 1, 2\}$.

Definition 1.2.: A 3-total product cordial labeling of a graph G is called 3-total super product cordial labeling if for each edge uv $|f(u) - f(v)| \leq 1$. A graph G is 3-total super product cordial if it admits 3-total super product cordial labeling.

2. Main Results

Theorem 2.1.: Path graph P_m is 3-total super product cordial.

Proof: Let P_m be the path $u_1, u_2, \dots, u_m$

Case I: $m \equiv 0 \pmod{3}$

Let $m = 3p$

If $p = 1$

Assign

$f(u_1) = 0$

$f(u_2) = 1$

$$f(u_3) = 2$$

Otherwise:

Define

$$f(u_i) = 0; 1 \leq i \leq p$$

$$f(u_{p+1}) = f(u_{p+2}) = 1$$

$$f(u_{p+2+i}) = 2; 1 \leq i \leq 2p - 2$$

Hence f is 3-total super product cordial labeling.

Case II: $m \equiv 1 \pmod{3}$

Let $m = 3p + 1$

Define

$$f(u_i) = 0; 1 \leq i \leq p$$

$$f(u_{p+1}) = 1$$

$$f(u_{p+1+i}) = 2; 1 \leq i \leq 2p$$

Hence f is 3-total super product cordial labeling.

Case III: $m \equiv 2 \pmod{3}$

Let $m = 3p + 2$

If $p = 0$ result is not true

If $p = 1$

$$f(u_1) = f(u_3) = 1$$

$$f(u_2) = 0$$

$$f(u_4) = f(u_5) = 2$$

Otherwise:

$$f(u_1) = f(u_3) = 1$$

$$f(u_2) = 0$$

$$f(u_{3+i}) = 2; 1 \leq i \leq 2p - 1$$

$$f(u_{2p+2}) = 1$$

$$f(u_{2p+3+i}) = 0; 1 \leq i \leq p - 1$$

Hence f is 3-total super product cordial.

Example 2.2.: The path P_6 and P_{10} are 3-total super product cordial graphs

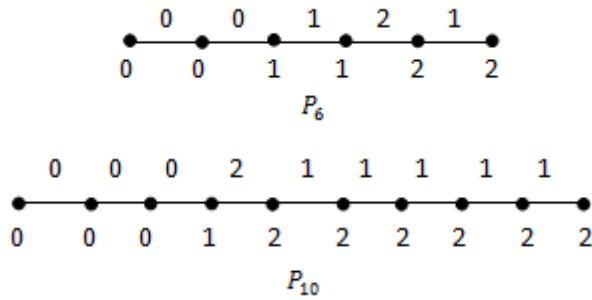


Figure 1: 3-total super product cordial labeling of path P_6 and P_{10}

Theorem 2.3.: $k_{1,m}$ is 3-total super product cordial. If $m \equiv 0 \pmod{3}$ and $m \equiv 2 \pmod{3}$.

Proof: Let $k_{1,m}$ be the complete bipartite graph we note that $|V(k_{1,m})| = m + 1$ and $|E(k_{1,m})| = m$

Let

$$V(k_{1,m}) = \{u, u_i : 1 \leq i \leq m\}$$

$$E(k_{1,m}) = \{uu_i : 1 \leq i \leq m\}$$

Case I: $m \equiv 0 \pmod{3}$

Let $m = 3p$

Assign

$$f(u) = 1$$

Define:

$$f(u_{3i+1}) = 0; 0 \leq i \leq p-1$$

$$f(u_{3i+2}) = 1; 0 \leq i \leq p-1$$

$$f(u_{3i+3}) = 2; 0 \leq i \leq p-1$$

Hence f is 3-total super product cordial.

Case II: $m \equiv 2 \pmod{3}$

Let $m = 3p + 2$

Assign

$$f(u) = 1$$

Define:

$$f(u_{3i+1}) = 0; 0 \leq i \leq p$$

$$f(u_{3i+2}) = 2; 0 \leq i \leq p$$

$$f(u_{3i+3}) = 1; 0 \leq i \leq p-1$$

Hence f is 3-total super product cordial.

