

Credit Risk Assessment: The Structural Approach (MERTON Model) Applied to the Moroccan Banking Sector

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List of Abbreviations and Acronyms

BAM: Bank Al Maghrib
FDIC: Federal Deposit Insurance Confederation (USA)
FGD : Fonds de Garantie des Dépôts
FCGD : Fonds collectif de garantie des dépôts (Maroc)
PD : Probability of Default "Probabilité de Défaut"
V.A : Random variable "Variable Aléatoire"
PPR : Absolute Priority Rule.
BCP : Central Populaire Bank "Banque Centrale Populaire".
BPR : Regional Populaire Bank "Banque Populaire Régionale".
TSDI: Subordinated notes of indefinite duration "Titres subordonnés à durée indéterminée".
FRBG: Funds for general banking risks "les fonds pour risques bancaires généraux".
PME: Small and medium enterprises "Petites et moyennes entreprises".
BMCE: Moroccan Foreign Trade Bank "Banque marocaine de commerce extérieur".
CIH: Crédit immobilier et hôtelier.
OFS: organismes financiers spécialisés.
SGMB: Société Générale Marocains de la banque.
A.D.I: Insurance Death Disability "l'Assurance Décès Invalidité".
AWB: attijariwafabank.
BMCI: Banque Marocaine de Commerce Intérieure
CDM: Crédit du Maroc
OTS: Office of Thrift Supervision.
FP: Equity "Fonds Propres"

Abstract: *A complete financial modeling of the firm is fundamentally based on several factors: the dynamics of the firm's assets, the contractual structure of priority between beneficiaries (the absolute priority of holders of debt on shareholders, for example), the recovery rate in case Of liquidation. The structural approach emerged in the early 1970s on the initiative of Fisher Black, Myron Scholes and Robert Merton. These authors have shown that any financing title could be identified with a derivative contract on the value of the firm's assets. Having made possible the modeling of the default risk, which made this approach very usable in many works. The problems facing the banks with regard to the quality of the borrower are as follows: How to calculate the probability of default of the counterparty through and that it would be the recovery rate in case of default? What if the interest rate is dynamic (flat rate curve)? How to calculate the capital needed to face this same probability of default?*

Keywords: Merton Model, Structural Model, Probability of Default (PD), Vasicek Model, Recovery Rate

1. Introduction

Any financial security holder is subject to the risk of default of the issuing company. In the "structural" approach, the default of a company is the result of a process that leads a firm in difficulty to the cease of payment. A thorough knowledge of this - from the point of view of its financial structure, its exposure to business and financial risks and the value of its assets potentially liquidated and the actual sharing between creditors - therefore becomes crucial.

« The bank is the only institution legally entitled to collect public savings in the form of refundable deposits » [Bale II et le risque de crédit Alain Verboomens l'ouis de bel page 4], These funds mainly use it to finance the economy through

the granting of credits, the purchase of financial securities more specifically shares and bonds. Knowing the origin of its resources, the bank must protect its savers but also the entire financial system that presents the heart of any economy developed through the management of various risks that can negatively affect its result and induce its bankruptcy. Hence the intervention of the Basel Committee [Le comité de Bâle sur le contrôle bancaire (ang. Basel committee on banking supervision, abrev. BCBS) a été institué en décembre 1974 par les gouverneurs des banques centrales des pays des Groupes des Dix (G10) sous l'appellation de « comité de Bale des règles et pratiques de contrôle des opérations bancaires » (ang. Committee on Banking Regulations and Supervisory Practices)] Which encourages banks to use external and, above all, internal

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models to identify, evaluate and manage the risks associated with its activities, in particular the credit risk which presents a major risk to any credit institution.

It is difficult to imagine that a bank could go bankrupt. Ideally, banking should only be used to finance the economy and individuals. The bank, by granting loans, allows companies to invest and individuals to buy real estate. Everything is implemented in a bank to secure the credit granted and minimize the risk incurred. So there is low risk of a bank going bankrupt because of the default of the borrowers. But, the subprime crisis [See Annex 1 for the free fall of the major international exchanges] Of 2008 reminds us that there are no impossibles in terms of finances, with the bankruptcy of the big bank Lehman Brothers, The Californian bank IndyMac, (see annex 2)...

We have to understand that banks, in order to pay for the money they deposit, have to invest this money, and are therefore subject to the risk, like any business, if investments go wrong, banks lose money. To improve the return on capital deposited, banks have long diversified their investments, not only limited to loans, but also to equity investments, for example.

A bank could have problems, like all businesses. These problems are of course exaggerated if large borrowings they had conceded were not repaid.

The great crisis of 1929 enabled the states to draw conclusions. The precipitate of savers to withdraw their money from the banks, sometimes threatened to lose everything because of hypothetical bank failures had only accelerated and amplified the crisis. It was to compensate for this problem that in 1933, the American state created the Federal Deposit Insurance Corporation (FDIC), ensuring up to \$ 100 000 in deposits. From this innovative system designed to restore confidence in the financial system and thus avoid new massive withdrawals of money from the banks were born the inspiration for the various deposit guarantee funds around the world like the FGD in France and the FCGD in Morocco.

The different approaches to credit risk management:

Prudential regulation of credit activities:

Prudential regulation encompasses all the constraints imposed on credit institutions for a good control of the risks they pose to all economic players and more particularly to their depositors. The main concern of the banking authorities is to limit the spread of bank failures that could hamper the overall financial system (systemic risk).

In 1988, credit risk led to the introduction of the Cooke ratio adopted by the Basel Committee. This ratio is intended to establish a floor of equity based on the risks initially limited to counterparty risk. Three categories of capital are distinguished: core or "core" or "tier one" capital (including capital, reserves, etc.), tier two ("soft core" or "tier two" Including subordinated notes of indefinite duration (TSDI)) and funds for general banking risks (FRBG).

1.1 Financial analysis

The purpose of the financial analysis is to conduct a thorough study of the state of financial health. It is a basic tool to find out if credit is available, but it can not determine the level of margin required. The expectation of gain in relation to the risk, in case of default, can not be modeled. This is therefore the most important limitation of financial analysis.

1.2 The rating of financial rating agencies

The rating is an independent assessment of the ability and willingness of a borrower to deal with its financial obligations, it is based on the probability of failure is also an increasing function of time, in other words, The risk increases with duration. In addition, probabilities of default are necessary to quantify possible losses and their volatility. Two major uses of the quantification of default rates: an estimate of the economic provisions to deal with future defaults. The estimate of the maximum losses on a portfolio of commitments. This technique is tailored to a particular clientele but difficult to use for a corporate clientele and the latter clientele which may represent a worrying credit risk.

1.3 The allocation of credit lines by counterparty:

In order to contain their risks to an acceptable level, banks must set exposure limits using risk-based risk-based systems, which set out authorizations for commitments by counterparty and market based on the quality of the counterparties and Of their financial situation. But mainly on the basis of the lender's own funds.

This technique requires ongoing monitoring to ensure that exposure limits are met.

1.4 Guarantees from insurance companies:

A lender may use credit insurance with an insurance company to hedge against the risk of his borrower's insolvency. However, this credit insurance covers only short-term commercial risk excluding political risks and natural disasters. Thus, the indemnification process is triggered only in the event of the debtor's insolvency.

1.5 The syndication

Syndication is a kind of "banking pools", called "banking syndicates". The entire loan is granted by all the banks involved in this syndicate. It is a technique of division of risks since each bank holds a fraction of the total of the receivables.

1.6 The taking of real or personal guarantees:

The endorsement and the guarantee constitute a personal guarantees, Mortgages and pledges are a real security.

1.7 Diversification

Asset diversification obviously reduces risk. Indeed, the overall risk of a portfolio is lower than the sum of its

individual risks (but it is necessary to see the correlation of these risks).

