

Power Allocation in OFDM-Based Cognitive Radio Systems

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Abstract: In this thesis, we develop an subcarrier transmission suboptimal power allocation algorithm and an underlay subcarrier transmission optimal power allocation algorithm for the orthogonal frequency division multiplexing (OFDM)-based cognitive radio (CR) systems with different statistical interference constraints imposed by different primary users (PUs). Given the fact that the interference constraints are met in a statistical manner, the CR transmitter does not require the instantaneous channel quality feedback from the PU receivers. First an alternative subcarrier transmission suboptimal algorithm with reduced complexity has been proposed and the performance has been investigated. Presented numerical results show that with our proposed suboptimal power allocation algorithm CR user can achieve 10 percent higher transmission capacity for given statistical interference constraints and a given power budget compared to the traditional suboptimal power allocation algorithms, uniform and water-filling power allocation algorithms. The proposed suboptimal algorithm outperforms traditional suboptimal algorithm, water-filling algorithm and uniform power loading algorithm. Second, We introduce an underlay subcarrier transmission optimal power allocation algorithms which allows the secondary users use the bandwidth used by PUs. And at the same time we consider the individual peak power constraint as the forth constraint added to the objective function which is the transmission capacity rate of the secondary users. Third, we propose suboptimal algorithm using GWF which has less complexity level than traditional water-filling algorithm instead of conventional water-filling algorithm in calculating the assigned power while considering the satisfaction of the total power constraint. The proposed suboptimal algorithm gives an option of using a low complexity power allocation algorithm where complexity is an issue.

Keywords: OFDM, GWF, Cognitive Radio, NULLING METHOD

1. Introduction

A Cognitive radio (CR) network means that a frequency band used by one or multiple primary users in a primary network can be operated by a secondary network which consists of one or multiple secondary users. To ensure the quality of service (QoS) of the Primary User (PU) and to maximize the transmission rate of the secondary users are one of the most important design issues for a Cognitive radio system. [1]. Orthogonal frequency division multiplexing (OFDM) is a promising candidate for cognitive radio systems. With OFDM, the SU has the ability to flexibly fill the spectral gaps left by PUs. CR should be designed to be able to detect spectrum holes and be able to use them while keeping the interference to the licensed user under specific limits. It also can determine its location; sense spectrums used by other devices, and change frequency, adjust output power or even alter transmission parameters and characteristics. It can meet the SUs communication needs while holding the FCC rules in US. Cognitive radio has three major tasks including: Radio scene analysis in which CR can detect spectrum holes, lightly used band and interference temperature; Channel state estimation, in which CR determine the channel capacity and the state of the channel; Spectrum management, in which CR make the spectrum sharing efficient.

2. OFDM

Orthogonal Frequency Division Multiplexing (OFDM) is a special form of multicarrier transmission. In OFDM, the available bandwidth is split into several narrow bands and data can be modulated into different subcarriers centered at these bands. The use of parallel transmission effectively increases the symbol duration and considerably reduces ISI

[2]. By choosing the subcarrier bandwidth to be less than the coherence bandwidth of the channel, they experience almost flat fading which makes equalization very simple. The subcarriers used in OFDM systems are orthogonal, to ensure proper retrieval of data at the receiver end and to minimize interference. The orthogonality of the subcarriers allows the spectra of individual subcarriers to overlap each other without any significant interference which allows for more efficient use of the bandwidth. Also, OFDM is very robust to narrow band interference because such interference can only affect a small percentage of the subcarriers. The subcarriers are separated by a frequency of $1/T_s$, where T_s is the symbol duration to ensure that the subcarriers are orthogonal over a symbol duration (in the absence of channel distortion). If the channel is slowly time varying, the capacity can be enhanced significantly by adapting the data rate per subcarrier according to the Signal to Noise Ratio (SNR) of that subcarrier [3]. The main difficulty in using the parallel data system is the complexity of the equipment needed for the implementation of the system. This is because OFDM requires very accurate frequency synchronization between the receiver and the transmitter; with frequency deviation, the subcarriers will no longer be orthogonal, causing Inter carrier Interference (ICI) which will reduce the system performance [4]. The employment of the Discrete Fourier Transform (DFT) to replace the banks of sinusoidal generators and the demodulators can significantly reduce the implementation complexity of OFDM modems [5]. This operation is equivalent to multiplying each bit by the desired subcarrier frequency; however, it ensures exact frequency spacing and phase coherence between subcarriers. The discrete Fourier transform can in turn be implemented using a Fast Fourier Transform (FFT) algorithm particularly when the number of subcarriers is large. The total number of subcarriers translates into the number of points of the FFT.

