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# Stability of Electrostatic Ion Cyclotron Harmonic Waves in a Multi-ion Plasma

**Noble P. Abraham**[1], **G. Sreekala**[2], **Sijo Sebastian**[3], **Manesh Michael**[4], **Prijil Mathew**[5], **Amir M.**[6], **R. Jayapal**[7], **G. Renuka**[8], **Chandu Venugopal**[9]

[1, 2, 3. 4, 5, 6, 9] School of Pure & Applied Physics, Mahatma Gandhi University, Kottayam. 686560, Kerala, India

[7] Department of Physics, Devaswom Board College, Thalayolaparambu, 686605, Kerala, India

[8]Kerala State Council for Science, Technology & Environment, Sasthra Bhavan, Pattom, Thiruvananthapuram - 695 004, Kerala, India

**Abstract:** ***Electrostatic ion cyclotron (EIC) waves in a multi-ion plasma is studied. Electrons and hydrogen ions drifting with velocities*** $V_{de}$ ***and*** $V_{dH^+}$ ***, respectively, along the ambient magnetic field and positively and negatively charged oxygen ions constitute the plasma under consideration. This composition very well approximates the plasma environment around a comet. Analytical expressions for the frequency and growth / damping rate of the EIC waves around the higher harmonics of hydrogen ion gyrofrequency have been derived. The EIC waves propagate at frequencies around the harmonics of the hydrogen ion gyrofrequency and the wave growth decreases rapidly for higher harmonics. We find that, the wave can be driven unstable by the hydrogen ion drift velocity*** $V_{dH^+}$ ***alone, at small*** $k_\perp \rho_{LH^+}$ ***as well as electron drift velocity*** $V_{de}$ ***at large*** $k_\perp \rho_{LH^+}$ ***. Also, the growth rate is dependent on the densities and temperature anisotropies of the various constituent ions.***

**Keywords:** electrostatic, ion cyclotron instability, multi-ion plasma

## 1. Introduction

A low frequency instability that can be driven unstable by electrons drifting parallel to the magnetic field is the electrostatic ion cyclotron (EIC) instability. This instability is important in a number of contexts as it has one of the lowest thresholds among current driven instabilities [1]. In an electron-ion plasma, the EIC wave has a frequency around the ion gyrofrequency $\omega_{ci}$ and propagates nearly perpendicular to the external magnetic field $B_0$; the wave is driven unstable by the drifting electrons due to the small, but definite wave vector component along the magnetic field [2].

Studies on EIC waves in multi-ion plasmas, including negative ions, have been carried out by a number of researchers. For example, both fluid and kinetic analyses of EIC waves in a negative ion plasma were carried out respectively by D'Angelo & Merlino [3] and Chow & Rosenberg [4]. Other aspects of these waves explored include the effect of a magnetic shear [5] and the effect of more than one type of positive ion [6]. Instability studies of higher harmonic EIC waves in a negative ion plasma has been done by Rosenberg & Merlino [7]. Similar studies, both on the fundamental [8] and higher harmonic [9] electrostatic dust cyclotron (EDC) waves have also been done.

Other relevant studies has been the effect of carrier heating on the modulation and demodulation of an electromagnetic wave, over a wide range of cyclotron frequencies, in a semiconductor plasma [10] and the effect of a parallel electric field on the electromagnetic ion cyclotron wave in a hot anisotropic plasma [11].

Low frequency electrostatic and electromagnetic waves have been observed in the plasma environments of comet Halley [12] and Giacobini-Zinner [13]. The electrostatic turbulence in the frequency range on 0-300 Hz has been suggested as being due to lower hybrid waves [14]. Ion-acoustic waves have also been observed by the ICE spacecraft about $2\times10^6$ km away from Giacobini-Zinner [15]. Also, the spacecraft Sakigake [16] has observed electromagnetic waves around the gyrofrequencies of $O^+$ (and $H_2O^+$).

It is now well accepted that a cometary plasma environment contains different species of ions such as $H^+$, $O^+$, $H_2O^+$, etc. [17]. In addition to these positive ions, negative ions such as $O^-$ have been unambiguously identified [18].

