International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2025: 7.089

Mathematical Modeling on Water Quality Management in Developing Countries Using Numerical Methods

Sathisha A B[1], Ashwini M Rao[2], K S Basavarajappa[3]

[1]Associate Professor, Department of Mathematics, Government First Grade College, Jagalur, Karnataka, India.

[2]Associate Professor, Department of Mathematics, Bapuji Institute of Engineering and Technology, Davanagere, Karnataka, India.

[3]Professor, Department of Mathematics, Bapuji Institute of Engineering and Technology, Davanagere, Karnataka, India.

Abstract: ***Water, with the chemical formula H_2O, is a transparent, tasteless, and odorless liquid that is crucial for life on Earth. It's composed of two hydrogen atoms and one oxygen atom, and it exists in three states: solid (ice), liquid, and gas (water vapor). Water is the main component of Earth's hydrosphere and is vital for all known forms of life. using mathematical equations and Numerical simulations to understand and predict how pollutants behave in water bodies, and to evaluate the effectiveness of different management strategies. Models simulate how pollutants like nutrients, bacteria, and dissolved oxygen move and change within a water system. Models can help pinpoint where pollution originates, whether from point sources like industrial discharges or non-point sources like agricultural runoff. In the present paper we analysed how the developing countries manage the quality of water for their domestic purpose. Numerical method is employed to analyse the water purity and to prevent water from pollution.***

Keywords: Water, bacteria, oxygen, nutrients, industry, quality

1. Introduction

Water pollution is one of the most devastating effects of industrialization. The potability and hygiene of drinking water have been affected by hazardous impurities that are released by industrial sectors, causing severe health effects to human, animal and aquatic life. Due to climatic changes, the occurrence of floods and draughts has become frequent in many parts of the world. On top of that, increasing water pollution from the waste released from various sectors like industry, agriculture, households and municipalities has greatly contributed to the decline of the quality and quantity of drinking water. Wastewater released from various sectors can be categorised into different types, such as, sewage wastewater, domestic wastewater and industrial wastewater. The wastewater produced by human household activities is known as domestic wastewater. The main source of this wastewater generally consists of two major waste streams: toilets, and the wastewater generated due to other household activities. Water with dissolved and suspended substances discharged from various industrial processes, such as the water released during manufacturing, cleaning and other commercial activities, is termed industrial wastewater. The nature of the contaminants present in industrial wastewater depends on the type of the factory and the industry. The various contaminants commonly found in industrial water outlets are chemicals, heavy metals, oils, pesticides, slits, pharmaceuticals and other industrial by-products. The main source of waterborne diseases such as gastroenteritis, dysentery, diarrhea, and viral hepatitis in this area is the water pollution of coliform bacteria.

Over the past few years, Mona Radwan et al [1] study the uncertainties associated with each of the above-mentioned uncertainty sources. Further, this can be used to identify the relative contributions of uncertainties associated with each source. A K Misra et al [2] proposed a nonlinear mathematical model to study the depletion of dissolved oxygen in a water body caused by industrial and household discharges of organic matters. J.B. Shukla et al [3] proposed a nonlinear mathematical model to study the depletion of dissolved oxygen in a water body caused by industrial and household discharges of organic matters. J.B. Shukla et al [4] modeled by considering the variables such as cumulative concentration of organic pollutants, the densities of bacteria, nutrients, algae, detritus and the concentration of DO. Busayamas Pimpunchat et al [5] motivated by the crucial problem of water pollution in many countries and specifically within the Tha Chin River in Thailand. For such real situations, simple models can provide decision support for planning restrictions to be imposed on farming and urban practices. W. J. S. Mwegoha et al [6] developed a model to predict the DO dynamics in fish ponds. Based on the model results, successful cultivation of healthy fish may require that retention time for water in the fish pond be 10 days. Hemant Pathak et al [7] obtained a model that have been discussed in light of role and effect on DO, and influences of other parameter on DO and keeping in view the composition of this standard resource an analysis of water samples of this city have been performed so as to derived a model for pollution and contamination with respect to DO and search their source as typical man- made activities. Witsarut Kraychang et al [8] calculated the pollutant level in a connected-pond reservoir system that had an entrance and an exit gate to open water of a canal and to determine the optimal pollutant levels in the wastewaters discharged from nearby industrial plants that would cost the plants minimally to pretreat.

