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# Two Fluid Cosmological Model in A Kantowski-Sachs Space-Time in General Relativity

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**Abstract :** ***A class of solutions of Einstein's field equations describing two fluid models of the universe in Kantowski-Sachs Space-time has been presented, in which the source of gravitational field consist of two commoving perfect fluids. One fluid is a radiation field, modeling the cosmic microwave background (CMB), while other is a matter field, modeling the material content of the universe. It is found that the both fluids are comoving in Kantowski –Sachs Space-time. The behavior of the radiation density, matter density, the ratio of the matter density to the radiation density and the pressure has been discussed. A subclass of solutions is found to describe models of a spatially homogeneous and partially isotropic evolving from a radiation dominated era to a pressure free matter dominated era. Further a table summarizes the asymptotic properties of all physically relevant variables.***

**Keywords:** Kantowski-Sachs.two fluid.general relativity

## 1. Introduction

The isotropic and homogeneous space-time due to Friedmann, Robertson and walker are simple models of the present stage of the expanding universe. Several researchers have studied the evolution of isotropic cosmological models filled with perfect fluids as well as viscous fluids. The discovery of 2.73*k* isotropic cosmic microwave background radiation (CMBR) motivated many researchers to investiga FRW with a two- fluid source [1-3]. In the two-fluid model, one fluid represents the matter content of the universe and another fluid is the radiation field corresponding to the observed cosmic microwave background (CMB) radiation [3-4]. The large scale matter distribution in the observable universe, largely manifested in the form of discrete structures, does not exhibit homogeneity of a high order. In contrast, the cosmic background radiation, which is significant in the microwave region is extremely homogeneous, however, recent space investigations detect anisotropy in the cosmic microwave background (CMB). In different angular scales, the observations from cosmic background explorer's differential microwave radiometer (COBE-DMR) have been detected and measured CMB an isotropies. These anisotropies are supposed to hide in their fold the entire history of cosmic evolution dating back to the recombination epoch and are being considered as indicative of the geometry and the content of the universe.

Universe is cooled sufficiently to form atomic hydrogen resulting into the release of photons in the recombination epoch. These have photons traveled freely through the universe forming the presently observed CMB. In 1992, COBE discovered temperature variations in the CMB level 1 part in 100,000. These small anisotropies are believed to have the information about the geometry and the content of the early universe. More about CMBR anisotropy is expected to be uncovered by the investigation of Microwave Anisotropy Probe (MAP) and COBRAS-SAMBA (Planck Surveyor) satellites. The observed anisotropies of CMB at the various angular scales make point of fresh look in the investigation of two-fluid models.

Bianchi type space-times exhibit spatial homogeniety and anisotropy. It is found that Bianchi Space-times of type I, II and $VI_0$ are asymptotically self similar into the past and the future [5]. Several cosmologists have constructed two-fluid cosmological models in general relativity.Coley and Dunn [6] investigated two-fluid Bianchi type $VI_0$ space- time.

The Kantowski–Sachs cosmological models containing perfect fluid with a zero cosmological constant was analyzed by Collins [7], he has carried out the qualitative study of the evolution of the Kantowski-Sachs model. Weber [8] applied Collin's method to a qualitative study of a Kantowski-Sachs models, in the presence of a nonzero cosmological constant $\wedge$ .

Recently, Pant and Oli [9] constructed two-fluid Bianch type-II cosmological models. Oli [10, 11] have studied two – fluid Bianchi type-I cosmological models with and withoutvariable G and $\wedge$ . Adhav et.al. [12-14] have studied anisotropic homogeneoue two-fluid cosmological models in Bianchi type-III and V space-times and also presented kantowski-Sachs model in presence of perfect fluid coupled with massless scalar field in general relativity.Katore et.al.[15] constructed Plane symmetric cosmological models with perfect fluid and dark energy. Two fluid cosmological models in higher dimensions have been presented by Mete et.al [16-18].

The main purpose of this paper is to construct two fluid Kantowski-Sachs space-time in the frame work of general relativity. In this paper, we have investigated physically sound co-moving two-fluid models in Kantowski-Sachs space-time.


