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Application of Fuzzy If-Then Rule in a Fuzzy Graph with Modus Ponens

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Abstract: ***In this work we discussed about fuzzy version of classical graph theory. Here we introduced fuzzy If-Then rules with Modus Ponens method and related results are proved.***

Keywords: Fuzzy set, Fuzzy graph, Modus Ponens, Connected Fuzzy graph.

1. Introduction

One of the remarkable mathematical inventions of the 20th century is that of Fuzzy sets by Lotfi.A.Zadeh in 1965. His aim was to develop a mathematical theory to deal with uncertainty and imprecision. The advantage of replacing the classical sets by Zadeh's fuzzy sets is that it gives more accuracy and precision in theory and more efficiency and system compactability in applications. So in systems with imprecision, a fuzzy set model is more valuable than a classic model. The distinction between set and fuzzy set is that the set divide the universal set into two subsets, namely members and non-members while fuzzy set assigns a sequence of membership values to elements of the universal set ranging from 0 to 1. That is partial memberships are allowed in the latter. Also fuzzy sets can be used effectively to study quality variables like intelligence, beauty, consistency, etc., Zadeh's paper "Fuzzy sets" also proved the way to a new philosophical thinking of Fuzzy logic which now, is an essential concept in artificial intelligence. In coming years, fuzzy logic is likely to grow in visibility, importance and acceptance.

Starting from the fact of fuzzy set and the If-Then rule, we can derive a new result is said to be the Modus Ponens method in a fuzzy set theory. The Modus Ponens is used in the forward inference. In general, the "inference" is a process to obtain new information by using existing knowledge. The representation of knowledge is an important issue in the inference.

The first definition of fuzzy graph by Kaufman(1973) was based on Zadeh's fuzzy relations (1971). But it was Azrial Rosenfeld (1975) who considered fuzzy relation on fuzzy sets and developed the theory of fuzzy graphs. The author introduced fuzzy analogues of several graph theoretic concepts such as subgraphs, paths and connectedness, cliques, bridges and cut nodes, forest and trees, etc. During the same time, Yeh and Bang (1975) also introduced fuzzy graphs independently and studied various connectedness concepts such as connectivity matrix, reachability matrix, degree of connectivity, edge connectivity, vertex connectivity etc. These results are applied directly to clustering analysis and modeling of information networks. The connectedness and acyclicity levels of a fuzzy tree are introduced by Delgado, verdegay and Villa (1975).

2. Preliminaries

Definition 2.1: Let S be a nonempty set. A fuzzy subset of S is a mapping $\sigma : S \to [0,1]$ which assigns to each element $x \in S$, a degree of membership, $\sigma(x)$, such that $0 \leq \sigma(x) \leq 1$.

In the special case where σ can only take on the value 0 and 1, it become the characteristic function of an ordinary subset of S.

Example2.1:

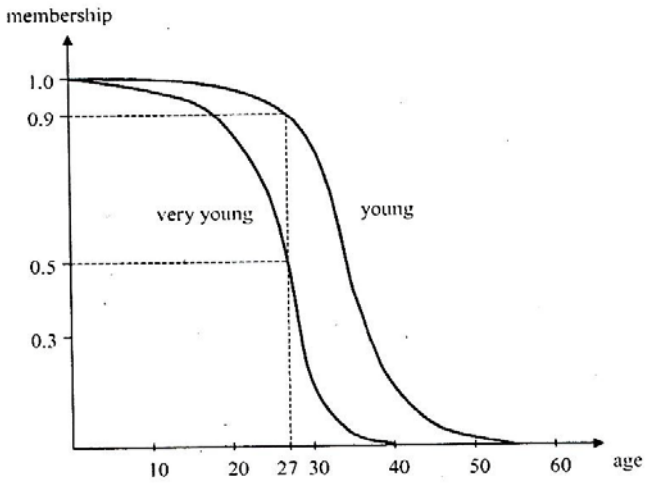


Figure 1: Fuzzy set

We consider statement "Kamalini is young". At this time, the term "Young" is vague. To represent the meaning of "vague" exactly, it would be necessary to define its membership function as in the above figure. When we refer "young", there might be age which lies in the range [0,80] and we can account these "young age" in these scope as a continuous set.

