

The Effect of Density of Sea Water on the Motion of Shock Waves

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Abstract: *Chester-Chisnell-Whitham method, a very well known theory in shock dynamics' is used to study the effect of density on underwater earthquake to analyse the tsunami. Neglecting the effect of overtaking disturbances, the analytical relations for shock strength, shock velocity, particle velocity and pressure are obtained for two cases viz. (1) when shock waves are diverging and (2) when they are converging. Finally, the flow variables are computed and discussed with the help of tables.*

Keywords: Shock waves, density of water, Tsunami

1. Introduction

Under water blasting is conducted for a number of uses such as rock excavation, demolition, grade preparation of foundations, structural rehabilitation, water way application (deepening channels/harbors, dike removal, and levee-raises during extreme flooding), geophysical exploration, fish sampling, metal forming, military operations, and many others. Since, **Taylor (1950)**, a considerable number of publications on the shock propagation have appeared in the literature, including treatises and reviews such as those of **Sedov (1959)**, **Sakurai (1965)**, **Lee et al. (1969)** and **Krobernikov (1971)**. The early studies of these phenomena [**Taylor (1950)** and **Sedov (1959)**] are based on self-similarity consideration and found in great agreement with experimental results. Analytical solution for the shock or blast wave propagation in homogenous and non-homogenous media have obtained by **Roger (1957)**, **Beach and Lee (1970)**, and many others. **Laumbach and Probststein (1969)**, **Sachdev (1971)** used an approach based on the shock propagation theory of **Brinkley and Kirkwood (1947)**.

For studying the flow behavior of water at high pressure and temperature, such as under water explosions, it is necessary to consider water as an idealized compressible fluid. Taking this into consideration, a solution of self-similar problem of strong explosion in water is given by **Anisimov and Kuznecov (1961)**. **Vishwakarma et al. (1988)** have been obtained similar solution for one dimensional unsteady adiabatic flow of water behind a diverging spherical shock wave assuming total energy of the flow to be time dependent. **Chisnell (1998)** described analytically the motion of converging spherical and cylindrical shock wave in ideal gas. **Yadav and Sharma (2004)** have used Chister-Chisnell- Whitham method to study the perturbation of water by strong converging shock waves in uniform medium and neglected the effect of overtaking disturbances. Recently, **Yadav et al. (2008)** studied the propagation of spherical blast wave in deep sea and investigated flow variables of perturbed sea water. Very recently, **Yadav et al. (2009)**. Investigated temperature variation of uniform gas atmosphere perturbed by strong spherical shock by Chester-Chisnell- Whitham method.

The purpose of present paper is to analysis the flow variables behind strong diverging and converging spherical shock waves propagating in water when effect of overtaking disturbances is neglected. Chester-Chisnell- Whitham method in used to obtain the analytical relations for freely propagation of shock. The dependence of flow variables on propagation distance(r) and specific heat index of water (n) have been numerically estimated and discussed through the figures and tables. Finally results obtained here compared with earlier results.

2. Basic Equations

The partial differential equations of motion in water governing the conservation of mass and momentum of the spherical flow are given by –

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial r}(\rho u) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho v) + \frac{\rho u}{r} + \rho v \frac{\cot \theta}{r} &= 0 \quad (1) \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} + \frac{1}{\rho} \frac{\partial p}{\partial r} &= 0 \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + \frac{1}{\rho r} \frac{\partial p}{\partial \theta} &= 0 \end{aligned}$$

Where, r is propagation distance u, v, p are respectively the particle velocity in radial direction, velocity in transverse direction, the pressure and 1 or 2 respectively for the cylindrical or spherical flow.

3. Boundary Conditions

The jump conditions are -

$$\begin{aligned} u &= \frac{2Ma_0}{(n+1)}, P = \frac{2a_0^2 M^2 \rho_0}{(n+1)}, \rho = \rho_0 \frac{(n+1)}{(n-1)} \quad (2) \\ \rho &= SMa_0 \left(\frac{n-1}{n+1} \right), S = \sqrt{\frac{2n}{n+1}} \end{aligned}$$

where $M = U/a_0$ is called the mach number ρ_0, a_0, P, ρ, n and U denote the undisturbed value of density, local sound velocity behind the shock, pressure, the density, specific heat index of water and shock velocity, respectively.

