

The Rigid Body Motion Table in a Matrix Form

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Abstract: Rigid body motion table is a celebrated piece of knowledge and educational cornerstone found in almost all theoretical mechanics textbooks. What these tables lack is the complete presentation of all quantities describing the rigid body motion, namely the orientation or the angular position of the rigid body. This omission is due to the vector formalism that is used. To correct this inconsistency the authors show how the same table should be written using matrix formalism. The authors also show how the fixed table helps students to much easier derive frequently used formulas and equations regarding theoretical mechanics and more specifically rigid body motion.

Keywords: Rigid body motion table, Rigid body kinematics, Rigid body dynamics

1. Introduction

The current paper presents the well-known rigid body motion table that is found in almost every theoretical mechanics textbook. The used vector formalism burdens the equation derivation and in most cases becomes the reason why in most rigid body motion tables the angular position of the rigid body is absent. This table summarizes all quantities describing the rigid body linear and rotational motion. There are two columns in the table: the first column describes the variables of the linear rigid body motion; the second column shows the variables of the rotational rigid body motion. The table then compares quantities in rows between linear motion and rotational motion.

2. Background

Here we shall not cite theoretical mechanics textbooks as such citations could be comprehended as negative advertisement for these wonderful books. We try to fill a small gap in the puzzle of theoretical mechanics education and not to criticize.

Instead, we shall mention that some novel free access online stereoscopic 3D simulations for e-learning mechanics [1] were created thanks to the table in matrix form. See Table 1 and Figure 1.

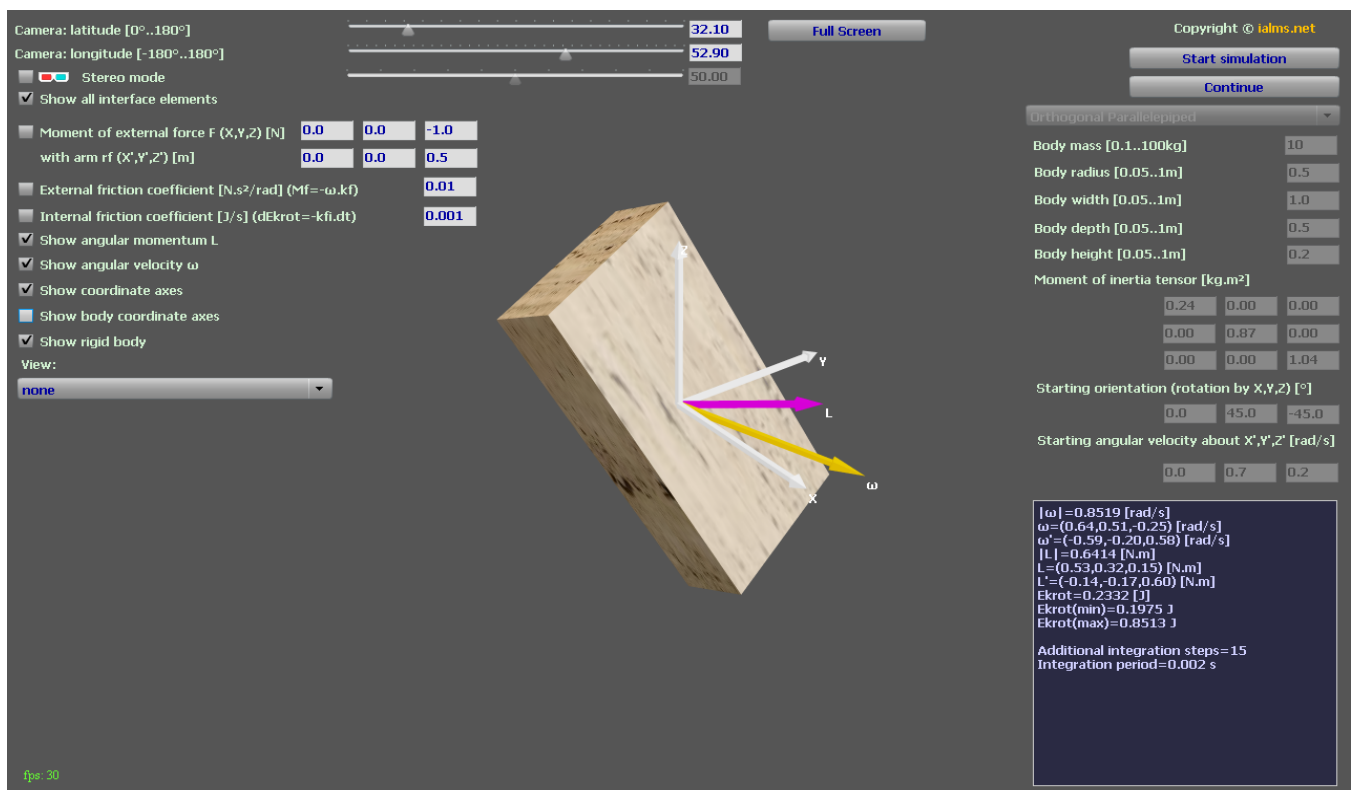


Figure 1: Simulation of rigid body motion available free at www.ialms.net.