We have, therefore, studied the stability of EIC waves in a multi-ion plasma of electrons (denoted by $e$), hydrogen ($H^+$) and positively and negatively charged oxygen (denoted respectively by $O^+, O^-$) ions. The electrons and hydrogen drift with velocities $V_{de}$ and $V_{dH}$ respectively. Expressions for the real frequency and the growth / damping rate for harmonics of EIC wave in such a plasma have been derived. We find that, the wave can be driven unstable by the hydrogen ion drift velocity $V_{dH^+}$ alone at small $k_\perp \rho_{LH^+}$ as well as electron drift velocity $V_{de}$ at large $k_\perp \rho_{LH^+}$ where $\rho_{LH^+}$ is the Larmour radius of the hydrogen ions.


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## 2. The Dispersion Formula

We consider, in this paper, the stability of EIC waves in a plasma composed of electrons ($e$), hydrogen ($H^+$) and positively ($O^+$) and negatively ($O^-$) charged oxygen ions; the electrons and hydrogen drifting with velocities $V_{de}$ and $V_{dH^+}$ respectively along $B_{0z}$, the background magnetic field oriented along the $z$-direction. The equilibrium distribution is then given by

$$f_{0\alpha} = \frac{n_{0\alpha}}{\pi^{3/2} v_{T\perp\alpha}^2 v_{T\|\alpha}} \exp\left(-\frac{v_\perp^2}{v_{T\perp\alpha}^2} - \frac{(v_\| - V_{d\alpha})^2}{v_{T\|\alpha}^2}\right); \quad (1)$$

$$\alpha = e, H^+, O^+, O^-$$

with $V_{de} \neq V_{dH^+} \neq 0$. Also

$$v_{T\perp\alpha} = \left(\frac{2k_B T_{\perp\alpha}}{m_\alpha}\right)^{1/2} \text{ and } v_{T\|\alpha} = \left(\frac{2k_B T_{\|\alpha}}{m_\alpha}\right)^{1/2} \quad (2)$$

In (2), $k_B$ is the Boltzmann's constant; $m_\alpha$ and $T_\alpha$ denote, respectively, the mass and temperature of species $\alpha$. Also the symbols $\|$ and $\perp$ indicate, respectively, the directions parallel and perpendicular to the magnetic field $B_0$.

Substituting (1) into the dispersion formula for electrostatic waves and carrying out the various integrations, we get the formula for electrostatic waves of frequency $\omega$ and wave vector $\vec{k}$ as [19]

$$\text{ò}(\vec{k}, \omega) = 1 + \sum_\alpha \sum_l \frac{\omega_{p\alpha}^2 \Lambda_l(\eta_\alpha)}{k^2 v_{T_{\perp\alpha}}^2} \times \left\{ \frac{T_{\perp\alpha}}{T_{\|\alpha}} \left[2(1 + \zeta_\alpha Z(\zeta_\alpha)\right] + 2\frac{l\omega_{c\alpha}}{k_\| v_{T_{\|\alpha}}} Z(\zeta_\alpha) \right\} = 0 \quad (3)$$

In (3), $\omega_{p\alpha} = \sqrt{4\pi n_{0\alpha} e^2 / m_\alpha}$ and $\omega_{c\alpha} = |e_\alpha| B_0 / (m_\alpha c)$ are respectively the plasma and gyrofrequencies of species $\alpha$; $|e_\alpha|$ their charges, $n_{0\alpha}$ their densities while $c$ is the velocity of light. Also, $\Lambda_l(\eta_\alpha)$ arises from the $dv_\perp$-integration and is defined as [20]

$$\int_0^\infty dv_\perp v_\perp J_n^2\left(\frac{k_\perp v_\perp}{\omega_{c\alpha}}\right) \exp\left(-\frac{v_\perp^2}{v_{T\perp\alpha}^2}\right) = \frac{v_{T\perp\alpha}^2}{2} \exp(\eta_\alpha) I_l(\eta_\alpha) = \Lambda_l(\eta_\alpha) \quad (4)$$

In (4), $I_l$ is the modified Bessel function with an argument,

$$\eta_\alpha = \frac{k_\perp^2 v_{T\perp\alpha}^2}{2\omega_{c\alpha}^2} \quad (5)$$