1.8 Securitization of receivables

Securitization entails making negotiable credits distributed by credit institutions recessed on a market. Example Assignments of receivables and asset swaps.

1.9 Discriminate analysis and credit scoring

Scoring techniques are numerous. The objective is to increase the effectiveness of decision-making. Financial analysis based on financial ratios.

Discriminate analysis is favored by score constructors. The most famous example of application of this technique is the Altman 1968 model called the prediction function Z:

$$Z = 1.2 X_1 + 1.4 X_2 + 3.3 X_3 + 0.6 X_4 + 1.0 X_5$$

X1: Working capital / total assets (liquidity)

X2: Retained earnings / total assets (age, profitability, conservatism)

X3: Earnings before interest and taxes / total assets (profitability)

X4: Market value of equity / book value of debt (capitalization)

X5: Sales / Assets (Productivity)

If the score is less than 1.81, the model predicts bankruptcy and if the score is greater than 1.81, then it predicts survival. The score thus reflects the risk of bankruptcy since it is largely derived from the level of liquidity and solvency of the company (in the sense of financial banking analysis).

1.10: The rating: a negotiated score expanded to qualitative variables:

The rating in the sense "financial rating agency", opened up more on the qualitative management of the company. Two major agencies in this sector exist Moody's and Standard and Poors. Most large companies now choose to be rated, especially when they have to rely on public savings.

1.11 The neural network

A neural network is a set of nodes connected by interconnections that carry precise information from one layer of neurons to another. The input of the basic information, thus the introduction of the values of the relevant indicators, is carried out by the first layer. Each node receives information, processes it by its nodal functions and retransmits the result to the next layer. The last layer provides the final result. The neural network method is a nonparametric approach, so the variables used do not need to follow a particular statistical distribution. The establishment of a network of neurons is generally carried out in four stages: choice of samples, elaboration of the structure, learning, validation.

Summary of section 1

The techniques used previously to assess credit risk, namely financial analysis that uses accounting data to judge the

health of the company, scoring that gives a quantitative-based score for decision-making, Granting loans while rating has integrated the qualitative to the development of the note, the same logic neural networks have developed a very advanced technique using imputed variables and an activation function. All these practices do not answer the real problem that it to calculate the probability of default of the issuer and the credit spread hence the need for a structural model.

Section 2: The MERTON model for assessing credit risk:

2. Reminder on Stochastics

2.1 Stochastic processes

A stochastic process is a sequence of random variables indexed by time which allows to model the random evolution of a variable over time.

There are two classes of stochastic processes: Discrete time processes and continuous time processes.

In a discrete (continuous) discrete time process, changes in the value of the variable occur at predetermined dates (at any time) and on a given date, the probability law of this variable is discrete (continuous). This can take finite values (in a range) that are part of a space called discrete (continuous) state space.

A process is called a Markov process if the only present value of a variable is useful to anticipate its future distribution. Indeed, predictions about the future value of this process do not depend on past values. The current value integrates all the information contained in the history of these values.

Let Z a Markov processes such that these expected increases are zero and the variance of these increases is equal to 1 per unit of time (usually one year). This process is called standard Wiener process or standard Brownian motion.

Z is a Wiener process, if it checks the following two properties:

- The variation ΔZ During a short time interval of length Δt Is expressed as follows:
 $\Delta Z = \varepsilon \sqrt{\Delta t}$ [formula 12.1 John Hull 6 edition page 267]
 or ε is a random variable that follows the normal distribution $N(0,1)$
- The values of ΔZ For two short length intervals Δt Are independent.

2.2. Processus and lemma's Itô

let Z A standard Wiener process, the general Wiener process is a variable X defined as a function of dz as following:

$$dx = a \cdot dt + b \cdot dz \text{ [formula 12.3 John Hull 6 edition page 270]} \quad (1.1)$$

Where:

- a is a constant called drift of X .

- b is a constant such that $b > 0$ called standard deviation of X .

Remarks

1. Suppose that $b.dz = 0$ so $dx = a.dt$ then $\frac{dx}{dt} = a$. By integrating this equation with respect to time, we obtain:

$$\int_0^T dx = \int_0^T a dt = x_0 + aT$$

where T a period of time and x_0 is the value of X on the date zero.

It is said that the value of x has increased by aT .

The term $b.dz$ is considered as an adding noise to the trajectory followed by X .

2. The variation $\Delta x = a.\Delta t + b.\varepsilon\sqrt{\Delta t}$ follows a normal distribution $N(a\Delta t, b\sqrt{\Delta t})$

The Itô Process x is a Winner general process whose parameters a and b are functions of x and t . It is written as follows:

$$dx = a(x, t)dt + b(x, t)dz \quad [\text{Formula 12.4 John Hull 6 edition page 271}] \quad (1.2)$$

Where :

- a is the drift of x
- b^2 is the variance of x

Remarks:

When the time t passes to $t + \Delta t$ the value x passes to $x + \Delta x$ where $\Delta x = a(x, t)\Delta t + b(x, t)\varepsilon\sqrt{\Delta t}$, the parameters $a(x, t)$ and $b(x, t)$ are constant during the time interval separating $t + \Delta t$.

Let x the Itô process, a function of f of x et t is expressed in the following form:

$$df = \left(\frac{\partial f}{\partial x} a + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} b^2 \right) dt + \frac{\partial f}{\partial x} b dz \quad (1.3)$$

where :

- z is the same process of Wiener as that which appears the process x
- f Is an Itô process of which:
 - The drift is equal:

$$\frac{\partial f}{\partial x} a + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} b^2$$

- The variance is equal :

$$\left(\frac{\partial f}{\partial x} \right)^2 b^2$$

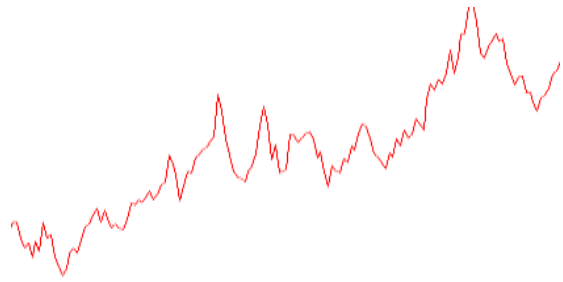


Figure 1: General Wiener process

Source : graphe établie par nos soins sous Matlab

2.3. The Black & Scholes model

Black and Scholes dealt with the issue of stock price movements and the valuation and hedging of a European call or put option on a non-dividend-paying share.

Black & Scholes offers a continuous time model with a risky asset and a risk-free asset.

The price of a risk-free asset S_t^0 is given by the following equation:

$$dV_t^0 = rV_t^0 dt$$

where r designate the interest rate.

Consider an action A Whose course evolution is modeled by a general Wiener process. The expectation of return required by investors, which is independent of the share price, is not taken into account by this model. This implies that the constant drift for the stock price is not suitable. This must be replaced by a constant drift for the course yield process.

Let V_t the value of the share on the date t whose drift is assumed to equal μV_t where μ represents the expected rate of return of the action in question.

The expected return of V_t For a short time interval of length Δt is equal to $\mu V_t \Delta t$.

Suppose that the volatility of the share price is zero, then this model is expressed as follows:

$$\Delta V_t = \mu V_t \Delta t$$

when $\Delta t \rightarrow 0$, the result is :

$$\frac{dV_t}{V_t} = \mu dt$$

By integrating this equation between $t = 0$ and $t = T$, we get :

$$V_T = V_0 e^{\mu T} \quad (1.4)$$

where V_0 and V_T Are respectively the share price on the dates 0 and T .