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## 2. Metric and Field Equations

We consider the Kantowski-Sachs space-time is given by
$ds^2 = dt^2 - R^2 dr^2 - S^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$,
where $R$ and $S$ are functions of time $t$ only.
The field equations are
$G_{ij} = T_{ij}^{(m)} + T_{ij}^{(r)}$ ,
where $G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R$ is the Einstein's tensor. $T_{ij}^{(m)}$ is the energy momentum tensor for matter field described by a perfect fluid with density $\rho_m$, pressure $p_m$ and four velocity
$u_i^{(m)} = (1,0,0,0)$ ,
where
$g^{ij} u_i^{(m)} u_j^{(m)} = 1$
$T_{ij}^{(r)}$ ,denotes the energy momentum tensor for the radiation field with density $\rho_r$, pressure $p_r = \frac{1}{3}\rho_r$ and four velocity $u_i^{(r)} = (1,0,0,0)$,
where
$g^{ij} u_i^{(r)} u_j^{(r)} = 1$
Thus,
$T_{ij}^{(m)} = (\rho_m + p_m) u_i^m \, u_j^m - p_m g_{ij}$
$T_{ij}^{(r)} = \frac{4}{3}\rho_r u_i^r \, u_j^r - \frac{1}{3}\rho_r g_{ij}$
The off diagonal equations of (2) together with energy conditions
$\rho_m + \rho_r > 0,\ \rho_r > 0,$ lead to
$u_1^{(m)} = u_2^{(m)} = u_3^{(m)} = u_1^{(r)} = u_2^{(r)} = u_3^{(r)} = 0$,
From (3), (4) and (7),
$$u_0^{(m)} = u_0^{(r)} = 1, \qquad (8)$$
In our model, matter field and radiation field both are comoving.
From (5), (6), (7) and (8) the surviving field equations (2) are
$$2\frac{\dot{R}\dot{S}}{RS} + \frac{\dot{S}^2}{S^2} + \frac{1}{S^2} = 8\pi(\rho_m + \rho_r)$$
$$2\frac{\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} + \frac{1}{S^2} = 8\pi\left(-p_m - \frac{\rho_r}{3}\right)$$
$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} = 8\pi\left(-p_m - \frac{\rho_r}{3}\right),$$
where overhead dot (.) denotes differentiation with respect to cosmic time $t$.

Out of these three equations in five unknowns $R$, $S$, $\rho_m$, $p_m$, and $\rho_r$ only three are independent which may be written as follows:

Substracting equation (10) from equation (11), we get
$$\frac{\ddot{R}}{R} - \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} - \frac{\dot{S}^2}{S^2} - \frac{1}{S^2} = 0$$
From equations (9), (10) and (11), we get
$$\rho_m = \frac{3}{2(4-3\gamma)}\left[\frac{7}{3}\frac{\dot{R}\dot{S}}{RS} + \frac{5}{3}\frac{\dot{S}^2}{S^2} + \frac{5}{3}\frac{1}{S^2} + \frac{3\ddot{S}}{S} + \frac{\ddot{R}}{R}\right] \qquad (1)$$
$$\rho_r = \frac{3}{2(4-3\gamma)}\left[\frac{(3-4\gamma)\dot{R}\dot{S}}{RS} + \frac{(1-2\gamma)\dot{S}^2}{S^2} + \frac{(1-2\gamma)}{S^2} - \frac{3\ddot{S}}{S} - \frac{\ddot{R}}{R}\right]. \qquad (2)$$
Here there are five unknowns viz. $R$, $S$, $\rho_m$, $p_m$, and $\rho_r$ .Thus to get a solution we require two additional relations among the field variables $R$, $S$ and fluid variable $\rho_m$, $p_m$, and $\rho_r$. One of the additional relations will invariably be the equation of state of the matter field.
We assume a $\gamma$ - law for the equation of state :
$$p_m = (\gamma - 1)\rho_m, \quad 1 \le \gamma \le 2 \qquad (3)$$
The field equations are highly non-linear .Therefore we require another relation between $R$ and $S$ to solve the field equation (12) . In the following sections we shall explore various possibilities in this respect so as to obtain physically meaningful models.

