

Bondage and Connectivity Analysis of Polar Satellite Systems Using Intuitionistic Fuzzy Graph Theory

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Abstract: *This paper presents a connectivity and risk analysis of polar satellite systems using Intuitionistic Fuzzy Graph (IFG). The model represents satellites and their communication links with membership and non-membership values to handle uncertainty in satellite connectivity and operational reliability. Graph connectivity, strong edges, and vulnerable links are analyzed to identify potential risks and critical communication pathways. The proposed approach provides a mathematical framework for assessing network robustness and supporting reliable satellite communication under uncertain conditions.*

Keywords: Intuitionistic Fuzzy Graphs, Polar Satellite Systems, Network Connectivity, Risk Analysis, Strong Edges, Satellite Communication, Network Reliability, Uncertainty Modelling

1. Introduction

Graph theory is an important branch of mathematics that studies relationships between objects using vertices and edges. It has wide applications in computer science, engineering, communication networks, transportation systems, and scientific modelling. Classical graph theory provides mathematical tools for analysing connectivity, shortest paths, cycles, and network reliability. However, in practical systems, relationships between objects may involve uncertainty, incomplete information, and varying levels of reliability. These limitations motivate the use of fuzzy graph models for representing complex real-world networks.

Fuzzy set theory, introduced by Lotfi A. Zadeh in 1965, provides a mathematical framework for representing uncertainty through degrees of membership. Building on this concept, A. Rosenfeld introduced fuzzy graph theory in 1975 by incorporating fuzzy sets into graph structures. Fuzzy graphs allow vertices and edges to possess different membership values, making them suitable for modelling relationships whose strength cannot be represented precisely using classical graphs.

Intuitionistic fuzzy set theory, introduced by K. T. Atanassov in 1986, extends fuzzy set theory by incorporating both membership and non-membership degrees. This framework provides a more comprehensive representation of uncertainty, subject to the condition that the sum of the membership and non-membership degrees does not exceed one. Intuitionistic fuzzy graphs (IFGs) apply these principles to graph structures, enabling the representation of both the strength and the uncertainty associated with connections between vertices. These graphs are useful in analysing complex systems in which connectivity and reliability are influenced by incomplete or imprecise information.

Polar satellite systems play an important role in Earth observation, environmental monitoring, weather forecasting, disaster management, navigation support, and scientific research. These systems involve satellites, ground stations,

communication links, and data transmission pathways that work together to provide essential services. Their performance may be affected by communication interruptions, equipment failures, orbital debris, signal degradation, and other operational risks. Therefore, analysing the connectivity and vulnerability of polar satellite systems is essential for understanding their reliability and identifying critical communication pathways.

The resulting model enables the analysis of network connectivity, strong edges, alternative communication routes, and potentially vulnerable connections. Such an approach can support the identification of critical links whose failure may affect communication and the overall performance of the satellite network.

In particular, connectivity analysis helps determine whether communication can be maintained through alternative pathways when certain links become unavailable. Risk analysis based on intuitionistic fuzzy graphs can further investigate how uncertainty in individual connections influences the structural robustness of the network. Concepts such as cycles, strong connectivity, and edge removal can be incorporated to examine the vulnerability of communication pathways and the resilience of the overall system.

The present study, entitled Connectivity and Risk Analysis of Polar Satellite Systems Using Intuitionistic Fuzzy Graph Theory, aims to develop an intuitionistic fuzzy graph model for representing the connectivity structure of polar satellite systems and analysing their associated risks. The study focuses on the representation of communication links through membership and non-membership values and the investigation of critical connections and alternative pathways. By applying graph-theoretical concepts within an intuitionistic fuzzy framework, the proposed approach seeks to provide a mathematical basis for evaluating network connectivity, identifying vulnerable links, and understanding the robustness of satellite communication systems under uncertainty.

2. Preliminaries

In this section, some basic definitions and theorems which are used in constructing the properties relating to IFG and bondage and non-bondage sets are given.