In reality one cannot imagine an action whose prices have zero volatilities. This leads to consider that the standard deviation of the variation of course in a length interval Δt Should be proportional to the share price from which the following result:

$$dV_t = \mu V_t dt + \sigma V_t dz$$

or

$$\frac{dV_t}{V_t} = \mu dt + \sigma dz \quad (1.5)$$

It is the model of the price evolution where :

- μ is a constant that indicates the expected return on the share price per unit of time ;
- σ Is a constant that indicates the volatility of the share price

The discrete time version of the general Wiener model or Brownian geometric motion of the action course is given by:

$$\frac{\Delta V_t}{V_t} = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t} \quad (1.6)$$

Or

$$\Delta S_t = \mu V_t \Delta t + \sigma V_t \varepsilon \sqrt{\Delta t}$$

Where:

- The term $\mu \Delta t$ represents the expected return;
- The term $\sigma \varepsilon \sqrt{\Delta t}$ represents the stochastic component of said return ;
- The term $\sigma^2 \Delta t$ the variance of the return .

According to equation (1.6) , the variable $\frac{\Delta V_t}{V_t}$ follows the normal distribution $N(\mu \Delta t, \sigma \sqrt{\Delta t})$.

Consider the process $y = \ln(x)$ where x Is an Itô process.

As $\frac{\partial y}{\partial x} = \frac{1}{x}$, $\frac{\partial^2 y}{\partial x^2} = -\frac{1}{x^2}$, $\frac{\partial y}{\partial t} = 0$ then by equation (1.3) we obtain:

$$df = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dz \quad (1.7)$$

where μ and σ are constants.

so the process y Is a general Winner process:

- Drift is equal $\mu - \frac{\sigma^2}{2}$
- The variance is equal σ^2

The variation of $\ln(x)$ between $t = 0$ and $t = T$ follows

a normal distribution $N\left[\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right]$ It means

that:

$$\ln(x_T) - \ln(x_0) \square N\left[\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right]$$

Or:

$$\ln(x_T) \square N\left[\ln(x_0) + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right] \quad (1.8)$$

2.4. The Merton model

Merton has developed a MEDAFTC equilibrium asset-time equilibrium model to describe the relationship of

instantaneous returns with the instantaneous covariances of the different securities. It assumes the log-normality of distributions of future stock prices.

According to the MEDAFTC model, there is a market portfolio M , whose value is V_m . Evolves following the Brownian geometric movement:

$$dV_m = \mu_m V_m dt + \sigma_m V_m dz_m \quad (1.9)$$

where:

- μ_m is the drift of the market portfolio.
- σ_m is the volatility of the market portfolio.
- z_m Is a standard Brownian process.

The prices of n risky assets change according to the following stochastic differential equations:

$$dV_i = \mu_i V_i dt + \sigma_{im} V_i dz_m + \sigma_i V_i dz_i \quad (1.10)$$

where:

- μ_i Is the instant drift of V_i ($i = 1, \dots, n$)
- z_i Is the Brownian motion mutually independent and independent standards of z_m , σ_m ($i = 1, \dots, n$)
- σ_i Is a positive constant ($i = 1, \dots, n$)

According to equation (1.9), each V_i ($i = 1, \dots, n$) Is described by a geometric Brownian motion, where:

- The instantaneous variance is equal to $\text{var}(V_i) = \sigma_{im}^2 + \sigma_i^2$
- The volatility of $i^{\text{ème}}$ share is equal to $\sigma_{V_i} = \sqrt{\sigma_{im}^2 + \sigma_i^2}$.
- The covariance of V_i and V_j is equal to $\text{COV}_{S_i, S_j} = \sigma_{im} \sigma_{jm}$.

The MEDAFTC model provides the equilibrium relationship between yields and instantaneous covariances as follows:

$$\mu_i - r_0 = \frac{\sigma_{im} \sigma_m}{\sigma_m^2} (\mu_m - r_0) \quad (i = 1, \dots, n); \quad (1.11)$$

where

- r_0 Is the return on the risk-free asset
- $\sigma_{im} \sigma_m$ is the covariance between V_i and V_m .

Equation (1.9), Equation (1.10) and Equation (1.11) characterize the continuous time market model.

In the following, we assume that $V_m(T)$ and $V_i(T)$ ($i = 1, \dots, n$) Are distributed according to a log-normal law, and that the equilibrium relations (1.11) are verified for all the titles. It is also assumed that the portfolio structure remains constant during $[0, T]$.

2.4.1 The assumptions of the model

- Perfect market: no transaction costs or taxes.
- Full market: risky assets (non-dependent) equals the number of risk sources.

- Absence of arbitrage opportunity: there are no profits without risk.
- Validity of Modigliani Miller's theorem: Independence of the value of the company in relation to its capital structure (in the absence of bankruptcy costs and income taxes)
- Constant interest rate (denoted r)

2.4.2 Assessment of bonds that may be default:

The idea behind the Merton model is the observation that corporate bonds can be considered as contingent claims on company assets and that the market value of the business is the underlying source of credit risk.

Asset (V)	Equity (B)
	Debt (K)

Equality between the assets and liabilities of the balance sheet [This equality is no longer verified in the time from which the appearance of Asset Liability Management (ALM)]

$$V = B + E$$

Merton considers a company whose market value V which can be considered as the market value of the assets. This company has a simplified balance sheet financed by own

funds and debts. This debt constitutes a zero-coupon bond with B-face value and maturity T .

The value of the firm is assumed to follow a geometric Brownian motion :

$$\text{with } \frac{dV_t}{V_t} = \mu \cdot dt + \sigma_V \cdot dZ_t, V_0 > 0 \dots (1.13)$$

μ : drift parameter of V .

σ_V : the volatility of V .

Z : Brownian movement.

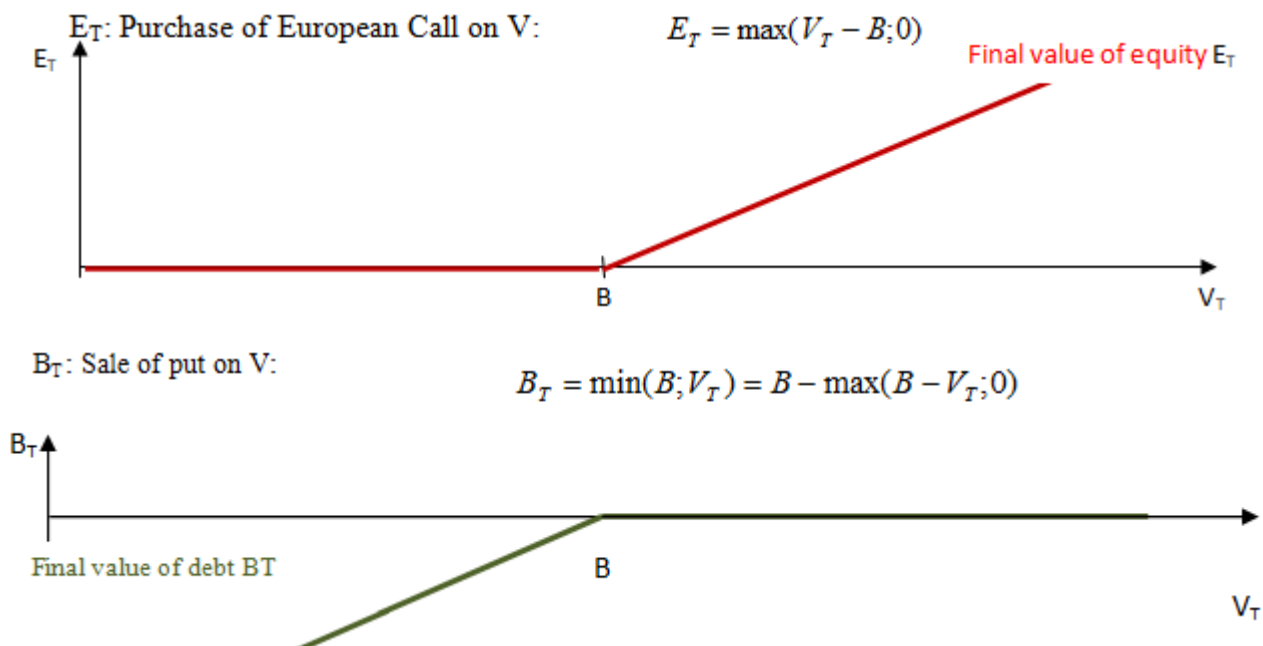
If $V_T > B$: The lenders (holders of the bonds) receive: K (total debt) and the shareholders receive $(V_T - B)$.