## 3. Power-Law Solution

Here we consider power law form for $R$ and $S$ : (4)
$$R(t) = At^m, \quad S(t) = Bt^n,$$
where $A$, $B$, $m$ and $n$ are constants. Equation (12) yield (5)
$$n = 1, \quad B = \frac{1}{\sqrt{m^2 - 1}}, \quad 0 < 1 < m \qquad (6)$$
The resulting solution is
$$R = At^m$$
$$S = \frac{1}{\sqrt{m^2 - 1}} t$$
$$\rho_m = \frac{1}{(4-3\gamma)}\left[2m(2m+1)\right]t^{-2} \qquad (7)$$
$$\rho_r = \frac{3}{(4-3\gamma)}\left[-\gamma m^2 + 2m(1-\gamma)\right]t^{-2}$$
The solution is physically insignificant as either of $\rho_m$, $\rho_r$ becomes positive in the range $0 < 1 < m$. (9)

## 4. A Class of Physically Meaningful Solutions

(10)
By the transformation
$R = \lambda(t) S$
Equation (12) reduces to (11)
$$\frac{d}{dt}\left(S^2\right) + 2\left(\frac{\ddot{\lambda}}{3\dot{\lambda}} + \frac{\dot{\lambda}}{3\lambda}\right)S^2 = \frac{2}{3\lambda^3\dot{\lambda}}, \quad \text{provided } \dot{\lambda} \neq 0$$
Here we have followed the method of Pant and Oli [9].
Equation (19) a is first order linear differential equation in $S^2$ and can be solved easily by a proper choice of $\lambda(t)$ corresponding to the other required additional relation. Here we assume a power law form for $\lambda$ :
$\lambda = t^{-n}$ ,
where $n$ is a non-zero arbitrary constant.


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The resulting solution is given by

$$R = \sqrt{\frac{C}{nV}}\; t^{\frac{1-2n}{3}} \qquad (21a)$$

$$S = \sqrt{\frac{C}{nV}}\; t^{\frac{n+1}{3}} \qquad (21b)$$

$$\rho_m = \frac{1}{9(4-3\gamma)}\left[6n^2 + 30V(n-1) - 6\right] t^{-2}, \qquad (21c)$$

$$\rho_r = \frac{1}{3(4-3\gamma)}\left[(4-3\gamma)V^2 + \{n(2-9\gamma) + 2(1+3\gamma)\}V + 3(\gamma-2)(n-1)\right] t^{-2}$$

, (21d)

where

$$V = C(n-2)\left[C + t^{\frac{-2(n-2)}{3}}\right]^{-1},$$

where $n \neq 0,2$ and $C$ is an arbitrary constant.The above is a class of solutions with $n$ as parameter.

From (21a, b) and (22), we have the following realistic conditions on $C$, $n$ and $V(t)$ :

$C > 0$, $n > 0$, $V(t) > 0$
$C > 0$, $2 < n < 0$, $V(t) < 0$
$C > 0$, $(n\text{-}2) < 0$, $V(t) < 0$

The set of conditions (23a) limits the validity of the solution (21) to a finite interval of time :

$$0 \le t < t_0 = C^{\frac{3}{-2(n-2)}},$$

On the other hand, the set of conditions (23 b, c) put a lower bound on $t$ :

$$t > t_0 = C^{\frac{3}{-2(n-2)}}. \qquad (24\ b)$$

Moreover,we note that since (15) is the equation of state for the matter distribution which is distinct from the radiation field, γ is restricted to the following range in view of (21 c, 21d) :

$$1 \le \gamma \le \frac{4}{3}$$

However, the solution (21) can also be used to construct another class of physically meaningful models for

$$\frac{4}{3} \le \gamma \le 2.$$

Our aim is to find suitable intervals of time, from the ranges provided by (24 a), (24 b), in which $\rho_m$ , $\rho_r$ , $p_m$ and $\frac{\rho_m}{\rho_r}$ behave reasonably.For a physically meaningful model $\rho_m \ge 0$, $\rho_r \ge 0$, $p_m \ge 0$. Also, for the evolution of the universe from a radiation dominated stage to a matter dominated stage, $\rho_r$ must decrease faster than $\rho_m$. Such property can be ascertained by the study of the function $\frac{\rho_m}{\rho_r}$ which, for a reasonable model of universe, must be a monotonically increasing function of time.