The horizontal axis shows age and the vertical one means the numerical value of membership function. The line shows

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possibility (value of membership function) of being contained in the fuzzy set "young". For example, if we follow the definition of "young" as in the figure, ten year-old boy may well be young. So the possibility for the "age ten" to join the fuzzy set of "young" is 1, also that of "age twenty seven" is 0.9. But we might not say young to a person who is over sixty and the possibility of this case is 0.

Now we can manipulate our last sentence to "Jenny is very young". In order to be included in the set of "very young", the age should be lowered and let us think the line is moved leftward as in the figure. If we define fuzzy set as such, only the person who is under forty years old can be included in the set of "very young". Now the possibility of twenty- seven year old man to be included in this set is 0.5.

That is, if we denote A = "young" and B = "very young", $\mu_A(27)=0.9$, $\mu_B(27)=0.5$.

Definition 2.2: Let X,Y⊆R be universal set, then R={((x,y), $\mu_R(x,y)$): x,y∈X×Y} is said to be a fuzzy relation from X to Y.

Definition 2.3: Let μ *be a fuzzy* relation on σ. *Then* μ is said to be reflexive if, μ(x,x) = σ(x), ∀ x∈S. If μ is reflexive, then it follows that $\mu(x,y) \leq \mu(x,x)$ and $\mu(y,x) \leq \mu(x,x)$, ∀ x∈S.

Definition 2.4: A fuzzy relation μ is said to be symmetric if $\mu(x,y) \leq \mu(y,x)$, ∀ x,y∈S.

Definition 2.5: A fuzzy graph(f-graph) is a pair G: (σ,μ) where σ is a fuzzy subset of a set S and μ *is a* fuzzy relation on σ. It is assumed that S is finite and nonempty, μ is reflexive and symmetric. Thus if G : (σ,μ) is a fuzzy graph, then $\sigma : S \to [0,1]$ and $\mu : S\times S \to [0,1]$ is such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u, $v \in S$.
Also, we denote the underlying graph of the fuzzy graph G: (σ,μ) by G*: (σ*,μ*) where σ* = {u ∈ S : σ(u) > 0} and μ^* = {(u,v) ∈ S×S : μ(u,v) > 0}. In examples, if σ is not specified, it is chosen suitably. Also G : (σ,μ) is said to be a trivial fuzzy graph if G*: (σ*,μ*) is trivial. That is σ^* is a singleton set.

Example2.5:

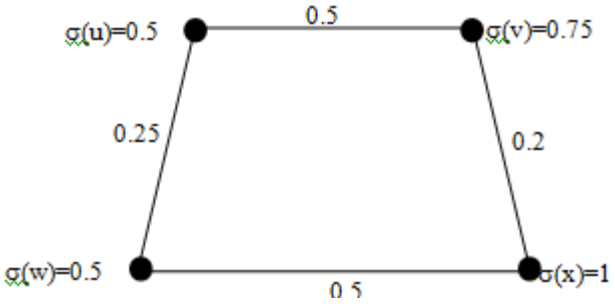


Figure 2: Fuzzy graph

Let G : (σ,μ) be with σ* = {u,v,w,x}. Let σ(u) = 0.5, σ (v) = 0.75, σ(w) = 0.5, σ(x) =1and μ(u,v) = 0.5, μ(v,x) = 0.2, μ(w,x) = 0.5, μ(w,u) = 0.25 . Then G is a fuzzy graph. since $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u,v ∈ σ*.

Definition 2.6: In a fuzzy graph G : (σ,μ), a path P of length n is a sequence of distinct nodes $u_0,u_1,\ldots,u_n$ such that $\mu(u_{i-1},u_i) > 0$, i=1,2,…,n and the degree of membership of a weakest arc is defined as its strength.