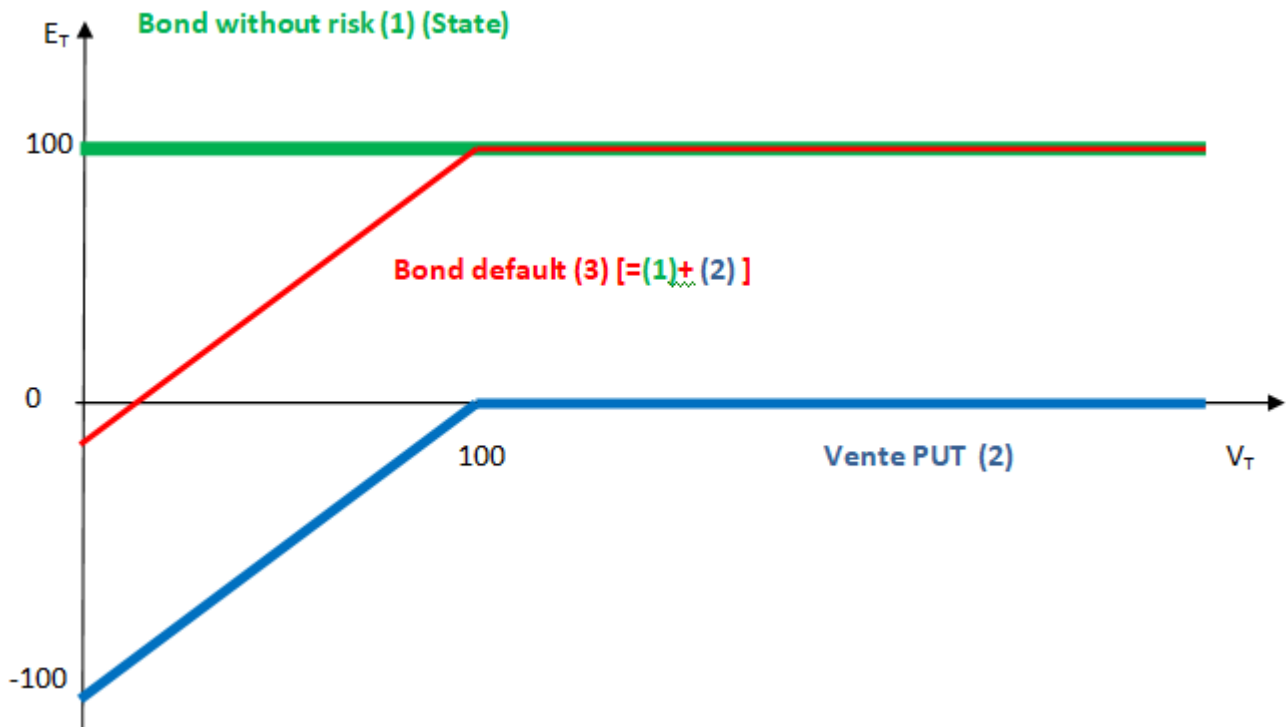
If $V_T < B$: the lenders receive: V_T And shareholders receive: 0
 According to the rule of Absolute Priority

According to these specifications, the defect time τ can be defined as a discrete random variable:

$$\tau = \begin{cases} T & \text{if } V_T < B \\ \infty & \text{if else.} \end{cases}$$

The key idea of Merton is that the final values (payoff) [BT of bonds and ET of equity] can be regarded as European options on the value of firm V with a strike price B and maturity T .





- The neutral risk position gives 100 (B-value of debt) regardless of the company's V_T value
- The defaultable bond: the holders of this bond begin to lose as soon as the value of the firm falls below the level of 100
- Bondholders sold a put on V
- The gain on the risky bond is the sum of the risk-free bond and the sale of the put
- Pay off of the bond

$$B_T = \min(B, V_T) = B - \max(B - V_T; 0) \quad (1.14)$$
- Pay off of equities (3) :

$$E_T = \max(V_T - B, 0) \quad (1.15)$$
- (2) $\Leftrightarrow$ Risk-free bond (B) – Pay off sale put on V with strike price B
- (3) $\Leftrightarrow$ Pay off of shareholders' equity = call option on V (B, T)

Another key and important aspect of this model is the assumption that the value of the firm is independent of the leverage ratio.

Demonstration using the parity of put -call:

$$C_T + Xe^{-rT} = P_T + S_0$$

[Relation 9.3 page 207 John HULL Options, Futures and other derivative assets]

Let us use the terms of Merton:

$$E_0 + Be^{-rT} = P_T + V_0 \dots et \dots E_0 = C^{BS}(V, B, r, T, \sigma)$$

$$\text{We will have: } E_0 + B_0 = V_0$$

Hence the value of the firm is independent of the financial structure of capital. And by the:

$$V_0 = E_0 + B_0$$

To assess the risky bond, the Black-Schools formula can be used to evaluate option (2.2) and (2.3).

At time $T=0$ The values of debt and equity are given successively by:

$$B_0 = Be^{-rT} - P^{BS}(V, B, r, T, \sigma) \dots (1.16)$$

[Formule 13.20 Options futures et autres actifs dérivés John Hull page 303 valeur du call Européen]

$$E_0 = C^{BS}(V, B, r, T, \sigma) \dots (1.17)$$

(2.4) $\Leftrightarrow$ The value of a risky bond calculated as the difference between the discounting of the value of a risk-free bond and the value of a LUT on the asset.

The PUT reflects the risk of obtaining a risky bond rather than a risk-free bond and can be considered as a reduction in the credit risk that bondholders need to assume that risk. So the risk of the bond must be reflected in the sale of PUT.

$$(1.17) \Leftrightarrow E_0 = V_0 N(d1) - B e^{(-rT)} N(d2) \dots (1.18)$$

$$\text{With: } d1 = \frac{\ln(S_0/B) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

[Options futures et autres actifs dérivés John Hull page 303] and

$$d2 = \frac{\ln(S_0/B) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d1 - \sigma\sqrt{T}$$

[Options futures et autres actifs dérivés John Hull page 304]

Debt assessment:

$$\text{We have: } V_0 = E_0 + B_0 \text{ where } B_0 = V_0 - E_0$$

At date T, the value of the debt is equal to $\min(BT; V_T)$

It therefore constitutes a zero-coupon without risk and a Put:

$$B_0 = Be^{(-rT)} + N(-d1)V_0 - B e^{(-rT)} N(-d2) \dots (1.19)$$

2.5. The interest rate differential:

$$\text{We have the relationship: } B_t \exp^{(j(t,T)(T-t))} = K$$

$$\text{So: } \exp^{(y(t,T)(T-t))} = \frac{K}{B_t}$$

then

$$y(t,T)(T-t) = \ln\left(\frac{K}{B_t}\right)$$

$$y(t,T) = \frac{1}{(T-t)} \ln\left(\frac{K}{B_t}\right)$$

$S(t,T)$ can be calculated by:

$$S(t,T) = y(t,T) - r_t \text{ and}$$

$$S(T,V) = y(T,V) - r = -\frac{1}{T} \ln\left(N(d_2) + \frac{V_0}{B.e^{-rT}} N(-d_1)\right)$$

[www.longin.fr]

And therefore the differential of interest rate (spread)