3. System Model

3.1 Underlay Model

We consider a downlink transmission scenario. It is assumed that the frequency bands of bandwidth $B_1, B_2, \dots, B_L$ have been occupied by PU1, PU2, ..., PUL. As in Figure 1, SUs can occupy either the spectrum of PUs or the adjacent spectrum of PUs. The available bandwidth for CR transmission is divided into N subcarriers based OFDM system, and the bandwidth for each subcarrier is Δf Hz. In the downlink transmission scenario, there are three instantaneous fading gains: between the SUs transmitter and SUs receiver for the i th subcarrier denoted as H_i^{ss} ; between the SUs transmitter and l th PU receiver denoted as h_l^{sp} transmitter and SUs receiver denoted as h_l^{ps} . We assume that these instantaneous fading gains are perfectly known at the SUs transmitter.

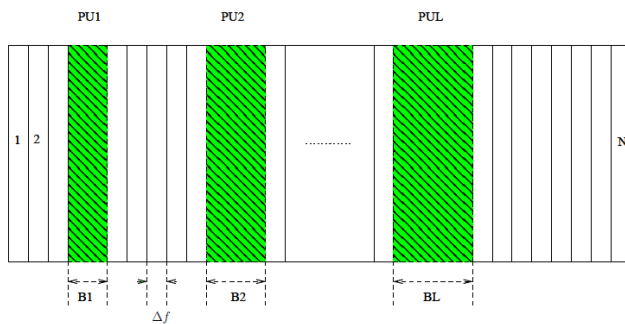


Figure 1: Illustration for Underlay Model

3.2 Geometric Water-Filling

In [6], a simple and elegant geometric water filling (GWF) approach is proposed to solve the un weighted and weighted radio resource allocation problems. Unlike the conventional water-filling (CWF) algorithm, it eliminated the step to find the water level through solving a non-linear system from the Karush-Kuhn-Tucker conditions of the target problem. The proposed GWF requires less computation than the CWF algorithm, under the same memory requirement and sorted parameters.

Fig. 2 (a) Illustration of water level step $k^* = 3$, allocated power for the third step s_3^* , and step/stair depth $d_i = 1/a_i$. (b) Illustration of $P_2(k)$ (shaded area, representing the total water/power above step k) when $k = 2$. (c) Illustration of $P_2(k)$ when $k = 3$. (d) Illustration of the weighted case [6].

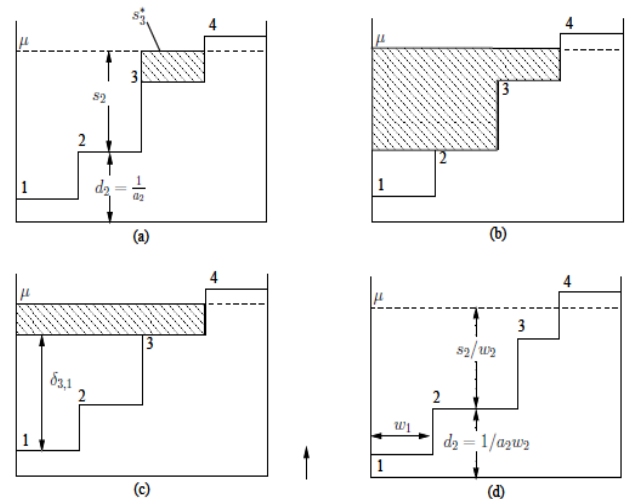


Figure 2: Illustration for the Geometric Water-Filling (GWF) algorithm.

$$P_2(k) = \left\{ P - \left[\sum_{i=1}^{k-1} \left(\frac{1}{a_k} - \frac{1}{a_i} \right) \right] \right\}^+ \\ = \left\{ P - \left[\sum_{i=1}^{k-1} \delta_{k,i} \right] \right\}^+, \text{ for } k = 1, \dots, K. \quad (1)$$

Due to the definition of $P_2(k)$ being the power (water volume) above step k , it can't be a negative number. Therefore we use $\{\cdot\}^+$ to assign 0 to $P_2(k)$ if the result inside the bracket is negative. The corresponding geometric meaning is that the k_{th} level is above water.