4. Theory For Diverging Shock

In the freely propagation description, the characteristic form of system of equations i.e. the form in which each equation contains derivatives in only direction in the (r, t) plane obtained as characteristic equation of diverging shock waves

$$U = \left\{ \frac{nK}{\rho'(1+xr)} \right\}^{1/2} K(1+xr)^{S/(4+2S)} r^{\frac{-\alpha S^2(n-1)}{(2+S)2+S(n-1)}} \quad (5)$$

$$u = \frac{2}{(n+1)} \left\{ \frac{nK}{\rho'(1+xr)} \right\}^{1/2} K(1+xr)^{S/(4+2S)} r^{\frac{-\alpha S^2(n-1)}{(2+S)2+S(n-1)}} \quad (6)$$

$$P = \frac{2\rho_0}{(n+1)} \left\{ \frac{nK}{\rho'(1+xr)} \right\}^{1/2} K^2(1+xr)^{2S/(4+2S)} r^{\frac{-\alpha S^2(n-1)}{(2+S)2+S(n-1)}} \quad (7)$$

5. Theory for Converging Shock -

In the freely propagation description, the characteristic form of system of equations for converging shock waves is

$$dp - \rho adu + \frac{\rho a^2 \alpha u}{(u-a)} \frac{dr}{r} = 0 \quad (8)$$

$$M' = K(1+xr)^{S/(4-2S)} r^{\frac{-\alpha S^2(n-1)}{(2-S)2-S(n-1)}} \quad (9)$$

$$U' = \left\{ \frac{nK}{\rho'(1+xr)} \right\}^{1/2} K(1+xr)^{S/(4-2S)} r^{\frac{-\alpha S^2(n-1)}{(2-S)2-S(n-1)}} \quad (10)$$

The expressions for shock strength, shock velocity, particle velocity and pressure just behind converging shock wave are

$$u' = \frac{2}{(n+1)} \left\{ \frac{nK}{\rho'(1+xr)} \right\}^{1/2} K(1+xr)^{S/(4-2S)} r^{\frac{-\alpha S^2(n-1)}{(2-S)2-S(n-1)}} \quad (11)$$

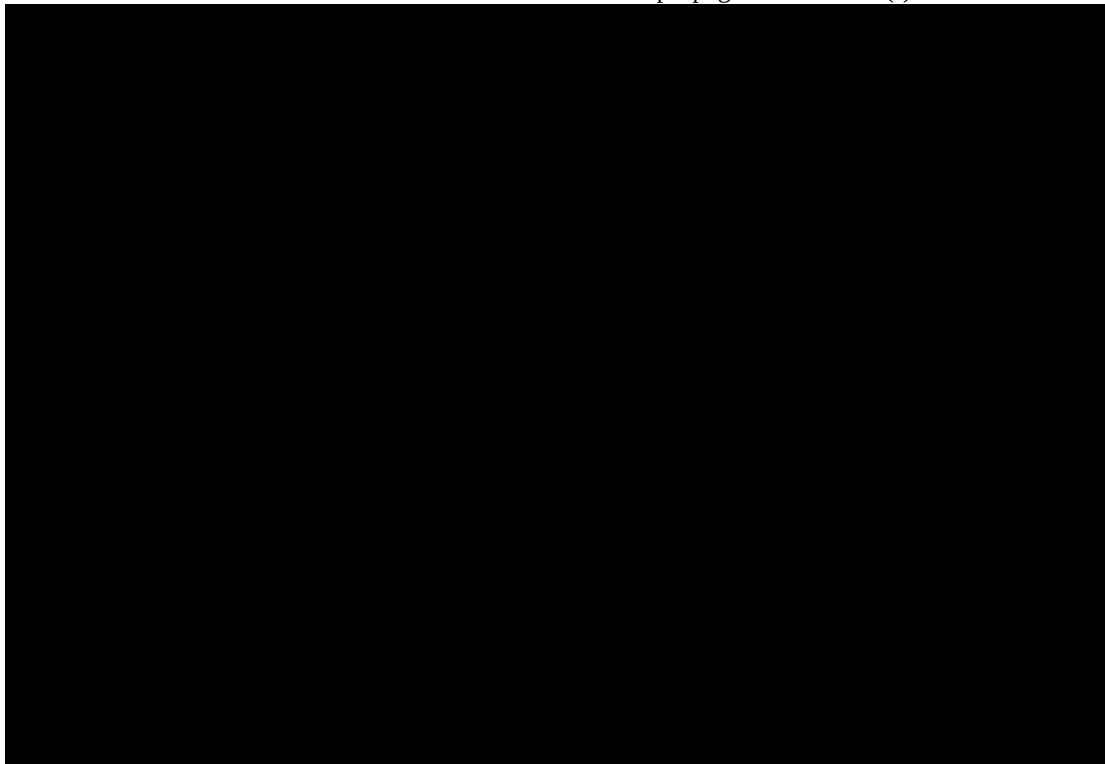
$$P' = \frac{2\rho_0}{(n+1)} \left\{ \frac{nK}{\rho'(1+xr)} \right\}^{1/2} K^2(1+xr)^{2S/(4-2S)} r^{\frac{-\alpha S^2(n-1)}{(2-S)2-S(n-1)}} \quad (12)$$