Example2.6:

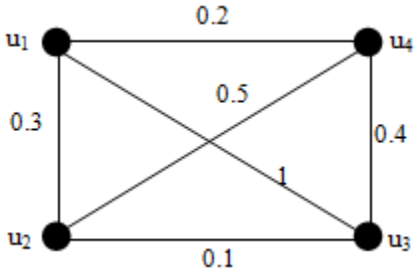


Figure 3: Fuzzy path

Let G: (σ,μ) be a fuzzy graph with σ* = (u_1,u_2,u_3,u_4). Let $\mu(u_1,u_2) = 0.3$, $\mu(u_2,u_3) = 0.1$, $\mu(u_3,u_4) = 0.4$, $\mu(u_4,u_1) = 0.2$. There are three different paths from u_1 to u_3 namely, $P_1 = u_1,u_2,u_3$; P_2 = arc (u_1,u_3); $P_3 = u_1,u_4,u_3$. Now strength of P_1 = min{0.3,0.1} = 0.1, strength of P_2 =1, and strength of P_3 = min{0.2,0.4} = 0.2. So the strongest path joining u_1 and u_3 is the arc (u_1,u_3) with strength 1.

Definition 2.7: A complete fuzzy graph (CFG) is a fuzzy graph G : (σ,μ) such that $\mu(x,y) = \sigma(x) \wedge \sigma(y)$ for all x and y. Note that if G is a Complete fuzzy graph, then G* is also a complete graph.

Example2.7:

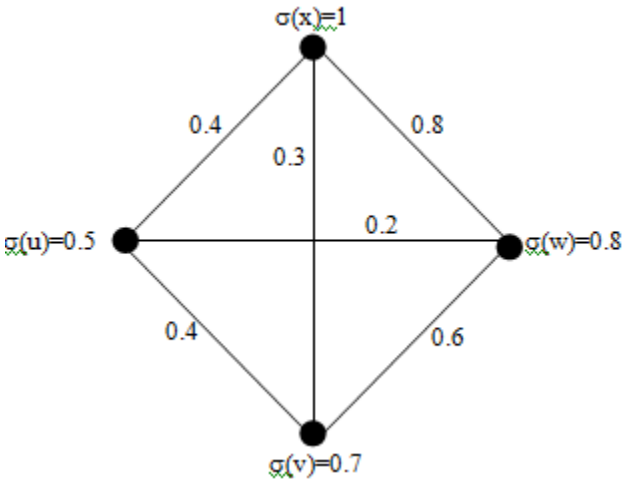


Figure 4: Complete Fuzzy graph

Let G: (σ,μ) be a fuzzy graph with σ* = (u,v,w,x). σ(u)=0.5, σ(v)=0.7, σ(w)=0.8, σ(x)=1. μ(u,v) = 0.5, μ(v,w) = 0.7, μ(w,x) = 0.8, μ(x,u) = 0.5, μ(u,w) = 0.5, μ(v,x) = 0.7. Then G is a Complete fuzzy graph.

Definition 2.8: The degree of any node $\sigma(v_i)$ of a fuzzy graph is sum of degree of membership of all those edges which are incident on a node $\sigma(v_i)$. And is denoted by $d(\sigma(v_i))$. A node of odd degree is an odd node and a node of even degree is an even node.

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Example 2.8:

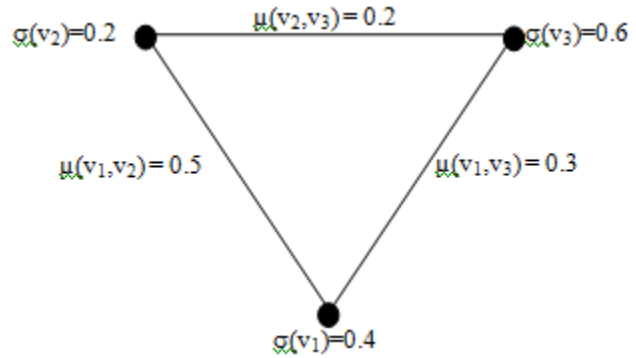


Figure 5: G

Degree of vertex $\sigma(v_2)$ = degree of membership of all those edges which are incident on a vertex $\sigma(v_2)$
$= \mu(v_1,v_2)+ \mu(v_2,v_3)$
$= 0.5+0.2=0.7$
i.e. $d[\sigma(v_2)] = 0.7$.