4. Suboptimal Power Allocation Algorithms Using Nulling Method

In this, we investigate the nulling method and GWF algorithms to the OFDM based subcarrier power allocation problems.

4.1 Problem Formulation and Optimal Power Allocation

Our design goal is to find power values for each subcarrier, $P_i (i = 1; 2; \dots; N)$ for given instantaneous fading gains h_i^{ss} , given fading statistics of h_i^{sp} and the total transmit power budget P_T . As such the total transmission rate of the CR user, C is maximized while the probability that the interference introduced to l th ($l = 1; 2; \dots; L$) PU band is kept below the threshold I_{th}^l ($l = 1; 2; \dots; L$), respectively, with the probability value α or above. Mathematically, the problem can be formulated as a constrained optimization problem as follows

$$C = \max_{P_i} \sum_{i=1}^N \log \left(1 + \frac{|h_i^{ss}|^2}{\sigma^2 + \sum_{l=1}^L J_i^{(l)}} \right), \quad (2)$$

subject to:

$$P_r \left(\sum_{i=1}^N I_i^{(l)}(d_{il}, P_i) \leq I_{th}^{(l)} \right) \geq \alpha, \quad \forall l, \quad (3)$$

$$P_i \geq 0, \quad \forall i, \quad (4)$$

$$\sum_{i=1}^N P_i \leq P_T, \quad (5)$$

where P_r : denotes the probability. Now the probabilistic interference constraint in Eq.(3) can be written as

$$P_r \left(|h_l^{sp}|^2 \sum_{i=1}^N K_i^{(l)} P_i \leq I_{th}^{(l)} \right) \geq \alpha, \quad \forall l, \quad (6)$$

4.2 Proposed Suboptimal Power Loading Schemes

Nulling Mechanism was discussed in [7]. The authors studied the effect of nulling the subcarriers and showed that the interference introduced to the PU band can be reduced by nulling those subcarriers (i.e., allocating zero power) that are adjacent to the PU band, since the adjacent subcarriers produce the maximum amount of interference. This procedure implies that for a given interference threshold, more power can be allocated to the far apart subcarriers than to the neighbouring subcarriers. Thus, one would expect that a higher transmission rate can be achieved using the same power. However, nulling the adjacent subcarriers loses frequency diversity, as the adjacent subcarrier is assigned with zero power even when it has a very good channel gain. Therefore, nulling creates a trade-off. Hence, here we study our proposed suboptimal scheme and classical uniform loading and water-filling schemes with the effect of nulling. It should be noted that we do not study effect of nulling on the optimal scheme, as it is already optimal, and automatically nulls the subcarriers that need to be nulled. By nulling we mean that for each side of the PU band, we null one subcarrier that is adjacent to it. Without loss of generality, we assume that each CR user band occupies N subcarriers. We assume the subcarriers are positioned around the PUs. Hence, we null $2L - 1$ subcarriers, namely, Now for studying the nulling mechanism, we load zero power in the subcarriers. For the remaining subcarriers, we load powers according to the scheme under study, such that the total interference introduced to the primary user band is equal to I_{th} , and the power profile for the schemes can be derived accordingly.

$$\emptyset \in \left\{ \left(\frac{N}{L}, \left(\frac{N}{L} + 1 \right), \left(\frac{2N}{L}, \left(\frac{2N}{L} + 1 \right), \dots, (N) \right) \right\} \quad (7)$$