Further,from equation (21), we observe that $\rho_r$ is quadratic function of $V$ ($t$), so that vanishing of each density will correspond to two values of $V$ ($t$) for given $n$ and γ.

$$\rho_m = 0 \Rightarrow V(t) = A, V(t) = B,\ A < B$$

$$\rho_r = 0 \Rightarrow V(t) = a, V(t) = b,\ a < b\ .$$

Now it is not difficult to explore the nature of $\rho_m$ in $A < V(t) < B$ and that of $\rho_r$ in $a < V(t) < b$. Having done so, we shall seek an interval for $V(t)$ or in other words an interval for t where $\rho_m$ and $\rho_r$ are non-negative functions :

$$V(t_1) \le V(t) \le V(t_2)$$

for

$$t_1 \le t \le t_2\ ,$$

where

$$t_1 = \left[C\left(\frac{(n-2)}{V(t_1)} - 1\right)\right]^{\frac{3}{-2(n-2)}} \qquad (22)$$

$$t_2 = \left[C\left(\frac{(n-2)}{V(t_2)} - 1\right)\right]^{\frac{3}{-2(n-2)}}.$$

Here the constant C provides a scaling factor for time. By proper choice of C, the interval of time $(t_1, t_2)$ is taken large enough. It is found that the solution (21) describes a class of reasonable models of the universe in a finite interval of time (30) for $n < -2.5$, provided $\gamma$ is restricted by (25). In case $\gamma$ obeys (26), one gets reasonable models for $n > 2.5$.

(23 a)
(23 b)
(23 c)
(24 a)

## 5. Investigation of Dust Models

Here we shall present a detailed study of the solution (21) with

$$\rho_m = 0 \text{ or } \gamma = 1 \qquad (31)$$

The solution for the exploration of physically meaningful dust models for different ranges of values of n , will be investigated as :

In view of (23), it is to be noted that for $V(t) > 0$, the possibility of the existence of a physically meaningful model lies only in the interval bounded by
$V(t) = b$, $V(t) = B$ ,

where as for $V(t) < 0$, for a meaningful investigation, our study will be limited to the interval bounded by
$V(t) = a$, $V(t) = A$

(25)
(26)


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**Table 1:** Various parameter associated with the dust models corresponding different values of *n*