Definition 2.1 [13] Intuitionistic fuzzy graph (IFG) is of the form $G = (V, E)$, where $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_i : V \rightarrow [0, 1]$ and $\nu_i : V \rightarrow [0, 1]$ denotes the degrees of membership and non-membership of the vertex $v \in V$ respectively and $0 \leq \mu_i + \nu_i \leq 1$, for every $v_i \in V$ ($i = 1, 2, \dots, n$). $E \subset V \times V$ where $\mu_{ij} : V \times V \rightarrow [0, 1]$ and $\nu_{ij} : V \times V \rightarrow [0, 1]$ are such that $\mu_{ij} \leq \min\{\mu_i, \mu_j\}$, $\nu_{ij} \leq \max\{\nu_i, \nu_j\}$, $0 \leq \mu_{ij} + \nu_{ij} \leq 1$, for all $(v_i, v_j) \in E$.

Note: Here the triple (v_i, μ_i, ν_i) denotes the degree of membership and degree of non-membership of the vertex v_i . The triple $(e_{ij}, \mu_{ij}, \nu_{ij})$ denotes the degree of membership and non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on E .

Definition 2.2 [9]

Let $G = (V, E)$ is said to be a complete intuitionistic fuzzy graph if $\mu_{ij} = \min\{\mu_i, \mu_j\}$, and $\nu_{ij} = \max\{\nu_i, \nu_j\}$ for every $(v_i, v_j) \in V$.

Definition 2.3 [9]

Let $G = (V, E)$ be an IFG on V . Let $v_i, v_j \in V$, v_i dominates v_j in G if there exists a strong edge between them.

Definition 2.4 [9]

Let $G = (V, E)$ be an IFG. A subset S of V is called a dominating set in G , if for every $v_j \in V - S$, there exists $v_i \in S$ such that v_i dominates v_j .

Definition 2.5 [7]

Let $G = (V, E)$ be an intuitionistic fuzzy graph (IFG), where $\mu_1(v_i)$ and $\nu_1(v_i)$ denote the membership and non-membership values of a vertex v_i , respectively, and $\mu_2(v_i, v_j)$ and $\nu_2(v_i, v_j)$ denote the membership and non-membership values of an edge (v_i, v_j) . The graph G is called a semi- μ strong intuitionistic fuzzy graph if, for every edge $(v_i, v_j) \in E$, $\mu_2(v_i, v_j) = \min\{\mu_1(v_i), \mu_1(v_j)\}$.

Definition 2.6 [7]

Let $G = (V, E)$ be an intuitionistic fuzzy graph. The graph G is said to be connected if there exists a path between every pair of distinct vertices in V . The strength of a path is determined by the strengths of its edges, taking into account their membership and non-membership values. The precise path-strength convention should follow the IFG framework adopted in the study.

Definition 2.7

Let $G = (V, E)$ be an intuitionistic fuzzy graph, and let $\eta(G)$ denote its domination number. A subset B of E_s , where E_s is the set of strong edges of G , is called a bondage set if removing the edges in B increases the domination number of G .

$$\eta(G - B) > \eta(G)$$

Definition 2.8

The bondage number of an intuitionistic fuzzy graph G , denoted by $b(G)$, is the minimum cardinality of a bondage set.

$$b(G) = \min\{|B| : B \subseteq E_s, \eta(G - B) > \eta(G)\}$$

Definition 2.9

Let $G = (V, E)$ be an intuitionistic fuzzy graph, and let $\eta(G)$ denote its domination number. A subset N of E_s is called a non-bondage set if deleting the edges in N does not increase the domination number of G .

$$\eta(G - N) \leq \eta(G)$$

Definition 2.10

The non-bondage number of an intuitionistic fuzzy graph G , denoted by $bn(G)$, is the maximum cardinality of a non-bondage set of strong edges.