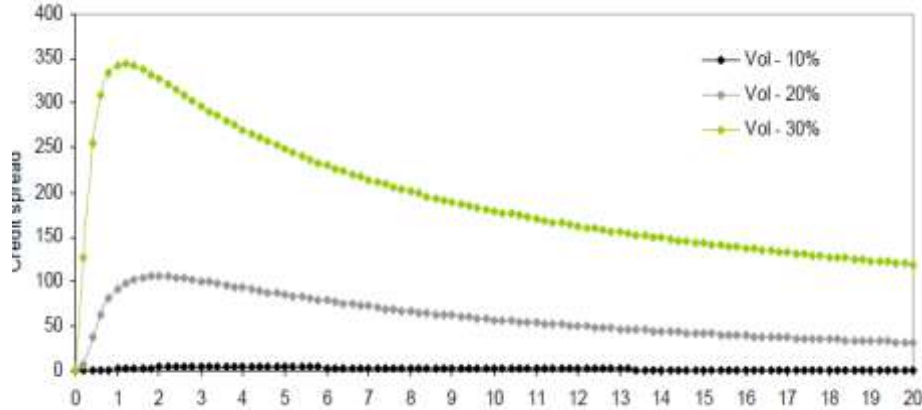


Figure 2: Credit spread according to maturity

Source: Graph established under Matlab using the Merton model $r = 5\%$ $L = 80\%$

The credit spread increases when volatility increases with a volatility of 10% all other things equal the spread is close to 0 then with a volatility of 30% the spread is close to 350 in

the first period also we note that this spread decreases when maturity increases

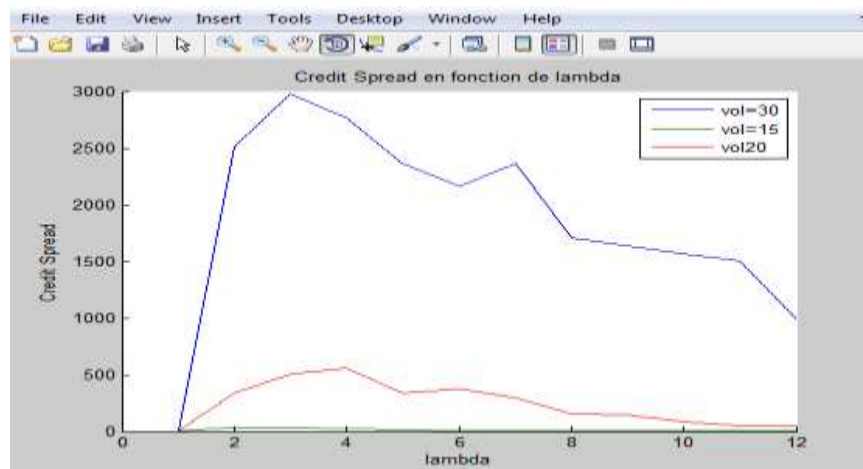


Figure 1: Credit spread based on the lambda debt ratio

Source: Graph established under Matlab using the Merton model $r = 5\%$ $L = 80\%$

The credit spread increases when volatility increases with a volatility of 10% all other things equal the spread is close to 0 then with a volatility of 30% the spread is close to 3000 in the third period also we note that this spread decreases when lambda increases so lambda has more impact on the spread than the maturity previously seen

2.6. The probability of default (PD)

It is assumed that the probability of default is equivalent to the probability that the value of the firm is less than the face value of the debt K at the maturity date T :

$$PD(T) = P(V_T < K) \dots \text{AVEC} \dots \frac{dV_T}{V_T} = \mu \cdot dt + \sigma_V \cdot dZ_t$$

$$PD(T) = P\left(V_0 e^{((\mu - \frac{1}{2}\sigma^2)T + \sigma Z_T)} < K\right)$$

$$= P\left(Z_T < \frac{\ln\left(\frac{K}{V_0}\right) - (\mu - \frac{1}{2}\sigma^2)T}{\sigma}\right)$$

$$PD(T) = \Phi\left(-\frac{\ln\left(\frac{K}{V_0}\right) - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) = N(-d_2) \dots (1.20)$$

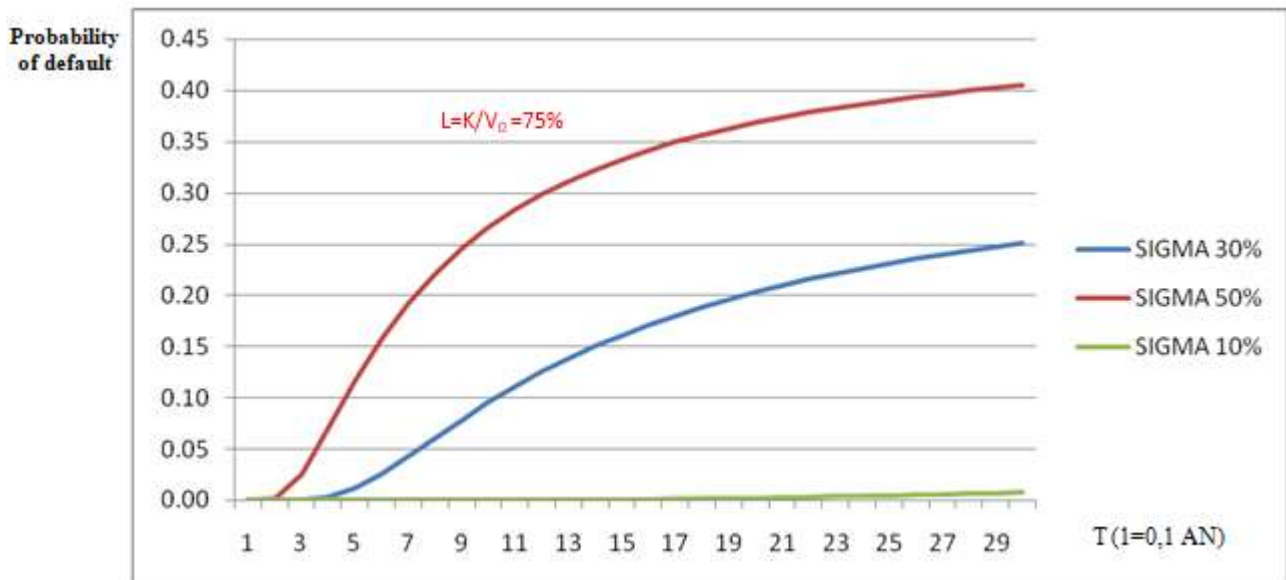


Figure 4: The default probability and the variance
Source: Graphic drawing by us using the formula (1.20)

Note: using a fixed leverage = 75%

1) An increase in the standard deviation implies an increase in the probability of default and vice versa.

2) If the maturity goes away then the probability of default increases and vice versa.

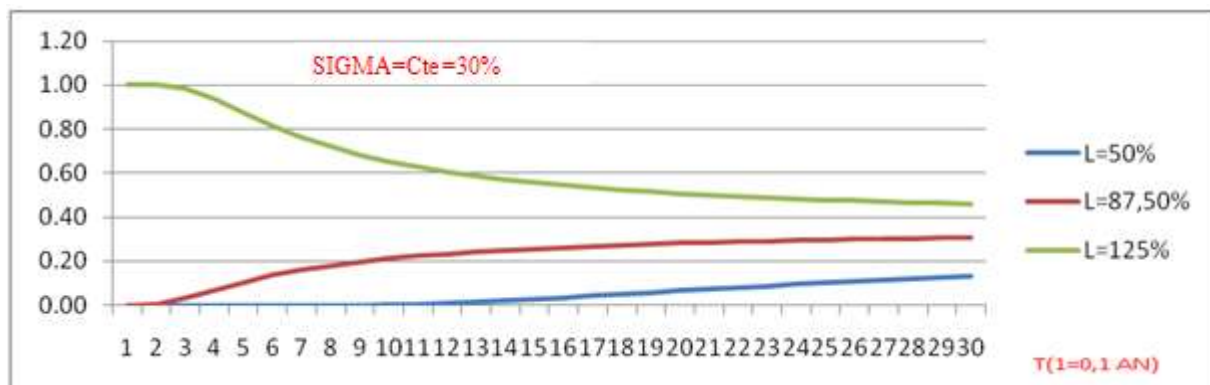


Figure 5: Probability of default and debt policy
Source: Graphic drawing by us using the formula (1.20)

With a leverage ratio of 125% in the very short term, the company is definitely in default because it does not have the time to recover from the current bad situation but with the time it can correct its situation is therefore the probability of default decreases. The curve can take several forms depending on the input parameters of the model.