6. Results and Discussion

Equations (4)&(9), (5)&(10),(6)&(11),(7)&(12) represent the shock strength, shock velocity, particle velocity and pressure of diverging and converging spherical shock wave propagating freely in non-uniform region of water. Initially taking, $M=12$ at $r=9.0$ for $x=1.22$, $n=7.5$, $a_0=1450$ and, we have computed shock strength using equations (4) and (9) and obtained results are presented in table (1).

Variation of shock strength, shock velocity, particle velocity and pressure when density ρ_0 is function of propagation distance (r) only-

The variation of flow variables with respect to initial density ρ_0 and propagation distance r are computed and shown in following table(1).

Table 1: Variation of flow variables with propagation distance (r)


Variation of shock strength, shock velocity, particle velocity and pressure when density ρ_0 is function of constant (x) only-

The variation of flow variables with respect to initial density ρ_0 and constant x are computed and shown in following table(2).

Table 2: Variation of flow variables with constant (x)

x	ρ_0	M	M'	U	U'	u	u'	P	P'
1.22	11.98	12	12	17400	17400	4094.118	4094.117647	10224055601	1.0224E+10
1.24	12.16	12.0357	12.1783	17451.86	17658.5468	4052.484	4154.9522	10596481527	1.0849E+10
1.26	12.34	12.0711	12.3565	17503.11	17917.0517	4012.184	4215.776875	10976701123	1.1502E+10
1.28	12.52	12.106	12.5348	17553.76	18175.5152	3973.146	4276.591819	11364760231	1.2184E+10
1.3	12.7	12.1405	12.713	17603.84	18433.9379	3935.307	4337.397172	11760704293	1.2896E+10
1.32	12.88	12.1747	12.8912	17653.35	18692.3205	3898.604	4398.193073	12164578360	1.3639E+10
1.34	13.06	12.2084	13.0694	17702.31	18950.6635	3862.982	4458.979655	12576427102	1.4413E+10
1.36	13.24	12.2418	13.2475	17750.73	19208.9674	3828.387	4519.757048	12996294816	1.5219E+10
1.38	13.42	12.2749	13.4256	17798.63	19467.2328	3794.771	4580.525377	13424225430	1.6059E+10
1.4	13.6	12.3075	13.6037	17846.02	19725.4602	3762.089	4641.284766	13860262518	1.6933E+10
1.42	13.78	12.3399	13.7818	17892.9	19983.6501	3730.296	4702.035335	14304449303	1.7843E+10
1.44	13.96	12.3719	13.9598	17939.3	20241.8031	3699.353	4762.7772	14756828666	1.8788E+10

Variation of shock strength, shock velocity, particle velocity and pressure with Specific heat index of water (n) only-
The variation of flow variables with specific heat index of water are computed and shown in the following table (3).