$$bn(G) = \max\{|N| : N \subseteq E_s, \eta(G - N) \leq \eta(G)\}$$

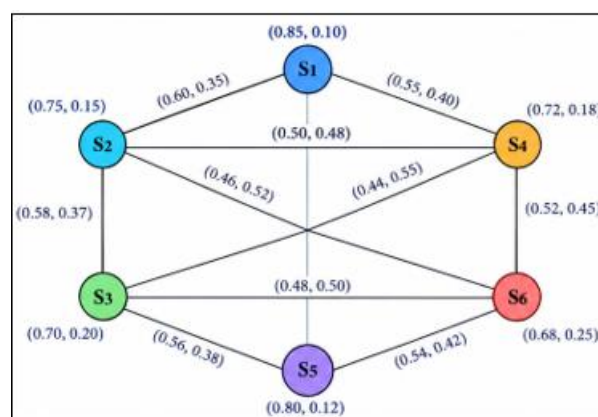
Notation. E_s denotes the set of strong edges of G . The symbols $|B|$ and $|N|$ denote the cardinalities of the corresponding edge sets, and $G - B$ (or $G - N$) denotes the graph obtained by deleting the specified edges.

3. Applications of Polar Satellite Systems in Connectivity

Polar satellite systems provide global connectivity for communication, environmental monitoring, weather forecasting, disaster management, and remote-area surveillance. They facilitate data transmission between satellites and ground stations, supporting reliable information exchange across geographically distant regions. Intuitionistic Fuzzy Graph Theory can model these systems by representing satellites as vertices and communication links as edges with membership and non-membership values. This approach helps analyse connectivity, identify vulnerable links, and evaluate network reliability under uncertainty.

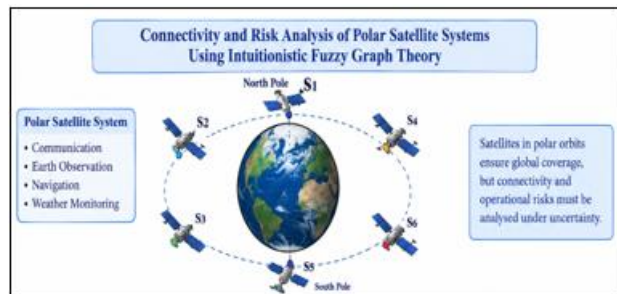
Use IFG for Polar Satellite Risk Analysis

The core advantage of Intuitionistic Fuzzy Graph over traditional fuzzy graphs lies in simultaneously characterizing membership degree (μ , e.g., link reliability/communication quality) and non-membership degree (ν , e.g., link failure probability/interference risk), with the constraint $\mu + \nu \leq 1$. The remaining portion $1 - \mu - \nu$ represents the hesitation margin, which can be used to characterize uncertainty in polar regions due to harsh environments, solar radiation, or sudden equipment failures.



For polar satellite systems, inter-satellite links often face:

- **High-latitude coverage overlap and handover:** Frequent switching between satellites.
- **Polar region physical limitations:** Inter-satellite link stability is affected by orbital convergence.
- **Ground station distribution is sparse:** Ground-satellite links are critical single points.



Node Attributes (μ, ν)

Node	Meaning	μ	ν
S1	Polar satellite 1	0.9	0.05
S2	Polar satellite 2	0.8	0.15
S3	Polar satellite 3	0.7	0.2
S4	Polar satellite 4	0.85	0.1
G	Ground gateway	0.95	0.02

Edge Attributes (μ, ν)

Edge	Meaning	μ	ν
S1-S2	Same-orbit ISL	0.85	0.1
S2-S3	Inter-orbit link	0.7	0.25
S3-S4	Same-orbit ISL	0.8	0.15
S1-S4	Cross-orbit backup	0.6	0.3
S2-G	Primary downlink	0.9	0.08
S4-G	Backup downlink	0.75	0.2

Risk Findings

- S1-S4 ($\nu = 0.30$) is the highest-risk link; hesitation margin 0.10.
- S2-S3 ($\nu = 0.25$) is the weakest inter-orbit link.
- S2-G is the most reliable critical downlink ($\mu = 0.90$, $\nu = 0.08$).
- S1 and G are the most stable nodes (highest μ).