$Z(\zeta_\alpha)$ is the plasma dispersion function which arises from the $dv_\|$-integration and is defined by [21]

$$Z(\zeta_\alpha) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-x^2)}{(x - \zeta_\alpha)} dx \quad (6)$$

with an argument

$$\zeta_\alpha = \frac{\omega - k_\| V_{d\alpha} - l\omega_{c\alpha}}{k_\| v_{T\|\alpha}} \quad (7)$$

## 3. The Dispersion Relation

We derive, in this section, expressions for real frequency for harmonics of the EIC wave around the hydrogen ion gyrofrequency and its growth / damping rate. Since the frequencies under consideration are $<<\omega_{ce}$, we retain only the $l = 0$ contribution to the imaginary term for electrons. Thus the $l = 0$ electron contribution is

$$\chi_e \approx -\frac{2\omega_{pe}^2 \Lambda_0(\eta_e)}{k^2 v_{T\perp e}^2} \frac{T_{\perp e}}{T_{\|e}} (1 + i\sqrt{\pi}\, \zeta_{e0} e^{-\zeta_{e0}^2}) \quad (8a)$$

where $\zeta_{e0} = (\omega - k_\| V_{de}) / (k_\| v_{T\|e})$.

We next consider the hydrogen contribution. For the $l = 0$ contribution we use the small parameter expansion of the plasma dispersion function and its asymptotic expansion for the $l \neq 0$ contributions. Using these [21], we can write down the hydrogen contribution as

$$\chi_{H^+} \approx -\frac{2\omega_{pH^+}^2}{k^2 v_{T\perp H^+}^2} \{\Lambda_0(\eta_{H^+}) \frac{T_{\perp H^+}}{T_{\|H^+}} \times (1 + i\sqrt{\pi}\ \zeta_{H^+0}\ e^{-\zeta_{H^+0}^2}) - \sum_l \frac{l\omega_{cH^+} \Lambda_l(\eta_{H^+})}{\omega^* - l\omega_{cH^+}}\} \quad (8b)$$

with $\omega^* = \omega - k_\| V_{dH^+}$ and $\zeta_{H^+0} = (\omega - k_\| V_{dH^+}) / (k_\| v_{T\|H^+})$.

Finally, since the oxygen ions are much colder than the electrons and hydrogen, we need the asymptotic expansion of the plasma dispersion function for these ions. Using it and simplifying, we can show that the $O^+$ contribution to the dispersion relation is

$$\chi_{O^+} \approx \sum_l \frac{2\omega_{pO^+}^2}{k^2 v_{T\perp O^+}^2} \frac{l\omega_{cO^+}}{\omega - l\omega_{cO^+}} \Lambda_l(\eta_{O^+}) \quad (8c)$$

The contribution of $O^-$ ions is similar to that of (8c).

Substituting the expressions for $\chi_\alpha$ ($\alpha = e, H^+, O^+, O^-$) into (3), we get the dispersion relation for the propagation of EIC waves as,


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$$1+\frac{2\omega_{pe}^2}{k^2 v_{T\perp e}^2}\Lambda_0(\eta_e)\frac{T_{\perp e}}{T_{\|e}}(1+i\sqrt{\pi}\,\zeta_{e0}e^{-\zeta_{e0}^2})$$
$$+\frac{2\omega_{pH^+}^2}{k^2 v_{T\perp H^+}^2}\{\Lambda_0(\eta_{H^+})\frac{T_{\perp H^+}}{T_{\|H^+}}(1+i\sqrt{\pi}\;\zeta_{H^+0}\;e^{-\zeta_{H^+0}^2})$$
$$-\sum_{l\neq 0}\frac{l\omega_{cH^+}}{\omega^*-l\omega_{cH^+}}\Lambda_l(\eta_{H^+})\} \qquad (9)$$
$$-\sum_m\frac{2\omega_{pO^+}^2}{k^2 v_{T\perp O^+}^2}\frac{m\omega_{cO^+}}{\omega-m\omega_{cO^+}}\Lambda_m(\eta_{O^+})$$
$$-\sum_m\frac{2\omega_{pO^-}^2}{k^2 v_{T\perp O^-}^2}\frac{m\omega_{cO^-}}{\omega-m\omega_{cO^-}}\Lambda_m(\eta_{O^-})=0$$