Finally, the probability of default with a very short maturity and even long maturity is 0 regardless of the degree of risk and the debt structure this can be formulated as follows:

$$PD(T) \rightarrow 0 \text{ pour } T \rightarrow 0 \quad (1.21)$$

$$PD(T) \rightarrow 0 \text{ pour } T \rightarrow \infty \quad (1.22)$$

Démonstration : see annex 3

2.7. The recovery rate

A characteristic of the Merton model is that the default can only arrive at the end, that is to say at maturity, from which the recovery rate is endogenous to the model.

In the event of default, the expected recovery rate R (%) of a risky bond is defined under Q :

$$R_0^Q = E^Q\left[\frac{V_T}{K}(1-\phi)e^{-rT}\right] \dots\dots 1 \geq R_0^Q \geq 0 \dots\dots (1.23)$$

ϕ = The costs of bankruptcy

We have:

$$\begin{aligned} E(R_0^Q) &= E\left[\frac{V_T}{K}(1-\phi)e^{-rT}\right] \\ &= E(V_T) \cdot \frac{1}{K}(1-\phi)e^{-rT} \end{aligned}$$

According to John HULL Future Options and Other Derivatives pages 289 (13.4) we have:

Hence

$$\begin{aligned} E(V_T) &= V_0 e^{rT} \\ E(R_0^Q) &= V_0 e^{rT} \cdot \frac{1}{K}(1-\phi)e^{-rT} \dots\dots (1.24) \end{aligned}$$

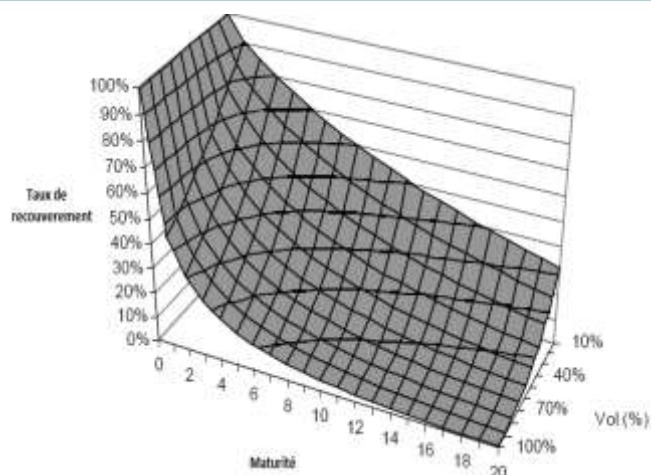


Figure 6: Recovery rate as a function of maturity and volatility

Source: Using the Merton model ($r = 5\%$ and $L = 90\%$)

- The increase in maturity implies a reduction in the expected recovery rate because, with the increase in maturity, the probability.
- The increase in sigma implies a reduction in the expected recovery rate, which is in line with economic reasoning (the increase in the risk of the asset will reduce the expected recovery rate).

2.8 Deficiency of MERTON model

- The balance sheet is generally much more complicated than is assumed in the model (it does not allow to pay coupons and it does not allow to issue new debts).
- Bonds often include clauses and options such as enforceability.
- Companies have off-balance sheet commitments that are difficult to measure and incorporate into the model.
- The Merton model assumes a constant risk-free interest rate, which is obviously unrealistic given that the observed yield curve is not flat and evolves over time.

Section 3: Application

DATA: The application covers a period from 2010 to 2013 of the various bank shares listed on the Casablanca stock exchange in order to predict their probability of default using the Merton's model described in section 2

3.1 Introduction of the Moroccan banking sector

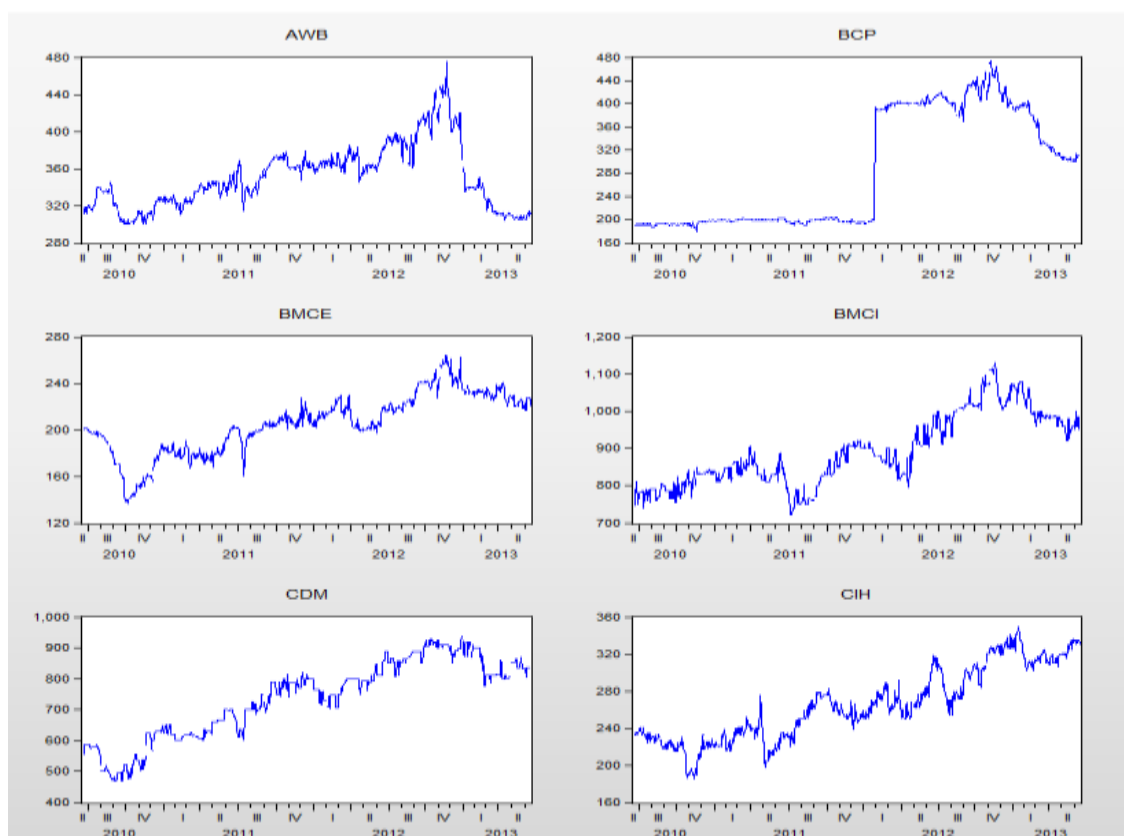


Figure 7: The evolution of Moroccan bank stocks

Source: Casablanca Stock Exchange

The shares of the 6 banks listed on the Casablanca stock exchange have a dynamic very volatile and shows an

upward trend for the entire period except for the BCP which experienced a spike of 200 to 400 in the second half of 2012.