Risk Interpretation

IFG Element	(μ, ν)	Role in Polar System
S1, G nodes	$(0.90, 0.05)$, $(0.95, 0.02)$	Core stable nodes
S1-S2 edge	$(0.85, 0.10)$	Reliable same-orbit ISL
S2-G edge	$(0.90, 0.08)$	Best downlink
S3-S4 edge	$(0.80, 0.15)$	Medium-risk ISL
S4-G edge	$(0.75, 0.20)$	Backup downlink
S2-S3 edge	$(0.70, 0.25)$	Weakest inter-orbit link
S1-S4 edge	$(0.60, 0.30)$	Highest-risk cross-orbit backup

Connectivity Analysis

- A path exists between any two satellites through strong links.
- The network is connected (in the intuitionistic fuzzy sense) with high path strengths.

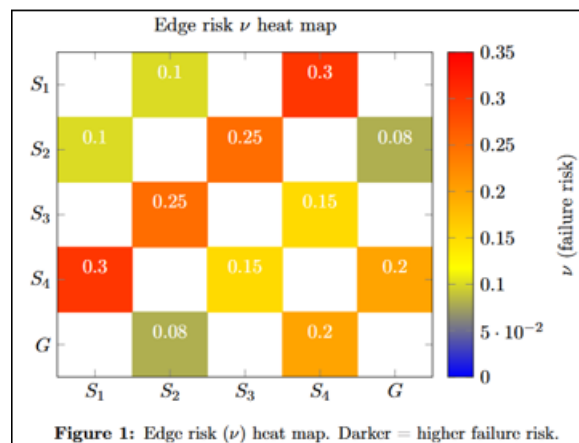
Risk Analysis

- Low membership / high non-membership values indicate higher risk.
- Critical edges/satellites are identified using bondage and non-bondage sets.
- Risk is assessed through edge/vertex removal and the effect on network connectivity and mission continuity.

Conclusion from the IFG: The polar convergence zone creates a **high-risk cluster** on the cross-orbit edges (S1-S4, S2-S3). The reliable backbone runs through S1-S2-G, so redundancy should be added along the S3/S4 side.

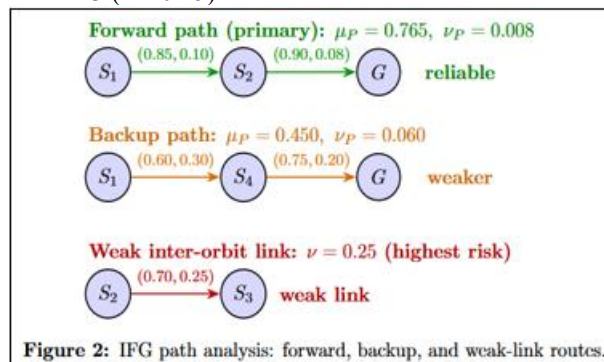
Edge Risk Heat Map

Figure shows the pairwise edge risk ν . Darker cells indicate higher failure risk; the S1-S4 and S2-S3 entries stand out



Path Analysis Diagram

Figure compares the forward path (S1-S2-G, $\mu_P = 0.765$, $\nu_P = 0.008$), the backup path (S1-S4-G, $\mu_P = 0.450$), and the weak inter-orbit link S2-S3 ($\nu = 0.25$).

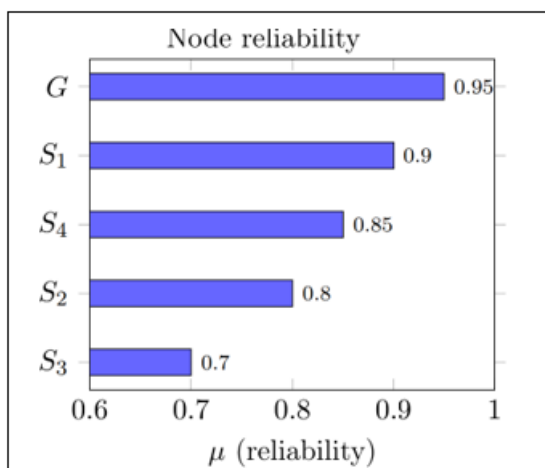
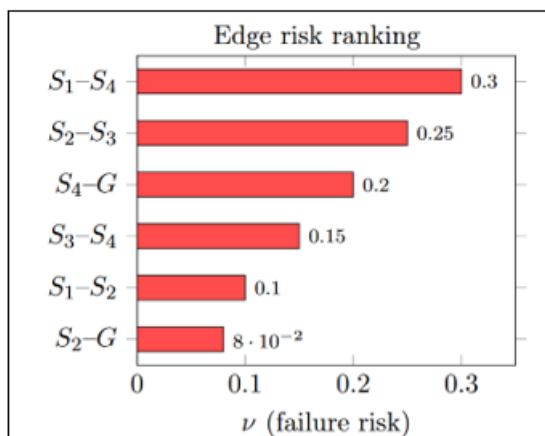