In this paper, a collocation approach based on ODE and its derivatives is utilized to convert the mathematical model of water pollutant into Numerical methods. Finally, the Newton method has been utilized to obtain the numerical solution. This method can be applied to a wider range of real-life mathematical models after minor revision, particularly those with more complicated systems and geometry. Using numerical method we analise the water quality management in pollutants water. We also demonstrate the water pollution leads the physiological effects in human beings as well as living beings.

2. Formulation

Proposed model for the transmission of water pollution is subdivided into four compartments represented as four ordinary differential equations. Let W be the concentration of water pollutants, S be the volume of solvable water pollutants, I be the volume of insolvable water pollutants and T be the insolvable water that is removed by treatment. Let A be the rate of water pollutants, α_1 be the transmission rate for solvable water pollutants, p be the rate of insolvable water pollutants becoming water pollution, α_2 be the transmission rate for insolvable water pollution, μ be the removable rate of water pollution, δ be the amount of treating insolvable water pollutants to solute θ_1 be the amount of solvable water pollutants treated and θ_2 be the amount of insolvable water pollutants treated. The rate at which the concentration of water pollutants is represented by the differentiable equation as

$$\frac{dW}{dt} = A - \alpha_1 WS - \alpha_2 WI + p\alpha_2 I - \mu W \quad (1)$$
$$\frac{dS}{dt} = \alpha_1 WS + \delta I - (\theta_1 + \mu)S \quad (2)$$
$$\frac{dI}{dt} = \alpha_2 WI - p\alpha_2 I - (\delta + \theta_2 + \mu)I \quad (3)$$
$$\frac{dI}{dt} = \theta_1 S + \theta_2 I - \mu T \quad (4)$$

Equation (1) describes the concentration of water pollutants. Equation (2) describes the concentration of volume of solvable water pollutants. Equation (3) describes the concentration of volume of insolvable water pollutants Equation (4) describes the concentration of insolvable water that is removed by treatment insolvable water that is removed by treatment.

In the present model, Ordinary differential equations have been used to study the physiological effect of water pollution on the dynamics of living beings or dynamics of diseases in the living beings. One of the main function of the immune systems is to prevent the body from developing different diseases.

3. Analysis

We present the approach to analyze the parameter values that have a high impact on the transmission rate. This will connect the early transmission of water pollutants to be soluble and insoluble in water and also the prevalence of water pollutants directly to the endemic point. This helps us to know how useful each parameter is to the transmission of water pollutants and the prevalence of pollutants. These parameters should be the focus of determining the way of finding control of water pollution in order to minimize the spread of water pollution in water bodies.

In this section, we analyze the dynamics of system of equations by carrying out numerical techniques and simulation using the parameter values. This is done because of its role in prediction of level of pollution by dissolving soluble and insoluble water pollutants into water bodies. The dynamics of water pollutant variables indicates that the accumulation of water contaminants does not shift dramatically as the amount of days rises. Also, soluble water contaminates rise rapidly in the initial stage then fall as time rises. Numerical techniques give evidence to variation of soluble and insoluble water pollutants the level of water pollution also varies.

4. Results and Discussions

Soluble water contaminates rise rapidly after a short fall down initially. It is likely that sharp spike in soluble water contaminates is due to interaction with water pollutants and transfer of insoluble water pollutants to solute. The decrease in soluble water contaminates, on the other hand, may be attributed to treatment of soluble water pollutants. Insoluble water contaminates rise slightly until the curve slopes downwards with time. Treatment of insoluble water contaminates to solute, as well as adequate hygine and schooling, may be adding to the down trend. Following graphs reveals that handling of water pollutants improves dramatically for several days before indreasingly declining as time passes. Treatment of soluble and insoluble water contaminates which account for the increase in treatment compartment.