Letting $\omega=l\omega_{cH^+}(1+\Delta)$, we can simplify the real part of (9) to yield

$$\Delta=\frac{k_\square V_{dH^+}(\Gamma_e-\Gamma_\pm)+l\omega_{cH^+}\Gamma_{H^+}}{l\omega_{cH^+}(\Gamma_e-\Gamma_\pm)-k_\square V_{dH^+}\Gamma_\pm} \qquad (10)$$

In (10),

$$\Gamma_e=k^2\lambda_{De}^2+\Lambda_0(\eta_e)+\tau_{eH^+}\delta_{H^+}\Lambda_0(\eta_{H^+}) \qquad (11a)$$
$$\Gamma_{H^+}=\sum_{l\neq 0}\tau_{eH^+}\delta_{H^+}\tau'_{H^+}\Lambda_l(\eta_{H^+}) \qquad (11b)$$
$$\Gamma_\pm=\sum_m\tau_{eO^+}\delta_{O^+}\tau'_{O^+}\Lambda_m(\eta_{O^+})\frac{m\omega_{cO^+}}{l\omega_{cH^+}}+\sum_m\tau_{eO^-}\delta_{O^-}\tau'_{O^-}\Lambda_m(\eta_{O^-})\frac{m\omega_{cO^-}}{l\omega_{cH^+}} \qquad (11c)$$

In (11a) to (11c),

$$\tau_{eH^+}=T_{\|e}/T_{\|H^+},\quad \tau_{eO^+}=T_{\|e}/T_{\|O^+},\quad \tau_{eO^-}=T_{\|e}/T_{\|O^-}$$
$$\tau'_{H^+}=T_{\|H^+}/T_{\perp H^+},\quad \tau'_{O^+}=T_{\|O^+}/T_{\perp O^+},\quad \tau'_{O^-}=T_{\|O^-}/T_{\perp O^-}$$
$$\delta_{H^+}=n_{0H^+}/n_{0e},\quad \delta_{O^+}=n_{0O^+}/n_{0e},\quad \delta_{O^-}=n_{0O^-}/n_{0e} \qquad (12)$$

Also $\lambda_{De}=\sqrt{(k_B T_{\|e}/4\pi n_{0e}e^2)}$ is the electron Debye length.

Using the formula $\gamma=-ImD/\frac{\partial ReD}{\partial\omega_r}$ we can, from (9), get the expression for the growth / damping rate as

$$\gamma=\frac{-\sqrt{\pi}\{\gamma_e+\gamma_{H^+}\}}{\gamma_{lH^+}+\gamma_{mO\pm}} \qquad (13)$$

with

$$\gamma_e=\Lambda_0(\eta_e)\left(\frac{\omega-k_\| V_{de}}{k_\| v_{T\|e}}\right)e^{-\zeta_{e0}^2}$$
$$\gamma_{H^+}=\tau_{eH^+}\delta_{H^+}\Lambda_0(\eta_{H^+})\left(\frac{\omega-k_\| V_{dH^+}}{k_\| v_{T\|H^+}}\right)e^{-\zeta_{H^+0}^2}$$
$$\gamma_{lH^+}=\tau_{eH^+}\delta_{H^+}\tau'_{H^+}\sum_{l\neq 0}\Lambda_l(\eta_{H^+})\frac{l\omega_{cH^+}}{(\omega^*-l\omega_{cH^+})^2}$$
$$\gamma_{mO\pm}=\sum_{s=O^+,O^-}\tau_{es}\delta_s\tau'_s\sum_m\Lambda_m(\eta_s)\frac{m\omega_{cs}}{(\omega-m\omega_{cs})^2}$$

## 4. Discussion

$\Delta$ is a measure of the deviation of the frequency of the EIC wave from the $l^{th}$ harmonic. We find that this deviation is independent of the drift velocity $V_{de}$ of the electrons but is dependent on $V_{dH^+}$, the drift velocity of the hydrogen ions. Also, if we let $T_{\|e}\to 0$, both $\Gamma_{H^+}$ and $\Gamma_\pm$ are equal to zero while $\Gamma_e=1$. Under these conditions, $\Delta$ is directly proportional to $V_{dH^+}$ and inversely proportional to $l$. Also from (13), both $V_{dH^+}$ and $V_{de}$ can contribute to the instability of the wave if they are greater than the phase velocity of the wave. Again the growth rate = 0 when the frequency of the wave is a harmonic of the heavier ion gyrofrequency, or the Doppler shifted frequency is a harmonic of the hydrogen ion gyrofrequency.