Risk and Reliability Rankings

Figure ranks edges by ν and nodes by μ , highlighting the critical S1-S4 link and the stable S1/G nodes.

Table 1: Path Strength Summary

Path	Edges	μ_{path}	ν_{path}
Forward (primary)	S_1-S_2-G	0.765	0.008
Backup	S_1-S_4-G	0.450	0.060
Weak link	S_2-S_3	0.700	0.250

**1 Path Strength Analysis**

Using the product t -norm of Eq. (1):

Forward Path

$$\text{Forward Path } S_1 \xrightarrow{(0.85, 0.10)} S_2 \xrightarrow{(0.90, 0.08)} G:$$

$$\mu_{\text{path}} = 0.85 \times 0.90 = 0.765,$$

$$\nu_{\text{path}} = 0.10 \times 0.08 = 0.008,$$

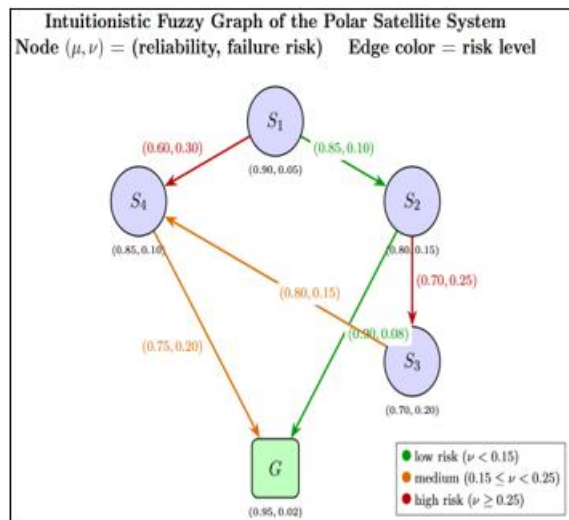
$$\text{Backup Path } S_1 \xrightarrow{(0.60, 0.30)} S_2 \xrightarrow{(0.75, 0.20)} G:$$

$$\mu_{\text{path}} = 0.60 \times 0.75 = 0.450,$$

$$\nu_{\text{path}} = 0.30 \times 0.20 = 0.060,$$

$$\text{Weak Link } S_2 \xrightarrow{(0.70, 0.25)} S_3$$

$\nu = 0.25$ (Highest inter-orbit risk in the constellation)



Intuitionistic Fuzzy Graph (IFG) representing the connectivity and risk of the polar satellite system. Nodes are labeled with their intuitionistic fuzzy attributes (μ, ν) , and edges are labeled with link reliability and failure risk. Edge colors encode risk: green (low), orange (medium), and red (high).

IFG Adjacency Matrix

The following matrix represents the membership μ (upper value) and non-membership ν (lower value) for each edge. A dash (–) indicates no direct link.

	S_1	S_2	S_3	S_4	G
S_1	–	0.85/0.10	–	0.60/0.30	–
S_2	0.85/0.10	–	0.70/0.25	–	0.90/0.08
S_3	–	0.70/0.25	–	0.80/0.15	–
S_4	0.60/0.30	–	0.80/0.15	–	0.75/0.20
G	–	0.90/0.08	–	0.75/0.20	–

Table 1: IFG adjacency matrix (edge μ/ν).