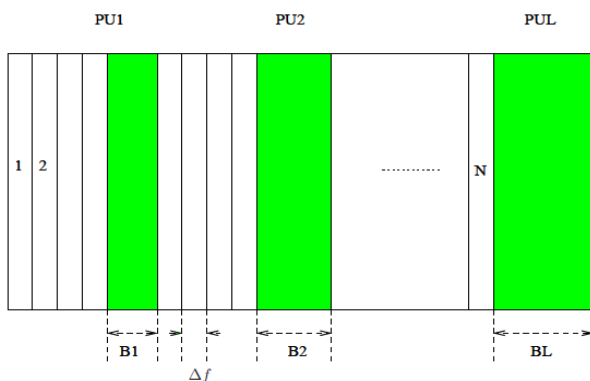


Figure 3: Illustration for the proposed suboptimal power loading scheme

4.2.1 Step 1

The power has to be assigned to N CR subcarriers such that the transmission rate of CR user can be maximized while all $L + 1$ total power constraint and L interference constraints from Eq. (4.7) are to be satisfied. The complexity of the optimal algorithm comes from the fact that these $L + 1$ constraints have to be met simultaneously. In order to reduce such complexity, we follow a three step procedure as follows. First, we keep only one of the $L + 1$ constraints and find power allocation sub optimally in each subcarrier. Without loss of generality, let us denote that the allocated power in the i th subcarrier due to the l th constraint by $P_i^{(l)}$. In order to satisfy l th interference constraint given in above Eq. , power is allocated according to nulling mechanism and ladder profile as in [8]. It is based on the heuristics that if a subcarrier is closer to a PU band, it introduces more interference. Hence, less power should be allocated to that particular subcarrier. We allocate zero power to the subcarrier which is the nearest to the PUs. In particular, we propose to allocate power in each CR subcarrier such that the allocated power is inversely proportional to the factor $K^{(l)}$ that depends on the spectral distance between the i th CR subcarrier and the l th PU band.

4.2.2 Step 2

In step 1 we calculate the L constraints from interference point of view. In step 2 we will calculate power values P_i^{L+1} according to the total power constraint. In order to reduce the computation complexity we use GWF instead of standard water-filling algorithm.

4.2.3 Step 3

The final allocated power according to our proposed suboptimal algorithm is the minimum of the power we obtained from step 2

$$P_i^{subopt} = \min \{ P_i^{(1)}, P_i^{(2)}, \dots, P_i^{(L)}, P_i^{(L+1)} \} \quad \forall i, \quad (8)$$

Through our proposed suboptimal algorithm, the resulted allocated power value selected from $L+1$ power values can satisfy both the interference and total power constraints.

4.3 Simulation Results

In this section we present simulation results where we assume that there are three PU bands ($L=3$), and there are twelve OFDM subcarriers ($N = 12$) for the CR user. The values of $T_s, \Delta f, B_1, B_2$, and B_3 have been assigned to be 4 seconds, 0.3125 MHz, 1 MHz, 2 MHz, and 5 MHz, respectively. AWGN variance, (σ^2) is assumed to be equal to $10^{-8}W$ and the channel fading gains are assumed to follow Rayleigh distribution. The average channel power gains for $|h_{ss}|^2, |h_l^{sp}|^2, |h_2^{sp}|^2, |h_3^{sp}|^2$ are assumed to be -10 dB, -5 dB, -7 dB, and -10 dB, respectively. The values of $J(l)$ are generated randomly with an average value of $1 \times 10^{-6}W$. The values of $I(1)_{th}$, and $I(3)_{th}$ have been assumed to be $1 \times 10^{-6}W$, and $5 \times 10^{-6}W$, respectively. Average transmitted data rates for different algorithms under consideration are obtained from 30, 000 independent simulation runs. In Figure 4, we plot the achievable maximum transmission rate for the CR user versus the total power budget for various algorithms. The value of $I(2)_{th}$ has been fixed to $2 \times 10^{-6}W$, and the value of α

has been considered to be 0.95. The highest curve is made by using nulling method. The curve below it is made using the method in [9]. The third curve is made by water-filling method, the lowest curve is made by using uniform power allocation method. From this figure, we observe that the proposed suboptimal algorithm is able to achieve higher transmission rate for a given power budget than the conventional suboptimal algorithms. It should be noted that as we increase the power budget for CR user, the interference constraint becomes dominant and the transmission rate of CR user does not increase as the power budget increases. This is expected as in this region the CR system operates in an interference limited scenario.