View	Proc	Object	Print	Name	Freeze	Sample	Sheet	Stats	Spec
				DAWB	DBCP	DBMCE	DBMCI	DCDM	DCIH
Mean				-0.001596	0.159574	0.026662	892.9001	0.372340	0.127660
Median				0.000000	0.000000	0.000000	866.0000	0.000000	0.000000
Maximum				24.70000	194.0000	14.00000	1130.000	51.00000	18.10000
Minimum				-23.70000	-22.00000	-14.85000	721.0000	-51.00000	-19.30000
Std. Dev.				5.335000	8.148633	3.722990	95.35228	14.92559	5.713757
Skewness				-0.017980	17.93195	-0.117915	0.420093	0.235216	0.133730
Kurtosis				6.402848	427.7168	5.598742	2.134548	6.205038	3.772798
Jarque-Bera				362.8609	5692344.	213.3510	45.58745	328.7987	20.95422
Probability				0.000000	0.000000	0.000000	0.000000	0.000000	0.000028
Sum				-1.200000	120.0000	20.05000	671460.9	280.0000	96.00000
Sum Sq. Dev.				21375.13	49866.57	10409.35	6828136.	167302.7	24517.91
Observations				752	752	752	752	752	752

Figure 8: Descriptive statistics of bank shares

Source: Casablanca stock exchange established by us under Eviews periode from 17/06/2010 to 18/05/2013

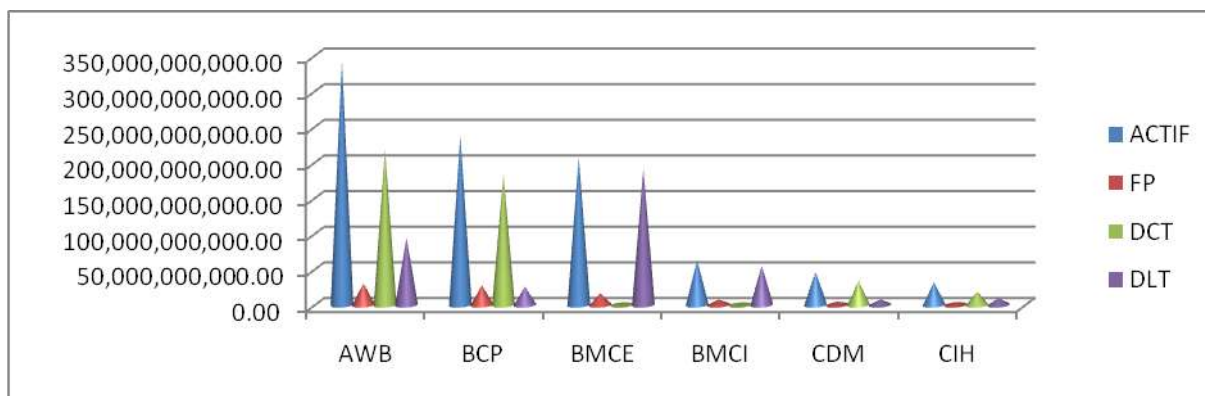


Figure 9: Balance Sheets of the various banks studied

Source Graph prepared by us using the data of the annual reports of the different banks

The graph shows the importance of each bank in terms of the size of the balance sheet, in which the AWB bank is at the top of the ranking by an asset total of 343452049000 DH followed by BCP which has a balance sheet total of 343452049000 DH comes After the BMCE by a total of

assets of 207988138000 this is the first three banks hold almost 85% of the active total of the various banks the other three come after as follows BMCI 207988138000, CDM 207988138000 and finally CIH with 32086000000.

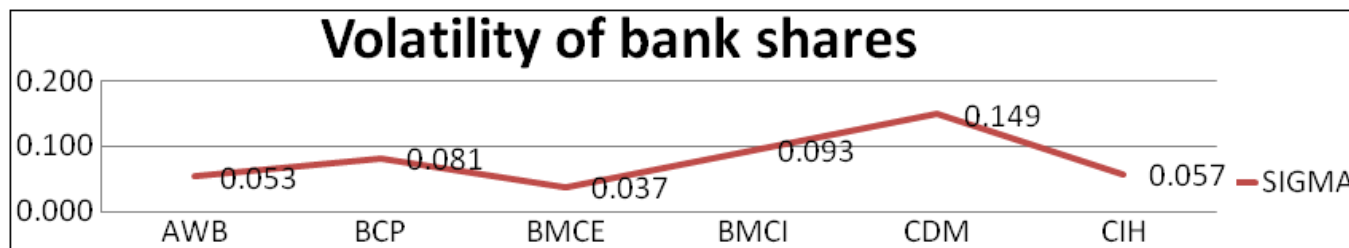


Figure 10: Volatility of the banking sector

Source : Graph prepared by us using from data stock exchange of Casablanca

The volatility of stock prices in the banking sector is less volatile except for the CDM which has a larger variance. This volatility has a great impact on the probability of

default as we will see on the overall graph of the probabilities of default of Moroccan banks

rating de depart	RATING APRES UN AN							
	AAA	AA	A	BBB	BB	B	CCC	DEFAULT
AAA	90,81	8,33	0,68	0,06	0,12	0,00	0,00	0,00
AA	0,70	90,65	7,79	0,64	0,06	0,14	0,02	0,00
A	0,09	2,27	91,05	5,52	0,74	0,26	0,01	0,06
BBB	0,02	0,33	5,95	86,93	5,30	1,17	0,12	0,18
BB	0,03	0,14	0,67	7,73	80,53	8,84	1,00	1,06
B	0,00	0,11	0,24	0,43	6,48	83,46	4,07	5,20
CCC	0,22	0,00	0,22	1,30	2,38	11,24	64,86	19,79

Source: Standard & Poor's CreditWeek (15 April 96)

Figure 11: Transition Rating Matrix

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The matrix of transition of the rating of the different banks gives us the probability of transition from one class to another class for example a bank begins the year in an AA class it has a probability of 90.65 to stay in the same class to

The end of the year and she a probability of 7.79 to move to class A and so on.

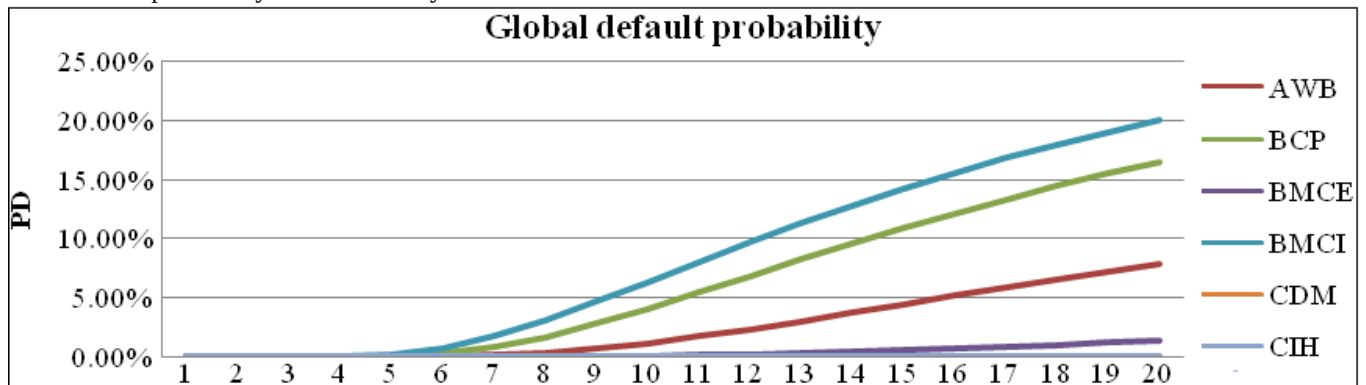


Figure 12: Probability of default of global banking's Moroccan sector
 Source: Established by us based on the Merton model

The model predicts a greater probability of default for BMCI followed by BCP and AWB. These banks have a very interesting dynamic in the market and have a greater volatility and thus a greater size in terms of the balance sheet whereas the others have a small Probability of default. These same probabilities of default increase for a very distant maturity and this for all the banks.