| S. | n | A | B | a | b | $V(t_1)$ | $V(t_2)$ | $t_1 C^{\frac{3}{2(n-2)}}$ | $t_2 C^{\frac{3}{2(n-2)}}$ | $\rho_r(t_1) C^{-\frac{3}{(n-2)}}$ | $\rho_m(t_2) C^{-\frac{3}{(n-2)}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.5 | -2.26974 | 0.5 | -2.28194 | 0.5 | 0.5 | 0.5 | 0 | 0 | 0 | 0 |
| 2 | 2.6 | -2.49285 | 0.29245 | -2.48877 | 0.30669 | 0.30669 | 0.29245 | 1.11808 | 0.88181 | 1.15384 | 6.94532 |
| 3 | 2.7 | -2.51044 | 0.34407 | -2.50151 | 0.34528 | 0.34528 | 0.34407 | 0.94383 | 0.93003 | 1.93089 | 7.10331 |
| 4 | 3 | -2.55344 | 0.47905 | -2.52874 | 0.48141 | 0.48141 | 0.47905 | 0.89440 | 0.88181 | 3.50669 | 10.9658 |
| 5 | 3.3 | -2.88364 | 0.63511 | -2.82343 | 0.63871 | 0.63871 | 0.63511 | 0.96070 | 0.94850 | 4.62243 | 12.7410 |
| 6 | 3.5 | -2.91854 | 0.73469 | -2.84403 | 0.75 | 0.75 | 0.73469 | 0.74157 | 0.95999 | 5.4375 | 14.7815 |
| 7 | 4 | -3.39305 | 0.99783 | -3.25423 | 1.00207 | 1.00207 | 0.99783 | 1.00310 | 0.99675 | 8.63271 | 20.1087 |
| 8 | 5 | -3.56865 | 1.46938 | -3.32442 | 1.66666 | 1.66666 | 1.46938 | 1.11802 | 0.97979 | 15.4074 | 37.0751 |
| 9 | 6 | -4.41878 | 1.94338 | -3.99448 | 1.96627 | 1.96627 | 1.94338 | 1.98742 | 0.99856 | 28.8826 | 56.7656 |
| 10 | 7 | -4.57930 | 1.98947 | -4.98761 | 2.38344 | 2.38344 | 1.98947 | 0.97239 | 0.85588 | 43.1739 | 94.6229 |
| 11 | 8 | -5.03997 | 2.91544 | -5.10664 | 2.92108 | 2.92108 | 2.91544 | 0.98692 | 0.98600 | 59.9003 | 113.173 |
| 12 | 9 | -5.58786 | 3.48600 | -5.44899 | 3.51951 | 3.51951 | 3.48600 | 1.00239 | 0.99828 | 80.0018 | 154.424 |
| 13 | 10 | -6.05589 | 3.77921 | -6.01983 | 3.86201 | 3.86201 | 3.77921 | 0.98714 | 0.99348 | 101.541 | 186.328 |
| 14 | 20 | -11.4333 | 8.91848 | -10.9957 | 10.5377 | 10.5377 | 8.91848 | 1.02917 | 0.99849 | 542.329 | 833.349 |
| 15 | 50 | -29.2304 | 21.7476 | -27.3036 | 23.6166 | 23.6166 | 21.7476 | 0.99900 | 0.99413 | 3286.24 | 5270.86 |
| 16 | 100 | -52.6098 | 47.4405 | -52.1817 | 47.9060 | 47.9060 | 47.4405 | 0.99931 | 0.99902 | 13446.5 | 22822.64 |
| 17 | 1000 | -569.400 | 447.477 | -448.775 | 567.780 | 567.780 | 447.477 | 1.00041 | 0.99968 | 1618862.5 | 1621615.05 |
| 18 | 10,000 | -5021.40 | 4480.62 | -4980.10 | 5685.09 | 5685.09 | 4480.62 | 1.000041 | 0.999968 | 162318775.8 | 162703324.8 |
| 19 | 10,0000 | -50236.6 | 44812.06 | -49794.6 | 56858.2 | 56858.2 | 44812.06 | 1.0000041 | 0.9999968 | 1.62392589 x $10^{10}$ | 1.627575403 x $10^{10}$ |

**Table 2:** Variation of radiation density matter density and ratio of matter density to radiation density in the dust model from the epoch $t = t_1$ to $t = t_2$ for $n = 3$

| S.No. | $C^{\frac{3}{2}}t$ | $C^{-3}\rho_m$ | $C^{-3}\rho_r$ | $\frac{\rho_m}{\rho_r}$ |
|---|---|---|---|---|
| 1. | $C^{\frac{3}{2}}t_1$=0.89440 | 10.6788 | 3.50669 | 3.04526 |
| 2. | 0.89356 | 10.6980 | 3.51223 | 3.04587 |
| 3. | 0.89272 | 10.7173 | 3.51779 | 3.04660 |
| 4. | 0.89188 | 10.7365 | 3.52336 | 3.04728 |
| 5. | 0.89104 | 10.7559 | 3.52893 | 3.04792 |
| 6. | 0.89020 | 10.7758 | 3.53452 | 3.04873 |
| 7. | 0.88936 | 10.7946 | 3.54011 | 3.04922 |
| 8. | 0.88852 | 10.8140 | 3.54571 | 3.04988 |
| 9. | 0.88768 | 10.8335 | 3.55132 | 3.05055 |
| 10. | 0.88684 | 10.8530 | 3.55693 | 3.05122 |
| 11. | 0.88600 | 10.8725 | 3.56256 | 3.05187 |
| 12. | 0.88516 | 10.8921 | 3.56820 | 3.05254 |
| 13. | 0.88432 | 10.9117 | 3.57384 | 3.05321 |
| 14. | 0.88348 | 10.9313 | 3.57949 | 3.05387 |
| 15. | 0.88264 | 10.9510 | 3.58515 | 3.05454 |
| 16. | $C^{\frac{3}{2}}t_2$=0.88181 | 10.9658 | 3.58940 | 3.055050 |