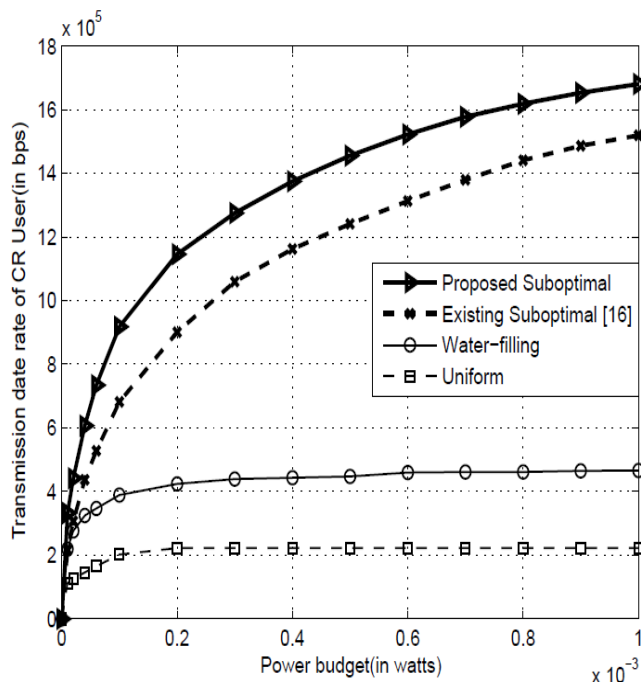


Figure 4: Maximum transmitted data rate vs. power budget (Total Power Constraint) for CR users

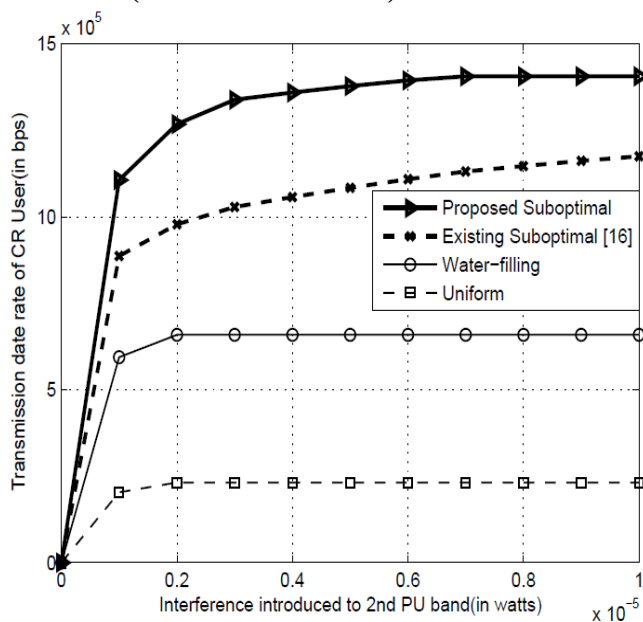


Figure 5: Transmission data rate of the CR user vs. interference threshold for 2nd PU band, I^2_{th}

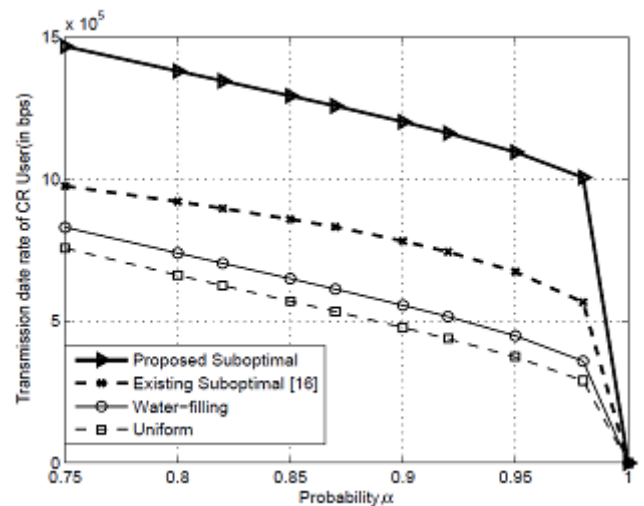


Figure 6: Transmission rate of the CR user vs. probability, α with which instantaneous interference introduced to the PU band remains below interference threshold, I_{th}

5. Subcarrier Power Allocation with Total Power, Interference and Individual Peak Power Constraints

In this, we propose an optimal power allocation algorithm for subcarrier power allocation with total power constraint, interference constraint and individual peak power constraint using underlay model where the SUs can use the band when the primary user uses the same band, but we keep the interference to the primary user under a threshold.