Table 3: Variation of flow variables with Specific heat index (n) of water

n	M	M'	U	U'	u	u'	P	P'
7.1	12.2433	13.3147	17752.82	19306.3577	4383.412	4767.0019	10224055601	932258674
7.2	12.1796	12.9416	17660.5	18765.3497	4307.439	4576.9145	9804882540	911336877
7.3	12.1179	12.6003	17571.01	18270.5497	4233.977	4402.5421	9416210579	891255025
7.4	12.058	12.2875	17484.21	17816.8939	4162.908	4242.1176	9054983809	871966260
7.5	12	12	17400	17400	4094.118	4094.1176	8718527849	853427012
7.6	11.9436	11.7352	17318.26	17016.0598	4027.502	3957.2232	8404493585	835596725
7.7	11.8888	11.4908	17238.88	16661.7512	3962.961	3830.2876	8110810386	818437608
7.8	11.8356	11.2649	17161.76	16334.1644	3900.401	3712.3101	7835647011	801914407
7.9	11.784	11.0556	17086.82	16030.7426	3839.735	3602.414	7577378800	785994205
8	11.7337	10.8615	17013.96	15749.2307	3780.88	3499.829	7334560020	770646238
8.1	11.6848	10.6811	16943.09	15487.6337	3723.757	3403.8755	7105900449	755841720
8.2	11.6373	10.5132	16874.15	15244.1811	3668.293	3313.9524	6890245484	741553700

It is found that in changing value of propagation distance (r) from 9.0 to 14.5, initial density ρ_0 are changes from 11.8 to 19.995 and corresponding strength of diverging and converging shock are decreases from 12.0000 to 9.6266 and increases from 12.0000 to 217.0882 respectively. Shock velocity decreases from 17400 to 13958.98 and increases from 17400 to 314777.8712. Particle velocity of diverging spherical shock wave are decreases from 4094.1176 to 3284.4659, while particle velocity of converging shock are increases from 4094.117646 to 74065.38147 and pressure of diverging shock are increases from 853427011.8 to 856895305.3 and pressure of converging spherical shock wave are increases from 1.0224E+10 to 8.14E+12.

In changing constant (x) from 1.22 to 1.44, initial density are changes to from 11.98 to 13.96 and obtained results are presented in table (2) and corresponding strength of diverging shock are increases from 12 to 12.3719 and converging shock are increases from 12 to 13.9598. Shock velocity of diverging shock are increased from 17400 to 17939.3, in case of converging shock velocity increases from 17400 to 20241.8031 and particle velocity decreases from 4094.118 to 3699.353(diverging shock), increases from 4094.117647 to 4762.7772(converging shock). Pressures of diverging shock are increases from 10224055601 to 14756828660, in case of converging shock pressure increases from 1.0224E+10 to 1.8788E+10.

In changing value of specific heat index of water (n) from 7.1 to 8.2 are obtained results are presented in table 3, the strength of diverging shock decreases from 12.2433 to 11.6373 and from 13.3147 to 10.5132(for converging

shock). Shock velocity of diverging shock decreases from 17752.82 to 16874.15 and shock velocity of converging shock decreases from 19306.3577 to 15244.1811. Particle velocity of diverging and converging shock are decreases from 4383.412 to 3668.2926 and 4767.0019 to 3313.9524. Pressure of diverging and converging shock are decreases from 10224055601 to 6890245484 and 932258674 to 741553700, respectively.

7. Conclusions

In this paper, the linear density distribution obeying $\rho_0 = \rho' (1+xr)$, which satisfy following points:

- 1) The strength of diverging shock decreases as density increases, whereas it increases in case of converging shock i.e. strength of tsunami increases with the density of water in case of converging shock only.
- 2) The particle velocity i.e. flow velocity behind diverging spherical shock decreases as initial density increases, whereas it increases with the increases in density. Thus it may be strongly concluded that diverging tsunamis are less damaging than converging tsunamis.
- 3) Shock velocity i.e. tsunami velocity decreases with initial density in diverging case, whereas it goes on increasing for converging shock waves.

Thus density of water plays as important and significant role in the motion of tsunami. The damage depends on the diverging or converging nature of the waves.

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