## 5. Results

A generally accepted model of a cometary plasma is one composed of hydrogen and positively charged oxygen ions [17,22,23]. An important and exciting discovery was the detection of negatively charged oxygen ions ($O^-$) [18]. We thus study the stability of electrostatic ion cyclotron waves in a plasma of this composition with hydrogen ions and electrons drifting, respectively with velocities $V_{dH^+}$ and $V_{de}$. The parameters used for our computations are as follows: an electron temperature $T_e=2\times10^5$ K, a hydrogen temperature of $T_{H^+}=8\times10^4$ K and oxygen temperatures of $T_{O^+}=T_{O^-}=1.16\times10^4$ K [18,24]. The densities used are: hydrogen density $n_{0H^+}=4.95\,\mathrm{cm}^{-3}$, positively charged oxygen ion density $n_{0O^+}=0.5\,\mathrm{cm}^{-3}$ and negatively charged oxygen ion density $n_{0O^-}=0.05\,\mathrm{cm}^{-3}$ [18,24]. Also the background magnetic field $B_0=75\times10^{-5}$ G [25], while the propagation angle was held a constant at $\theta=88^\circ$ and temperature anisotropies $\tau'_s=1$.

Figure (1) is a plot of the normalised real frequency $\omega_r/\omega_{cH^+}$ versus $k_\perp\rho_{LH^+}$ ($\rho_{LH^+}$ is the hydrogen ion gyroradius) for the parameters given above. The solid lines are for $V_{dH^+}/v_{T\perp H^+}=2.0$ while the dashed lines are for $V_{dH^+}/v_{T\perp H^+}=0$. The figure shows that the wave


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propagates very near the harmonics of the hydrogen ion gyro-frequency $\omega_{cH^+}$ at low $k_\perp \rho_{LH^+}$. The deviations which begin to increase as $k_\perp \rho_{LH^+}$ increases tend to flatten at very high $k_\perp \rho_{LH^+}$.

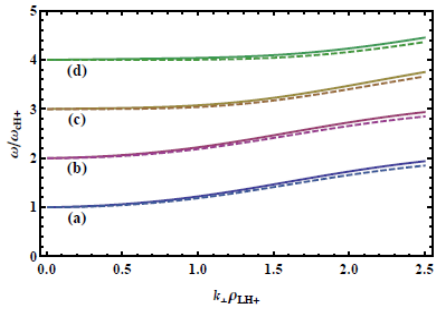



**Figure 1:** Plot of the normalised real frequency $\omega_r / \omega_{cH^+}$ versus $k_\perp \rho_{LH^+}$. The solid lines are for $V_{dH^+} / v_{T\perp H^+} = 2.0$ and dashed lines are for $V_{dH^+} / v_{T\perp H^+} = 0$. Curve (a) is for the first harmonic, curve (b) for the second harmonic, curve (c) for the third harmonic and curve (d) is for the fourth harmonic.

Figure (2) is a plot of the normalised growth rate $\gamma / \omega_{cH^+}$ versus $k_\perp \rho_{LH^+}$ for the densities given above with $V_{dH^+} / v_{T\perp H^+} = 2$ and $V_{de} / v_{T\perp H^+} = 40$. We find the maximum growth rate occurs for the first harmonic; the wave growth decreases rapidly and also shifts towards higher $k_\perp \rho_{LH^+}$ for higher harmonics.

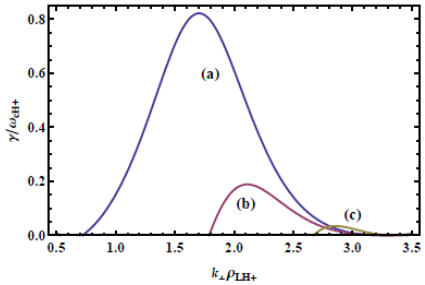



**Figure 2:** Plot of the normalised growth rate $\gamma / \omega_{cH^+}$ versus $k_\perp \rho_{LH^+}$ for the harmonics of the EIC wave. The hydrogen and electron drift velocities are, respectively, $V_{dH^+} / v_{T\perp H^+} = 2$ and $V_{de} / v_{T\perp H^+} = 40$. Curve (a) is for the first harmonic, curve (b) for the second harmonic and curve (c) is for the third harmonic.