5.1 Simulation Results

In this section we present a numerical example where we assume that there are three PU bands ($L=3$), and there are twelve OFDM subcarriers ($N = 12$) for the CR user. The values of $T_s; \Delta f; B_1; B_2$, and B_3 have been assigned to be 4 seconds, 0.3125 MHz, 1 MHz, 2 MHz, and 5 MHz, respectively. AWGN variance, (σ^2) is assumed to be equal to $10^{-8}W$ and the channel fading gains are assumed to follow rayleigh distribution. The average channel power gains for $|h_{i^{ss}}|^2$; $|h_{i^{ps}}|^2$; $|h_{2^{ps}}|^2$; and $|h_{3^{ps}}|^2$; are assumed to be -10 dB, -5 dB, -7 dB, and -10 dB, respectively. The values of $J(l)i$ are generated randomly with an average value of $1 \times 10^{-6}W$. The values of $I(1)_{th}$, and $I(3)_{th}$ have been assumed to be $1 \times 10^{-6}W$, and $5 \times 10^{-6}W$, respectively. Average transmitted data rates for different algorithms under consideration are obtained from 30, 000 independent simulation runs. In Figure 7, we plot the achievable maximum transmission rate for the CR user versus individual power constraints for various algorithms. The value of $I(2)_{th}$ has been fixed to $2 \times 10^{-6}W$, the total power P_T has been fixed to be $5 \times 10^{-4}W$, and the value of α has been considered to be 0.95. From this figure, we observe that the proposed optimal algorithm is able to achieve higher transmission rate for a given power budget than the suboptimal algorithm. It should be noted that as we increase the individual power constraint for CR user, the transmission rate of CR user does increase as the individual peak power constraint increases. This is expected as individual subcarrier can be allocated more power.

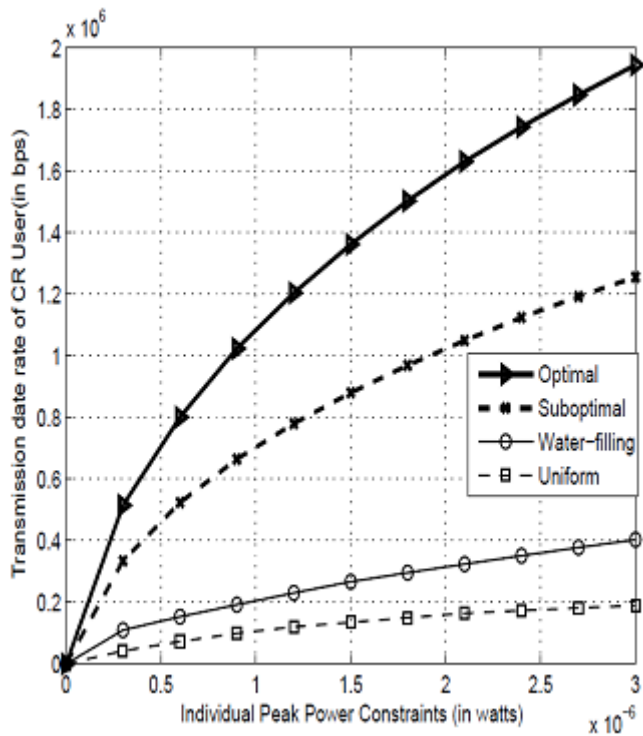


Figure 7: Transmission rate of the CR user vs. Individual power constraints, with which instantaneous interference introduced to the PU band remains below interference threshold, I_{th}