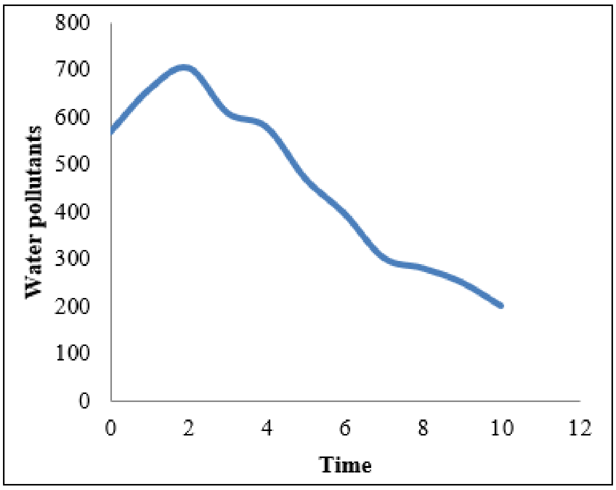

Figure 1: Water pollutants V/S Time

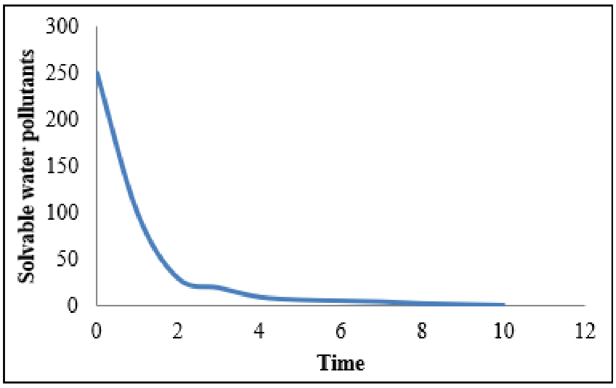

Figure 2: Solvable Water pollutants V/S Time

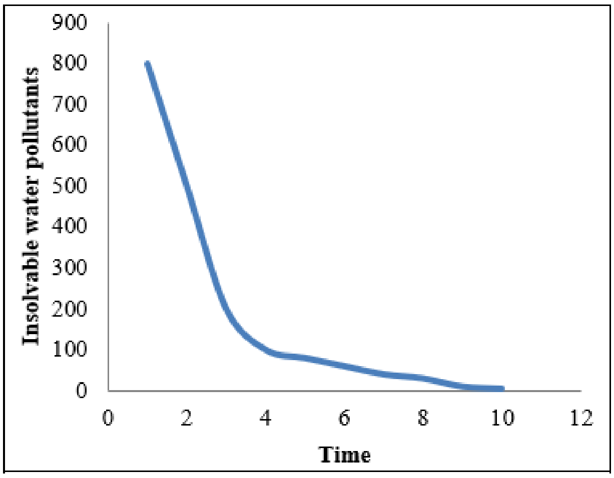

Figure 3: Insolvable Water pollutants V/S Time

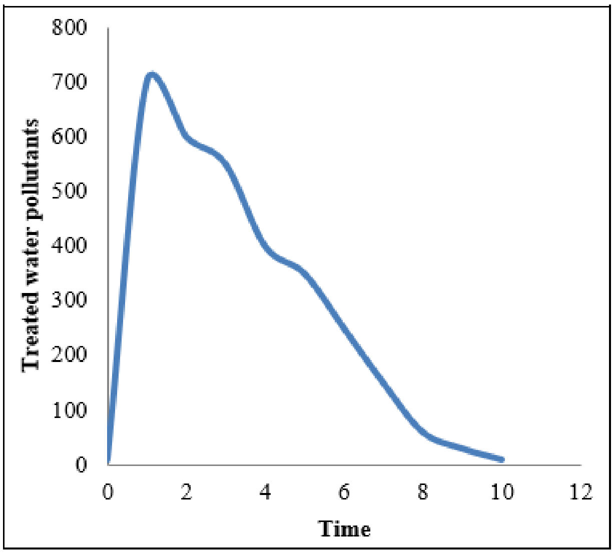

Figure 4: Treated Water pollutants V/S Time

References

[1] Mona Radwan, Patrick Willems and Jean Berlamont [2004], "Sensitivity and uncertainty analysis for river quality Modeling". Journal of Hydroinformatics, Volume 6, Issue 2, pp. 83 – 99.