Definition 2.9: Let G : (σ,μ) be a fuzzy graph. The strength of connectedness between two nodes x and y is defined as the maximum of the strength of all paths between x and y and is denoted by $CONN_G(x,y)$. An x-y path P is called a strongest x-y path if its strength equals $CONN_G(x,y)$. An fuzzy graph G:(σ,μ) is connected if for every x,y in σ^*, $CONN_G(x,y) > 0$. A maximal connected fuzzy subgraphs are called components. Clearly if G is connected, then any two nodes are joined by a path.

Example2.9:

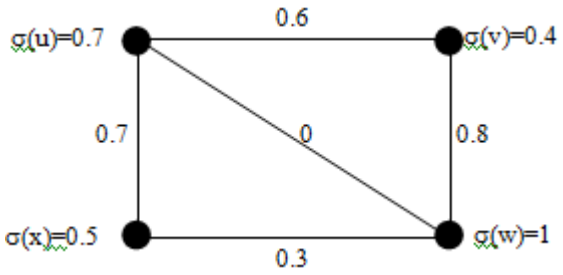


Figure 6: Connected Fuzzy Graph

The path between u and v nodes are {uv, uxwv, uwv}
$CONN_G(u,v) = \max \{ uv, uxwv, uwv \}$
$CONN_G(u,v) = \max [0.6, 0.3, 0\}$
$CONN_G(u,v) = 0.6$. Similarly, $CONN_G(u,w)=0.6$, $CONN_G(u,x)= 0.7$, $CONN_G(v,w) = 0.8$, $CONN_G(v,x) = 0.6$, $CONN_G(w,x) = 0.6$

Definition 2.10: The modus ponens (MP)
Fact : x is A
Rule : IF x is A THEN y is B
Result : y is B
is a well- known deduction rule in (Boolean) logic. From the fact "x is A" and the IF- THEN rule "IF x is A THEN y is B", we can derive a new fact, namely "y is B,". However, if we do not exactly known that "x is A", we cannot make any deduction concerning y, even if we would have tons of other information on x. This implies that if we want to develop a useful derivation system (e.g.. for a computer- controlled car) based on this MP, we have provide an IF-THEN rule for each possible A. Needless to say this would be highly inefficient, if not impossible.

3. Fuzzy If-Then Rule with Modus Ponens Method

Theorem3.1: If a fuzzy graph G : (σ,μ) has exactly two nodes of odd degree, then they must be connected by a path.

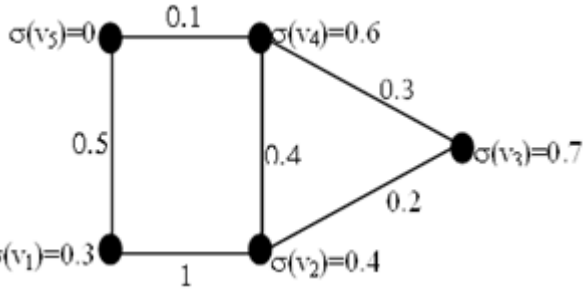
Proof:
We can prove this theorem by using Modus ponens Method:

Rule 1:
Fact : x is a fuzzy graph G : (σ,μ) with all its nodes of even degree, except for $\sigma(v_1)$ and $\sigma(v_2)$ which are of odd degree and also G_1 is the component with $\sigma(v_1)$.

Rule :**IF** x is a fuzzy graph G : (σ,μ) with all its nodes of even degree, except for $\sigma(v_1)$ and $\sigma(v_2)$ which are of odd degree and also G_1 is the component with $\sigma(v_1)$ **THEN** y is a fuzzy graph in which G_1 has an even number of nodes of odd degree.

Result: y is a fuzzy graph in which G_1 has an even number of nodes of odd degree.

Example:



$d[\sigma(v_1)] = 1+0.5 =1.5$
$d[\sigma(v_2)] = 1+0.4+0.2 =1.6$
$d[\sigma(v_3)] = 0.2+0.3 =0.5$
$d[\sigma(v_4)] = 0.4+0.3+0.1 =0.8$
$d[\sigma(v_5)] = 0.5+0 .1 =0.6$

Here the nodes $\sigma(v_1)$ and $\sigma(v_2)$ are of odd degree and the remaining nodes $\sigma(v_3),\sigma(v_4),\sigma(v_5)$ are of even degree.