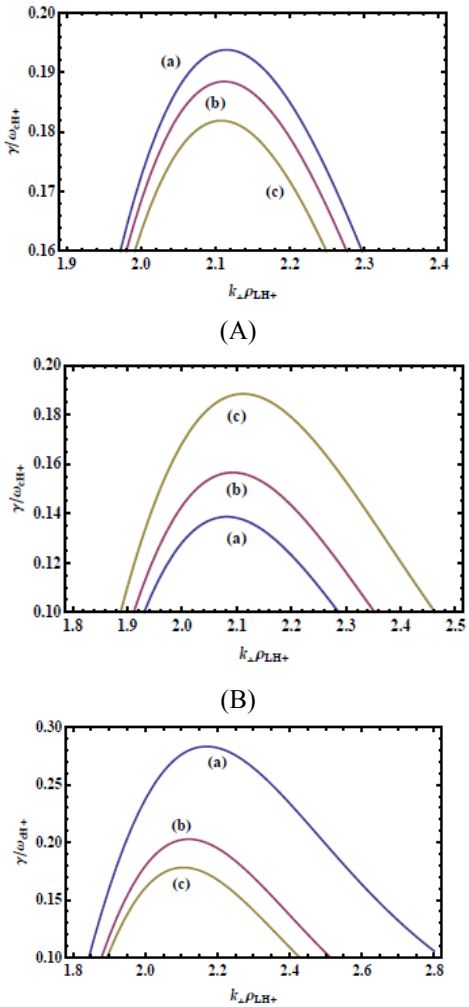


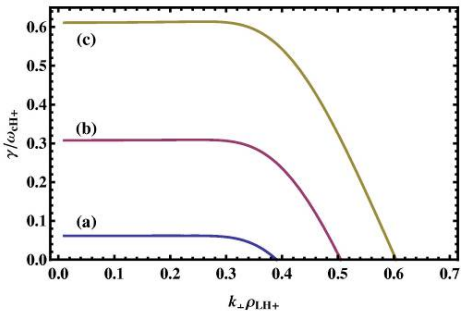

**Figure 3.** Variation of the normalized growth rate $\gamma / \omega_{cH^+}$ versus $k_\perp \rho_{LH^+}$ with the density of various ions for the second harmonic and for the drift velocities $V_{dH^+} / v_{T\perp H^+} = 2$ and $V_{de} / v_{T\perp H^+} = 40$. The top panel depicts the growth rate as a function of the negatively charged oxygen density (curve (a) is for $n_{0O^-} = 0.01\,\mathrm{cm}^{-3}$, curve (b) is for $n_{0O^-} = 0.05\,\mathrm{cm}^{-3}$ and curve (c) is for $n_{0O^-} = 0.1\,\mathrm{cm}^{-3}$, while $n_{0H^+} = 4.95\,\mathrm{cm}^{-3}$ and $n_{0O^+} = 0.5\,\mathrm{cm}^{-3}$). The middle panel depicts the variation of the growth rate with $O^+$ densities: curve (a) is for $n_{0O^+} = 0.1\,\mathrm{cm}^{-3}$, curve (b) is for $n_{0O^+} = 0.25\,\mathrm{cm}^{-3}$ and curve (c) is for $n_{0O^+} = 0.5\,\mathrm{cm}^{-3}$, while $n_{0H^+} = 4.95\,\mathrm{cm}^{-3}$ and $n_{0O^-} = 0.05\,\mathrm{cm}^{-3}$. The lower panel depicts the variation with the hydrogen density $n_{0H^+}$ (curve (a) is for $n_{0H^+} = 2.0\,\mathrm{cm}^{-3}$, curve (b) is for $n_{0H^+} = 4.0\,\mathrm{cm}^{-3}$ and curve (c) is for $n_{0H^+} = 6.0\,\mathrm{cm}^{-3}$, while $n_{0O^+} = 0.5\,\mathrm{cm}^{-3}$ and $n_{0O^-} = 0.05\,\mathrm{cm}^{-3}$ ).