[2] A K Misra, Peeyush Chandra and J B Shukla [2006], "Mathematical modeling and analysis of the depletion of dissolved oxygen in water bodies". Nonlinear Analysis: Real World Applications, Vol 7, pp 980 – 986.

[3] J.B. Shukla, A.K. Misra and Peeyush Chandra [2007], "Mathematical Modeling of the Survival of a Biological Species in Polluted Water Bodies". Differential Equations and Dynamical Systems, Vol. 15, Nos. 3 & 4, pp. 209-230.

[4] J.B. Shukla, A.K. Misra, Peeyush Chandra [2007], "Mathematical modeling and analysis of the depletion of dissolved oxygen in eutrophied water bodies affected by organic pollutants". Nonlinear Analysis: Real World Applications, Volume 9, pp.1851 – 1865.

[5] Busayamas Pimpunchat, Winston L. Sweatman, Graeme C. Wake, Wannapong Triampo, Aroon Parshotam [2009], "A mathematical model for pollution in a river and its remediation by aeration". Applied Mathematics Letters, Volume 22, pp. 304 – 308.

[6] W. J. S. Mwegoha, M. E. Kaseva and S. M. M. Sabai [2010], "Mathematical modeling of dissolved oxygen in fish ponds". African Journal of Environmental Science and Technology Vol. 4, Issue 9, pp. 625-638.

[7] Hemant Pathak And Sudhir Nagesh Limaye [2011], "A mathematical modeling with respect to DO for environmentally contaminated drinking water sources of Sagar city (M.P.), India: A case study". Ovidius University Annals of Chemistry Volume 22, Number 2, pp. 87-93.

[8] Fathi M Allan and Emad Elnajjar [2012], "The Role of Mathematical Modeling in Understanding the Groundwater Pollution". International Journal of Thermal & Environmental Engineering, Volume 4, Issue 2, pp.171-176.

[9] Hemanth Pathak [2013], "An Environ-Economical Mathematical Modeling Of Water Pollution Impact Assessment in Reference to Indian Scenario". Analele UniversitaŃii din Oradea – Seria Geografie. Volume 1(2013), pp. 168 – 171.

[10] Urvasini Singh, Vandana Sharma, Shruthi Bhandari, Jayashri Vajpai and Sunita Kumbhat [2014], "Absorbance Based Model for Determination of Biochemical Oxygen Demand". British Journal of Applied Science & Technology, Volume 4, Issue 31, pp 4408 – 4419.

[11] Rajat Kaushik [2015], "Mathematical modelling on water pollution and self-purification of river Ganges". Advances in Applied Science Research, Volume 6, Issue 7, pp 57 – 64.

[12] Witsarut Kraychang and Nopparat Pochai [2016], "Implicit Finite Difference Simulation of Water Pollution Control in a Connected Reservoir". International Journal of Applied Mathematics, Volume 46, Issue 1, pp 1-12.

[13] Y Yamin, A Permanasari, S Redjeki and W Sopandi [2017], "Application of Model Project Based Learning on Integrated Science in Water Pollution". International Conference on Mathematics and Science Education (ICMScE) IOP Publishing.

[14] Delong Wan and Huiping Zeng [2018], "Water environment mathematical model mathematical algorithm". 2nd International Symposium on Resource Exploration and Environmental Science IOP Publishing.

[15] M Z Mokhtar, M S Sofiana, W O WanMaznah and S Y The [2019], "Water quality assessment of Bukit Merah Reservoir, Malaysia using mathematical modeling". International Conference on Tropical Limnology, IOP Conf. Series: Earth and Environmental Science 535 (2020) 012023.

[16] Ebenezer Bonyah, Paul Agbekpornu and Canan Unlu [2021], "Mathematical Modeling of Transmission of Water Pollution". Journal of Prime Research in Mathematics, Volume 17, Issue 2, pp 20 – 38.