3. Conclusion

However, in the early 1990s, another class of models emerged. The so-called "reduced form" models abandon any attempt at financial explanation of the cessation of payment and consider that the default occurs "by chance". The arrival date of the fault is modeled using an unpredictable random time or in an equivalent manner using the first stopping time of a stochastic process. While it admits many practical advantages, the latter approach appears less intuitive than the structural approach.

Strategic extensions in which shareholders and creditors can renegotiate debt to avoid default.

The intermediate models between structural specification and reduced specification, which we will call "mixed". Finally, the last part deals with the empirical performances of the structural approach.

In the approach of Black, Scholes and Merton, the company is not "under surveillance" before the maturity of the debt. Such an assumption appears unrealistic. Mixed structural modeling also relies on this assumption of an early detection threshold for company default.

In the case of Black and Cox (1976), this default threshold is interpreted as the amount to be provisioned to repay part of the debt or to cover the costs associated with it.

The original model of Merton (1974) is based on a perfect market hypothesis. In this context, all spreads between corporate and government bonds are attributable to credit risk. In practice, a large part of the spreads is due to the liquidity differential between the two classes of debt. Ericsson and Renault (2001) propose to incorporate the liquidity phenomena in a Merton model (1974).

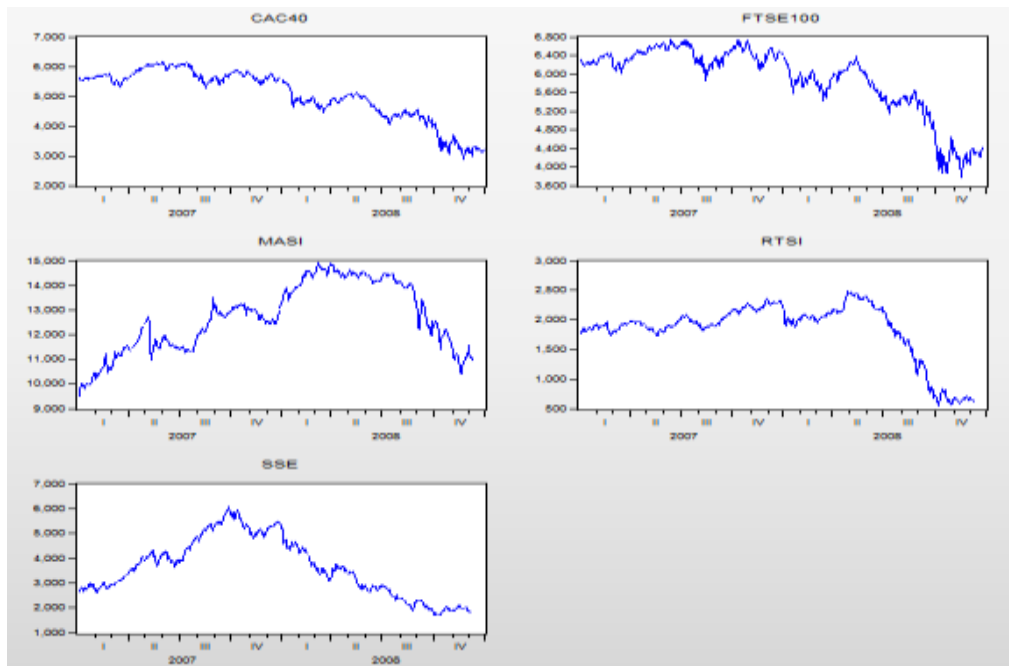
Another aspect of market perfection in the original model is that the liquidation of the firm is done at no cost, with creditors fully grasping the value of the firm in the event of bankruptcy. Bankruptcy costs (reflecting court costs, deteriorating corporate image, loss of clients, etc.) are nevertheless an important consequence of collective proceedings. They have been taken into account by Leland (1994), Leland and Toft (1996) and Briys and de Varenne (1997) in a frame with early revelation of the default.

The assumption of an exogenous recovery rate has also proved very useful for the development of the structural approach since only the probability of default must be calculated.

The assumption of Gaussian profitability for the firm's assets has also been criticized, based on the empirical results of Sarig and Warga (1989). These authors highlight structural slopes by strictly decreasing lines of credit that are difficult to model in the original model for low-debt companies. Bhattacharya and Mason (1981), Zhou (1997), Barone-Adesi and Colwell (1999) and Moraux (1999) and Li (2001) have considered multiple processes for the value of the firm's assets.

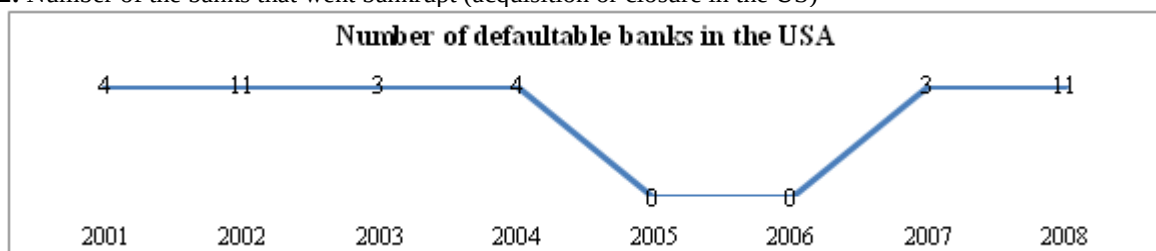
Annex 1: Black September on the global stock exchanges

The bad news coming from the United States has heckled the stock markets of the whole world. A fall that aggravates a trend already very dark. A brief overview of the stock market collapse.



Source: (graph prepared by us under Eviews), CAC40 Paris Stock Exchange, RTSI: Moscow Stock Exchange, SSE: Shanghai Stock Exchange, FTSE100: London Stock Exchange, MASI: Casablanca Stock Exchange

Annex 2: Number of the banks that went bankrupt (acquisition or closure in the US)



Source: FDIC (Federal Deposit Insurance Corporation)

Annex 3

First, we know (from formula 1.20) that the probability of default in neutral risk Q is written in the form:

$$PD(T)^Q = \Phi\left(\frac{\ln\left(\frac{K}{V_0}\right) - \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

Secondly we have for a normal distribution:

a) $\Phi(X) \rightarrow 0$ when..... $X \rightarrow -\infty$

b) $\Phi(X) \rightarrow 1$ when..... $X \rightarrow \infty$

when $T \rightarrow \infty$

$$\lim_{T \rightarrow \infty} PD(T)^Q = \lim_{T \rightarrow \infty} \Phi\left(\frac{\ln\left(\frac{K}{V_0}\right) - \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

If we notice that $T \geq \sqrt{T}$ we get:

$$\lim_{T \rightarrow \infty} \frac{\ln\left(\frac{K}{V_0}\right)}{\sigma\sqrt{T}} = 0 \text{ and } \lim_{T \rightarrow \infty} \frac{-\left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = -\infty$$

Consequently

$$\text{so: } \lim_{T \rightarrow \infty} \left(\frac{\ln\left(\frac{K}{V_0}\right) - \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right) \rightarrow -\infty$$

$$\lim_{T \rightarrow \infty} PD(T)^Q = \lim_{T \rightarrow \infty} \Phi\left(\frac{\ln\left(\frac{K}{V_0}\right) - \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right) = 0$$

when $T \rightarrow 0$

We suppose that $\frac{K}{V} < 1$

$$\lim_{T \rightarrow 0} \frac{\ln\left(\frac{K}{V_0}\right)}{\sigma\sqrt{T}} \rightarrow -\infty \text{ and } \lim_{T \rightarrow 0} \frac{-\left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \rightarrow 0$$

from where

$$\lim_{T \rightarrow 0} \frac{\ln\left(\frac{K}{V_0}\right) - \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \rightarrow -\infty$$

Then

$$\lim_{T \rightarrow 0} PD(T)^Q = \lim_{T \rightarrow 0} \Phi\left(\frac{\ln\left(\frac{K}{V_0}\right) - \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right) = 0$$

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