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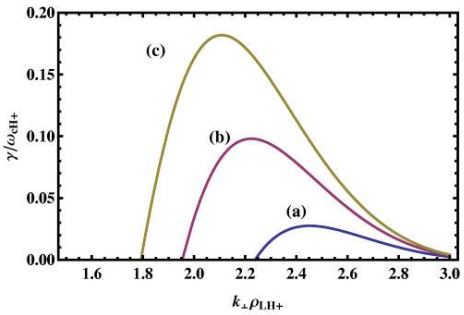
Figure (3) is a plot of the normalized growth rate $\gamma / \omega_{cH^+}$ versus $k_\perp \rho_{LH^+}$ for the second harmonic and for the drift velocities $V_{dH^+} / v_{T\perp H^+} = 2$ and $V_{de} / v_{T\perp H^+} = 40$; the other parameters are as mentioned earlier. The lower panel depicts the variation with the hydrogen density $n_{0H^+}$ (curve (a) is for $n_{0H^+} = 2.0\,\mathrm{cm}^{-3}$, curve (b) is for $n_{0H^+} = 4.0\,\mathrm{cm}^{-3}$ and curve (c) is for $n_{0H^+} = 6.0\,\mathrm{cm}^{-3}$, while $n_{0O^+} = 0.5\,\mathrm{cm}^{-3}$ and $n_{0O^-} = 0.05\,\mathrm{cm}^{-3}$). We find that the growth rate decreases with increasing hydrogen densities. The middle panel depicts the variation of the growth rate with $O^+$ densities: curve (a) is for $n_{0O^+} = 0.1\,\mathrm{cm}^{-3}$, curve (b) is for $n_{0O^+} = 0.25\,\mathrm{cm}^{-3}$ and curve (c) is for $n_{0O^+} = 0.5\,\mathrm{cm}^{-3}$, while $n_{0H^+} = 4.95\,\mathrm{cm}^{-3}$ and $n_{0O^-} = 0.05\,\mathrm{cm}^{-3}$. We find that the growth rate increases with increasing $n_{0O^+}$. Finally, the top panel depicts the growth rate as a function of the negatively charged oxygen density (Curve (a) is for $n_{0O^-} = 0.01\,\mathrm{cm}^{-3}$, curve (b) is for $n_{0O^-} = 0.05\,\mathrm{cm}^{-3}$ and curve (c) is for $n_{0O^-} = 0.1\,\mathrm{cm}^{-3}$, while $n_{0H^+} = 4.95\,\mathrm{cm}^{-3}$ and $n_{0O^+} = 0.5\,\mathrm{cm}^{-3}$). We find the growth rate again decreases with increasing negatively charged oxygen ion densities.

The dependence of drift velocities of hydrogen ions and electrons is studied next. In figure (4), the panel on the top depicts the variation of the normalized growth rate $\gamma / \omega_{cH^+}$ with the drift velocity of hydrogen ions $V_{dH^+} / v_{T\perp H^+}$ for the second harmonic. Here the drift velocity of electrons $V_{de} / v_{T\perp H^+} = 0$. The densities used are: $n_{0H^+} = 4.95\,\mathrm{cm}^{-3}$, $n_{0O^+} = 0.5\,\mathrm{cm}^{-3}$ and $n_{0O^-} = 0.05\,\mathrm{cm}^{-3}$. The other parameters remain unchanged. Curve (a) is for $V_{dH^+} / v_{T\perp H^+} = 0.01$, curve (b) is for $V_{dH^+} / v_{T\perp H^+} = 0.05$ and curve (c) is for $V_{dH^+} / v_{T\perp H^+} = 0.1$. We find that smaller drift velocities of hydrogen ions, in the absence of electron drift, can drive the wave unstable at smaller $k_\perp \rho_{LH^+}$ and that this growth increases with increasing hydrogen ion drift velocities. The bottom panel shows the variation of the normalized growth rate $\gamma / \omega_{cH^+}$ with the drift velocity of electrons $V_{de} / v_{T\perp H^+}$ for the second harmonic. Here the drift velocity of hydrogen ions is fixed at $V_{dH^+} / v_{T\perp H^+} = 2.0$. The densities and the other parameters remain unchanged. Curve (a) is for $V_{de} / v_{T\perp H^+} = 35.0$, curve (b) is for $V_{de} / v_{T\perp H^+} = 38.0$ and curve (c) is for $V_{de} / v_{T\perp H^+} = 40.0$. We find that electron drift drives the wave unstable at large $k_\perp \rho_{LH^+}$ and the growth rate increases with increasing electron drift velocities.