Consider the component G_1

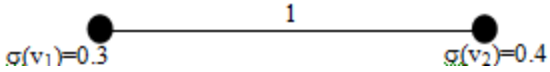


Figure 8:G_1

Here $\sigma(v_1) \in G_1$ and G_1 has an even number of nodes of odd degree.

Rule 2:
Fact: x is a fuzzy graph having component G_1 with node $\sigma(v_1)$.

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Rule: **IF** x is a fuzzy graph having component G_1 with node $\sigma(v_1)$ **THEN** y is also a fuzzy graph having component G_1 must contain the node $\sigma(v_2)$, the only other node of odd degree.

Result: y is also a fuzzy graph having component G_1 must contain the node $\sigma(v_2)$, the only other node of odd degree.

Example:

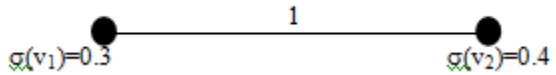


Figure 9:G_1

i.e. $\sigma(v_2) \in G_1$
$d[\sigma(v_1)] = 1$
$d[\sigma(v_2)] = 1$

Rule3:
Fact: x is a fuzzy graph with exactly two nodes $\sigma(v_1)$ and $\sigma(v_2)$ are of odd degree.

Rule: **IF** x is a fuzzy graph with exactly two nodes $\sigma(v_1)$ and $\sigma(v_2)$ are of odd degree **THEN** y is also a fuzzy graph in which of $\sigma(v_1)$ and $\sigma(v_2)$ are must be connected by a path.

Result: y is also a fuzzy graph in which of $\sigma(v_1)$ and $\sigma(v_2)$ are must be connected by a path.

Example:

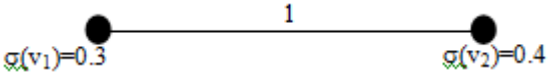


Figure 10: G_1

i.e. $\sigma(v_1), \sigma(v_2) \in G_1$.
Since a component is connected, there is a path between $\sigma(v_1)$ and $\sigma(v_2)$.

Hence a fuzzy graph G : (σ,μ) of the graph G : (V,E) has exactly two nodes of odd degree, they must be connected by a path.

Hence proved the Theorem.

Lemma3.2: For any set of positive integers $n_1,n_2,\ldots,n_k$. If a fuzzy graph G : (σ,μ) of a graph G : (V,E). Then
$\sum_{i=1}^{k}\sigma(n_i^2) \leq (\sum_{i=1}^{k}\sigma(n_i))^2 - (k-1)\,(2\sum_{i=1}^{k}\sigma(n_i) - k)$.

Proof:

We have, $\sum_{i=1}^{k}(\sigma(n_i) - 1) = \sum_{i=1}^{k}\sigma(n_i) - k$

Squaring on both sides, we get

$[\sum_{i=1}^{k}(\sigma(n_i) - 1)]^2 = [\sum_{i=1}^{k}\sigma(n_i) - k]^2$,

or $[(\sigma(n_1)-1) + (\sigma(n_2)-1) + \ldots + (\sigma(n_k)-1)]^2 = [\sum_{i=1}^{k}\sigma(n_i)]^2 - 2k\sum_{i=1}^{k}\sigma(n_i) + k^2$,

$\text{or}(\sigma(n_1)-1)^2+(\sigma(n_2)-1)^2+\ldots+(\sigma(n_k)-1)^2+\sum_{i=1}^{k}\sum_{\substack{j=1\\ i\neq j}}^{k}\sigma(n_i - 1)$

$\sigma(n_j - 1) = [\sum_{i=1}^{k}\sigma(n_i)]^2 - 2k\sum_{i=1}^{k}\sigma(n_i) + k^2$

Therefore,
$\sum_{i=1}^{k}\sigma(n_i^2) - 2\sum_{i=1}^{k}\sigma(n_i) + k + \sum_{i=1}^{k}\sum_{\substack{j=1\\ i\neq j}}^{k}\sigma(n_i - 1)\,\sigma(n_j - 1)$
$= [\sum_{i=1}^{k}\sigma(n_i)]^2 - 2k\sum_{i=1}^{k}\sigma(n_i) + k^2$,

or $\sum_{i=1}^{k}\sigma(n_i^2) \leq 2\sum_{i=1}^{k}\sigma(n_i) - k + [\sum_{i=1}^{k}\sigma(n_i)]^2 - 2k\sum_{i=1}^{k}\sigma(n_i) + k^2$,

or $\sum_{i=1}^{k}\sigma(n_i^2) \leq [\sum_{i=1}^{k}\sigma(n_i)]^2 - (k-1)\,(2\sum_{i=1}^{k}\sigma(n_i) - k)$.

