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Some New Bi-Measures of Fuzzy Entropy

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Abstract: A bi-measure of fuzzy entropy is the sum of two functions, at least one of which must be a measure of entropy. Some new parametric measures of fuzzy entropy have been deduced.

Keywords: Fuzzy Entropy, Bi- Measures

1. Introduction

In 1965, Zadeh [5] introduced fuzzy sets. A fuzzy set A is represented as $A=\{x_i / \mu_A(x_i): i=1,2,...,n\}$ where $\mu_A(x_i)$ is a membership function.

A measure of fuzzy entropy $F(\mu_A(x_i))$ in a fuzzy set A should satisfy at least the following properties.

1. It should be defined for all $\mu_A(x_i)$ in the range of $0 \le \mu_A(x_i) \le 1,\ i=1,2,.....n$.
2. $F(\mu_A(x_i))$ should be continuous in this region.
3. $F(\mu_A(x_i))=0$ when $\mu_A(x_i)=0$ or 1.
4. $F(\mu_A(x_i))=F(1-\mu_A(x_i))$.
5. $F(\mu_A(x_i))$ should be maximum when $\mu_A(x_i)=\frac{1}{2},\ i=1,2,....,n$.
6. It should be increasing function of $\mu_A(x_i)$ when $0 \le \mu_A(x_i) \le \frac{1}{2}$ and other variables are kept fixed and it should be decreasing function of $\mu_A(x_i)$, when $\frac{1}{2} \le \mu_A(x_i) \le 1$ and other variables are kept fixed.

Fuzzy entropy corresponding to Shannon's [4] measure of entropy is:

$$F_1(A) = -\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\mu_A(x_i) + (1-\mu_A(x_i))\ln(1-\mu_A(x_i))\right].$$

Fuzzy entropy corresponding to Havrda and Charvat's [1] measure of entropy is:

$$F_2((A)) = \frac{1}{1-\alpha}\left(\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)-1\right),\ \alpha\neq 1,\ \alpha>0.$$

Fuzzy entropy corresponding to Ranyi's [3] measure of entropy is:

$$F_3((A)) = \frac{1}{1-\alpha}\left(\sum_{i=1}^{n}\ln\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)\right),\ \alpha\neq 1,\ \alpha>0.$$

Fuzzy entropy corresponding to Kapur's [2] measure of entropy is:

$$F_4(A) = -\sum_{i=1}^{n}(\mu_A(x_i)\ln\mu_A(x_i)+(1-\mu_A(x_i))\ln(1-\mu_A(x_i)) + \frac{1}{a}\sum_{i=1}^{n}((1+a\mu_A(x_i))\ln(1+a\mu_A(x_i))+(1+a(1-\mu_A(x_i)))\ln(1+a(1-\mu_A(x_i)))) \text{ and } -2\frac{1}{a}(1+a)\ln(1+a),\ a\ge -1.$$

$$F_5(A) = -\sum_{i=1}^{n}(\mu_A(x_i)\ln\mu_A(x_i)+(1-\mu_A(x_i))\ln(1-\mu_A(x_i)) + \frac{1}{a^2}\sum_{i=1}^{n}((1+a\mu_A(x_i))\ln(1+a\mu_A(x_i))+(1+a(1-\mu_A(x_i)))\ln(1+a(1-\mu_A(x_i)))) -2\frac{1}{a^2}(1+a)\ln(1+a),\ a\ge -1.$$

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2. Some new Bi- Measures of fuzzy entropy

Consider the measure

$$F_{a,b,k}(A) = -\sum_{i=1}^{n}\left(\mu_A(x_i)\ln\mu_A(x_i)+(1-\mu_A(x_i))\right)\ln(1-\mu_A(x_i)) +$$

$$\frac{b}{a^k}\sum_{i=1}^{n}\left((1+a\mu_A(x_i))\ln(1+a\mu_A(x_i))+(1+a(1-\mu_A(x_i)))\ln(1+a(1-\mu_A(x_i)))\right)$$

$$-2\frac{b}{a^k}(1+a)\ln(1+a)$$

where a, b, k are parameters.

Here $F_{a,b,k}(A)$ is defined for all $\mu_A(x_i)$, in the range $0 \le \mu_A(x_i) \le 1,\ i = 1, 2, ..., n$ and it should be continuous in this range.

$F_{a,b,k}(A) = 0,$ *when* $\mu_A(x_i) = 0$ *or* $\mu_A(x_i) = 1.$

$F_{a,b,k}(\mu_A(x_i)) = F_{a,b,k}(1-\mu_A(x_i)).$

Now $F_{a,b,k}(A)$ can be written as $\sum_{i=1}^{n} f(x_i)$, where

$$f(x) = -x\ln x+(1-x)\ln(1-x)+\frac{b}{a^k}\left((1+ax)\ln(1+ax)+(1+a(1-x))\ln(1+a(1-x))\right)$$

$$-2\frac{b}{a^k}(1+a)\ln(1+a)$$

Then

$$f'(x) = -\ln x+\ln(1-x)+\frac{b}{a^k}\left(a\ln(1+ax)-a\ln(1+a(1-x))\right) \text{ and}$$

$$f''(x) = -\frac{1+(a-c)x}{x(1+ax)}-\frac{1+(a-c)(1-x)}{(1-x)(1-a(1-x))};\ c=\frac{b}{a^k}.$$

Hence $F_{a,b,k}(A)$ will be concave, if $1+(a-c)x \ge 0$ *and* $1+(a-c)(1-x) \ge 0$

But it is true when

$1 \ge c$ *or* $a \ge c$ *i.e. when* $a^{k-2} \ge b$ *or* $a^{k-1} \ge b.$ .....(1)

$(k-2)\ln a \ge 0$ *or* $(k-1)\ln a \ge 0.$ .....(2)

But $(k-2)\ln a \ge 0$, *if* $a \ge 1$ *or* $k \ge 2$ *or* $0 < a < 1$, $k < 2;$

$(k-1)\ln a \ge 0$, *if* $a \ge 1$ *or* $k \ge 1$ *or* $0 < a < 1$, $k < 1.$

3. Special Cases:

1) When $a^{k-2} \ge b$ or $a^{k-1} \ge b$, $F_{a,b,k}(A)$ represent a three parametric bi- measure of fuzzy entropy.
2) When $b = 1, k = 1,$ $F_{a,b,k}(A) = F_4(A).$
3) When $b = 1, k = 2,$ $F_{a,b,k}(A) = F_5(A).$
4) When $0 < b \le 1,$ then (1) will be hold if
5) When $b > 1,$ we can find suitable value of a, k using (2). Thus in any case, the family of bi-measure of fuzzy entropy is much larger than $F_4(A)$ *and* $F_5(A).$

References

[1] Havrda, J.H. and Charvat, F. (1967): Quantification method of classification Process: Concept of structural α entropy. Kybernatica, 3, 30-35.
[2] Kapur, J.N. (1986): Four families of Measures of Entropy. Ind. Jour. Pure and App. Math's, 17(44), 429-449.
[3] Renyi, A. (1961): Measure of Entropy and Information. In Proceeding Fourth Berkeley Symp. Math. Statist. Probab. , 1, 547-561.

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[4] Shannon, C.E. (1948): A Mathematical Theory of Communication. Bell system Tech Journal, 27, 379-423, 623-659.
[5] Zadeh, L.A. (1965): Fuzzy sets. Infor. And Control, 8(3), 338-353.

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