Finally, in figure (5), the dependence of the normalized growth rate $\gamma / \omega_{cH^+}$ with temperature anisotropies is depicted. Here the temperatures used are: an electron temperature $T_{\|e} = 2\times10^5$ K, a hydrogen temperature of $T_{\|H^+} = 8\times10^4$ K and oxygen temperatures of $T_{\|O^+} = T_{\|O^-} = 1.16\times10^4$ K. The drift velocities are $V_{dH^+} / v_{T\perp H^+} = 2$ and $V_{de} / v_{T\perp H^+} = 40$ and the densities remaining unchanged. For the second harmonic, curve (a) shows the case for temperature isotropic ions i.e., $\tau_s^{'} = 1.0$ and curve (b) depicts the case for temperature anisotropic ions $\tau_s^{'} = 0.1$. We find that temperature anisotropies of the constituents enhance the growth rate of electrostatic ion cyclotron harmonic waves.




(A)




(B)

**Figure 4**. Variation of the normalized growth rate $\gamma / \omega_{cH^+}$ versus $k_\perp \rho_{LH^+}$ with drift velocities of hydrogen ions and electrons. The top panel depicts the variation with the drift velocity of hydrogen ions $V_{dH^+} / v_{T\perp H^+}$ for the second harmonic. Here $V_{de} / v_{T\perp H^+} = 0$. Curve (a) is for $V_{dH^+} / v_{T\perp H^+} = 0.01$, curve (b) is for $V_{dH^+} / v_{T\perp H^+} = 0.05$ and curve (c) is for $V_{dH^+} / v_{T\perp H^+} = 0.1$. The bottom panel shows the variation


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with the drift velocity of electrons $V_{de} / v_{T\perp H^+}$ for the second harmonic. Here $V_{dH^+} / v_{T\perp H^+} = 2.0$. Curve (a) is for $V_{de} / v_{T\perp H^+} = 35.0$, curve (b) is for $V_{de} / v_{T\perp H^+} = 38.0$ and curve (c) is for $V_{de} / v_{T\perp H^+} = 40.0$.

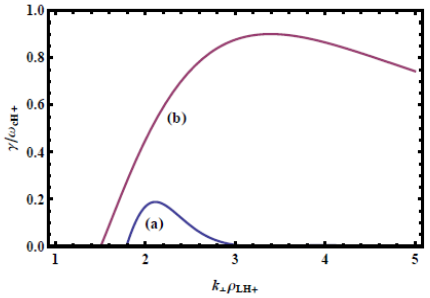



**Figure 5.** Variation of the normalized growth rate $\gamma / \omega_{cH^+}$ versus $k_\perp \rho_{LH^+}$ with temperature anisotropies of the constituents. For the second harmonic, curve (a) is for $\tau_s^{'} = 1.0$ (temperature isotropic ions and electrons) and curve (b) is for $\tau_s^{'} = 0.1$.

## 6. Conclusions

We have, in this paper, studied the electrostatic ion cyclotron harmonic waves in a multi-ion plasma consisting of electrons and hydrogen ions drifting along the ambient magnetic field, with velocities $V_{de}$ and $V_{dH^+}$, respectively and positively and negatively charged oxygen ions. This composition very well approximates plasma environment around a comet. Analytical expressions for the frequency and growth / damping rate of the EIC waves around higher harmonics of hydrogen ion gyrofrequency have been derived. Computation shows that the waves propagate around harmonics of the hydrogen ion gyrofrequency.

We find that, the growth rate increases with increasing positively charged oxygen ion density $n_{O^+}$, while it decreases with increasing hydrogen ion densities $n_{H^+}$ and negatively charged oxygen ion densities $n_{O^-}$. The wave can be driven unstable by the hydrogen drift velocity $V_{dH^+}$ alone, at lower $k_\perp \rho_{LH^+}$ as well as electron drift velocity $V_{de}$ at large $k_\perp \rho_{LH^+}$. We also find that the growth rate increases with increasing temperature anisotropies of the plasma constituents.

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