**Case (i) :** $0 < n < -2.5$. In this case $\rho_m > 0$, $\rho_r > 0$ in the interval $a < V(t) < A$.

**Case (ii) :** $n = -2.5$. Here $a < A$. Thus no interval exists for a meaningful model.

**Case (iii) :** $n_1 < n < n_2$, $n \neq 2$, where $n_1 < -2.5$ and $n_2 < 0$ are the roots of equation.

$$0.66666n^2 + 1.0223n - 1.68896 = 0.$$

In this range of values of *n*; *A* and *B* becomes imaginary and $\rho_r$ is positive for all *t*.

**Case (iv) :** $n = n_1 . n_2$. In these cases $B > A$ and $\rho_m \geq 0$,

**Case (v) :** $-2.5 < n < n_1$, $\rho_m > 0$, $\rho_r > 0$ in the interval $a < V(t) < A$.

**Case (vi) :** $n = 2.5$. Here $B = b$. Thus no interval exists for a meaningful model.

**Case (vii) :** $n > 2.5$. In this case $A < a < B < b$.

We find $\rho_m \geq 0$, $\rho_r \geq 0$ in the following interval for $V(t) > o$:

$$b = V(t_1) \leq V(t) \leq V(t_2) = B.$$

Since $\rho_r(t_1) = 0$, $\rho_m(t_2) = 0$ and $t_1 > t_2$, the solution describes a set of models where in the early stages matter dominates radiation and in later stages radiation dominates matter. This does not agree with the actual universe.


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**Case (viii) :** $n < -2.5$ . In this case $A < a < b < B$ and $\rho_m \geq 0$, $\rho_r \geq 0$ , in the interval

$$A = V(t_1) \leq V(t) \leq V(t_2) = a \ .$$

Here $\rho_m(t_1) > 0$, $\rho_r(t_2) > 0$ and $t_1 < t_2$ . This corresponds to a class of models where the matter density increases with time, whereas the radiation density which is increases. For $n > 2.5$, the solution (21) provides a class of dust models in which, for the time interval $\left(C^{\frac{3}{2(n-2)}}t_1, C^{\frac{3}{2(n-2)}}t_2\right)$. $\rho_m$ and $\rho_r$ are positive and the ratio $\rho_m / \rho_r$ is monotonically increasing.

## 6. Conclusion

In this paper we present a class of solutions of Einstein's field equations describing two fluid models of the universe in Kantowski-Sachs Space-time. In these models one fluid is the radiation distribution which represents the cosmic microwave background and the other fluid is the perfect fluid representing the matter content of the universe. It is found that the both fluids are comoving in Kantowski – Sachs Space-time.

A numerical study of the model has been presented by calculating A, B, a, b, $V(t_1)$, $V(t_2)$, $C^{\frac{3}{2(n-2)}}t_1$, $C^{\frac{3}{2(n-2)}}t_2$, $\rho_m(t_2)$, $\rho_r(t_1)$ for some values of $n > 2.5$ (Table – I). As an illustration of the physical behaviour of the model, the variation of $\rho_m$, $\rho_r$, $\rho_m / \rho_r$ in the interval $\left(C^{\frac{3}{2(n-2)}}t_1, C^{\frac{3}{2(n-2)}}t_2\right)$ is obtained for $n = 3$ (Table -II). A qualitative study of two fluid Kantowski-Sachs Space-time has been presented.

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