4. Future Scope

- Theoretical Extensions:** Extend scalar (μ, ν) to interval-valued, Pythagorean, neutrosophic, and hesitant IFGs to better capture polar-link uncertainty.
- Dynamic & Temporal IFG:** Introduce time-varying $\mu(t)$, $\nu(t)$ per orbital pass and temporal IFG (TIFG) for intermittent polar links; add predictive IFG using orbital mechanics and space-weather data.
- Machine Learning on IFG:** Apply Graph Neural Networks (GNNs) with (μ, ν) edge features for failure prediction, reinforcement learning for autonomous routing, and IFG-based autoencoders for anomaly detection.
- Multi-Criteria Decision Making:** Use IFG-TOPSIS, IFG-AHP, and IFG-VIKOR for constellation selection, ground-station siting, and routing-path ranking.
- Resilience Metrics:** Develop IFG connectivity index, domination integrity, edge/node-connectivity, and vulnerability index as single-scalar resilience scores.
- Real-Time Autonomy:** Implement an on-board IFG router (FPGA/ASIC), live IFG dashboard, and IFG-based

handover for seamless polar connectivity; integrate with SDN.

- 7) **Quantum & Secure Comms:** Rank QKD links by μ (key rate) and ν (eavesdrop risk); apply IFG for anti-jamming and post-quantum routing.
- 8) **Multi-Constellation IFG:** Combine Starlink, OneWeb, and Iridium polar shells into a cross-operator IFG; integrate with 6G non-terrestrial networks.
- 9) **Validation & Benchmarks:** Publish open IFG datasets from polar missions, build a benchmark suite, and validate on a CubeSat polar flight.

5. Conclusion

The IFG framework provides a unified representation of connectivity and risk for polar satellite systems. By simultaneously encoding reliability (μ) and failure risk (ν), the model identifies the cross-orbit links S1–S4 and S2–S3 as the critical risk cluster and confirms S1–S2–G as the most reliable forward path. These results support redundancy planning, adaptive routing, and ground-station placement decisions for polar constellations.

References

- [1] K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20(1), 1986, 87–96.
- [2] K. Atanassov, Intuitionistic fuzzy sets, *Theory and Applications*, Heidelberg, Physical-Verlag, New York, 35(1) 1999, 1–137.
- [3] Arya Sebastain, J.N. Mordeson and Sunil Mathew, Generalized fuzzy graph connectivity parameters with application to human trafficking, *MDPI*, 8(3), 2020, 424–433.
- [4] R. Buvaswari and K. Umamaheswari, Bondage and non-bondage sets in regular intuitionistic fuzzy graphs, *Notes on Intuitionistic Fuzzy Sets*, 29(3), 2023, 318–324.
- [5] R. Jemimal Chrislight and Y. Therese Sunitha Mary, The non-split bondage number of central graph, *Malaya Journal of Matematik*, 1, 2020, 133–137.
- [6] M. G. Karunambigai, M. Akram and R. Buvaswari, Strong and super strong vertices in intuitionistic fuzzy graphs, *Journal of Intelligent and Fuzzy Systems*, 30, 2016, 671–678.
- [7] M.G. Karunambigai, R. Parvathi and R. Buvaswari, Arcs in intuitionistic fuzzy graphs, *Notes on Intuitionistic Fuzzy Sets*, 18(4), 2012, 48–58.
- [8] A. Nagoorgani, K. Prasanna Devi and Muhammad Akram, Bondage and non-bondage
- [9] number of a fuzzy graph, *Int. J. Pure Appl. Math.*, 103, 2015, 215–226.
- [10] L. A. Zadeh, *Fuzzy sets and their applications to cognitive and decision processes*, Academic Press, 1975, 125–149.
- [11] H. Zehui Shao and T. Saeed Kosari, New concepts in intuitionistic fuzzy graph application in water supplier systems, *MDPI*, 8(8), 29, 2020, 1–17.
- [12] H. Zehui Shao and T. Saeed Kosari, A study on domination in vague incidence graph and its application in medical sciences, 12(11), 2022, 85–97