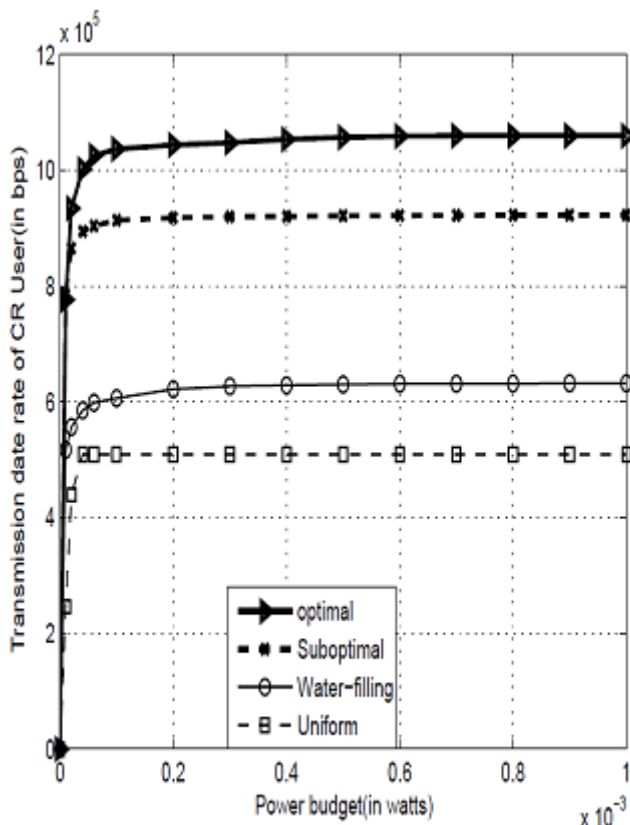


Figure 8: Maximum transmitted data rate vs. power budget (PT) for CR users

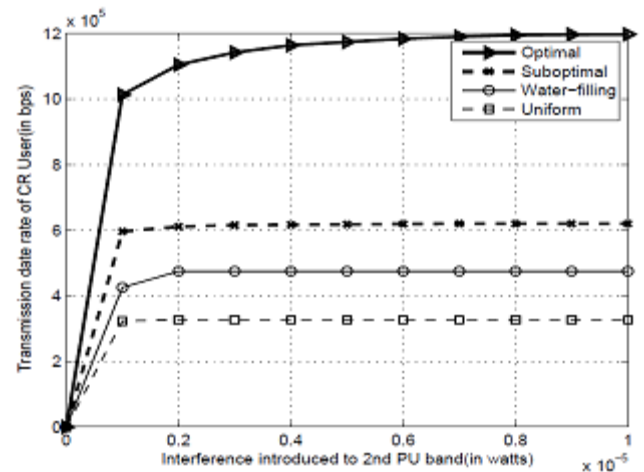


Figure 9: Transmission data rate of the CR user vs. interference threshold for the 2nd PU band, I^2_{th}

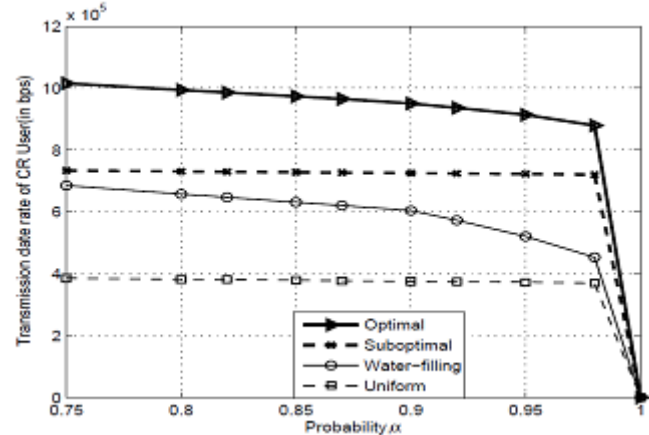


Figure 10: Transmission rate of the CR user vs. probability, α with which instantaneous interference introduced to the PU band remains below interference threshold, I_{th}

6. Conclusions

In this thesis, first we develop an novel cognitive radio concept using underlay channel band and an optimal power allocation algorithm for the orthogonal frequency division multiplexing (OFDM)-based CR system with total power, interference and individual peak power constraints. Also we developed an suboptimal power allocation algorithm using nulling method for this problem in order to reduce the complexity of computation. As such the transmission rate of the CR user is maximized for a given total power budget, probabilistic interference and individual peak power constraints. Instead of instantaneous channel fading gains between the PU receivers and the CR transmitter, the developed power allocation algorithm requires the fading statistics and corresponding parameters to be known at the CR transmitter. Second, we propose and investigate performance of a low complexity suboptimal power allocation algorithm for subcarrier power allocation with total power and interference constraints. Simulation results have shown that our proposed optimal and suboptimal power allocation algorithms can achieve higher transmission rate for CR user compared to the existing power allocation algorithms namely, the uniform and water-filling power

allocation algorithms. The proposed low complexity suboptimal algorithm achieves better performance than existing suboptimal, uniform and water-filling power loading algorithms.

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