Apply Modus Ponens Method:

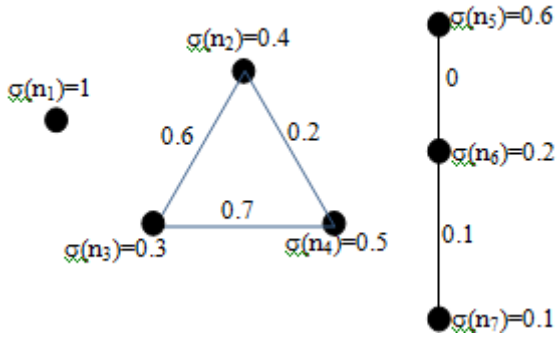


Figure 11: G

Figure 11: Gives a disconnected fuzzy graph with three components.

Rule1:
Fact: x is $\sum_{i=1}^{k}\sigma(n_i^2)$
Rule: **IF** x is $\sum_{i=1}^{k}\sigma(n_i^2)$ **THEN** y is 1.91
Result: y is 1.91
Example:

$\sum_{i=1}^{k}\sigma(n_i^2) = \sigma(n_1)^2 + \sigma(n_2)^2 + \sigma(n_3)^2 + \sigma(n_4)^2 + \sigma(n_5)^2 + \sigma(n_6)^2 + \sigma(n_7)^2$
$= (1)^2 + (0.4)^2 + (0.3)^2 + (0.5)^2 + (0.6)^2 + (0.2)^2 + (0.1)^2$
$= 1 + 0.16 + 0 .09 + 0.25 + 0.36 + .04 + 0.01$
$= 1.91$

Rule2:
Fact: x is $(\sum_{i=1}^{k}\sigma(n_i))^2 - (k-1)\,(2\sum_{i=1}^{k}\sigma(n_i) - k)$
Rule**: IF** x is $(\sum_{i=1}^{k}\sigma(n_i))^2 - (k-1)\,(2\sum_{i=1}^{k}\sigma(n_i) - k)$ **THEN** y is 3.21
Result: y is 3.21.
Example:

$(\sum_{i=1}^{k}\sigma(n_i))^2 - (k-1)(2\sum_{i=1}^{k}\sigma(n_i) - k)$
$= (1+0.4+0.3+0.5+0.6+0.2+0.1)^2 - (3-1)(2(3.1)-3)$
$= (3.1)^2 - (2)(6.2-3)$
$= 9.61 - (2)(3.2)$
$= 9.61 - 6.4$
$= 3.21$

Rule3:
Fact: x is $\sum_{i=1}^{k}\sigma(n_i^2)$

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Rule: **IF** x is $\sum_{i=1}^{k}\sigma(n_i^2)$ **THEN** y is lessthan or equal to $\sum_{i=1}^{k}\sigma(n_i))^2 - (k-1)\ (2\ \sum_{i=1}^{k}\sigma(n_i) - k))$.
Result: y is lessthan or equal to $(\ \sum_{i=1}^{k}\sigma(n_i))^2 - (k-1)\ (2\ \sum_{i=1}^{k}\sigma(n_i) - k))$.

Example:

$\sum_{i=1}^{k}\sigma(n_i^2) \leq [\ \sum_{i=1}^{k}\sigma(n_i)\ ]^2 - (k-1)\ (2\ \sum_{i=1}^{k}\sigma(n_i) - k)$.
$1.91 \leq 3.21$.

Hence prove the Lemma.

4. Conclusion

Finally fuzzy If-Then rule with Modus Ponens method applied on the fuzzy graph results are efficient than crisp results.

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