Example 2.4.: The stars $k_{1,5}$ and $k_{1,9}$ are 3-total super product cordial graphs.

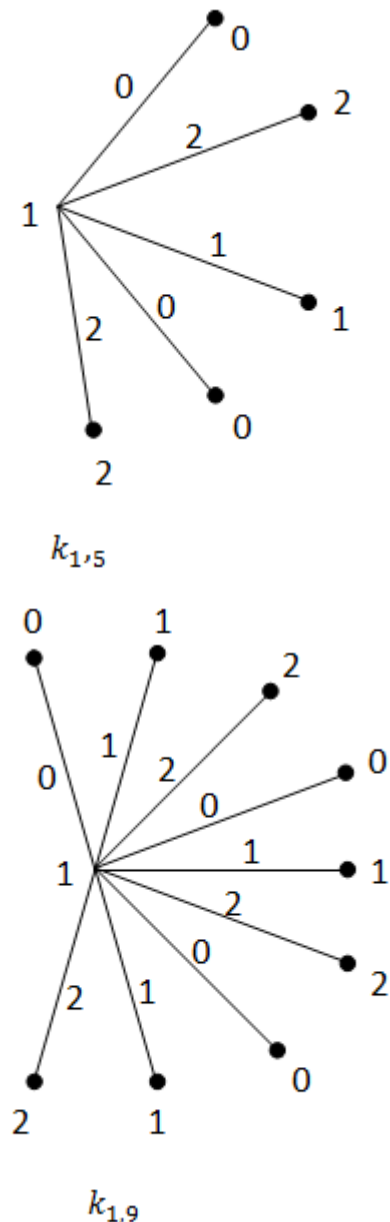


Figure 2: 3-total super product cordial labeling of the stars $k_{1,5}$ and $k_{1,9}$

Theorem 2.5.: Cycle graph c_m is 3-total super product cordial labeling. If $m \equiv 1 \pmod{3}$ and $m \equiv 2 \pmod{3}$

Proof: Let c_m be the cycle graph. We note that $|V(G)| = m$ and $|E(G)| = m$.

Case I: $m \equiv 1 \pmod{3}$

Let $m = 3p + 1$

$$f(u_i) = 0; 1 \leq i \leq p$$

$$f(u_{p+1}) = 1$$

$$f(u_{p+1+i}) = 2; 1 \leq i \leq 2p-1$$

$$f(u_{3p+1}) = 1$$

Hence f is 3-total super product cordial.

Case II: $m \equiv 2 \pmod{3}$

Let $m = 3p + 2$

Define:

$$f(u_i) = 0; 1 \leq i \leq p$$

$$\begin{aligned} f(u_{p+1}) &= 1 \\ f(u_{p+1+i}) &= 2; 1 \leq i \leq 2p \\ f(u_{2p+2}) &= 1 \end{aligned}$$

Hence f is 3-total super product cordial.

Example 2.6.: The cycle c_7 and c_8 are 3-total super product cordial graphs.

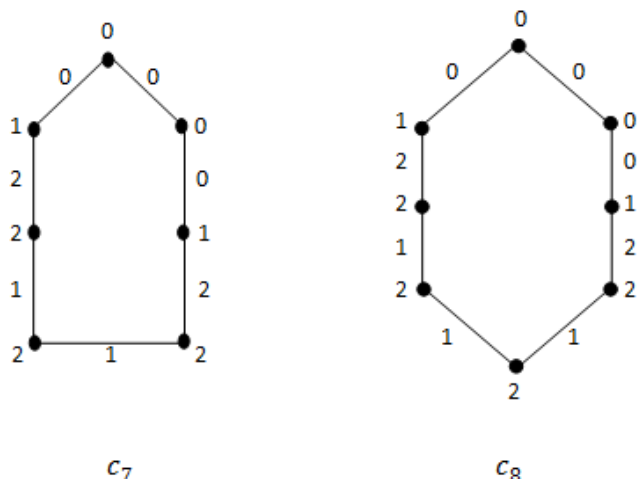


Figure 3: 3-Total super product cordial labeling of c_7 and c_8

3. Conclusion

Every 2-total product cordial graph is 2-total super product cordial graphs.

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