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3. The Rigid Body Motion Table

The description of the current approach would require a concise overview of the basic terms and formalism utilized in order to clarify the presentation. When talking about rigid body motion, a non-inertial reference frame needs to be defined. This reference frame is connected to the body and is called the body frame. The rigid body does not move, nor rotate in respect to the body frame. Thus the motion of the body equals the motion of the body frame in respect to the inertial space frame. This motion is linear and rotational with 6 degrees of freedom [2]. Table 1 shows the variables, describing the two motions (linear and rotational) of the rigid body. All variables are compared one by one. Such tables are common in theoretical mechanics textbooks, but very often they miss fundamental variables and relationships such as rotational position (orientation). Table 1 is generalized and utilizes the matrix presentation of the state variables, describing the rigid body state.

We shall make a brief overview of a few notations and equations known from the linear algebra university course. These notations and equations will be used in the current paper extensively. Three dimensional vectors are preferably presented in matrix form, either as three element row-matrix or three element column-matrix, as follows:

$$(1.) \quad \vec{b} \Leftrightarrow \mathbf{b} = \begin{bmatrix} b_x & b_y & b_z \end{bmatrix}$$

Matrices are denoted with bold characters. Transposed matrix of matrix $\mathbf{R}$ is denoted with $\mathbf{R}^{\sim}$ and the derivative in respect to time is expressed with the dot notation $\dot{\mathbf{R}}$. In this paper, the anti-symmetric matrix of a vector will be used extensively.

If vectors $\vec{a}$ and $\vec{b}$ are given in matrix form $\mathbf{a}$ and $\mathbf{b}$, then their inner product (dot product) is given by:

$$(2.) \quad \vec{a} \cdot \vec{b} \Leftrightarrow \mathbf{a} \mathbf{b}^*$$

If vector $\vec{b}$ is given in matrix form $\mathbf{b}$, then its anti-symmetric matrix is denoted with $\mathbf{b}^*$ and is equal to:

$$(3.) \quad \mathbf{b}^* = \begin{bmatrix} b_x & b_y & b_z \end{bmatrix}^* = \begin{bmatrix} 0 & -b_z & b_y \\ b_z & 0 & -b_x \\ -b_y & b_x & 0 \end{bmatrix}$$

Matrix $\mathbf{b}^*$ has three degrees of freedom and is isomorphic to the 3D vector $\mathbf{b}$. The asterisk notation may be better understood as *asterisk operator*. Thus the *inverse asterisk operator* is trivial:

$$(4.) \quad \mathbf{b} = \begin{bmatrix} \mathbf{b}_{32}^* & \mathbf{b}_{13}^* & \mathbf{b}_{21}^* \end{bmatrix}$$

When vectors are presented in matrix form, the equivalence between vector product (cross product) and the multiplication with anti-symmetric matrix of a vector is as follows:

(5.)

$$\vec{a} \times \vec{b} \Leftrightarrow \mathbf{a} \mathbf{b}^* = \begin{bmatrix} a_x & a_y & a_z \end{bmatrix} \begin{bmatrix} 0 & -b_z & b_y \\ b_z & 0 & -b_x \\ -b_y & b_x & 0 \end{bmatrix}$$

The outer product of two vectors, presented in matrix form, should also be recalled:

(6.)

$$\mathbf{a} \otimes \mathbf{b} = \begin{bmatrix} a_x b_x & a_x b_y & a_x b_z \\ a_y b_x & a_y b_y & a_y b_z \\ a_z b_x & a_z b_y & a_z b_z \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \begin{bmatrix} b_x & b_y & b_z \end{bmatrix} = \mathbf{a} \tilde{\mathbf{b}}$$

Note that both matrix products $\mathbf{b} \otimes \mathbf{b}$ and $\mathbf{b}^{*2}$ yield symmetric matrices as follows:

$$(7.) \quad \mathbf{b} \otimes \mathbf{b} = \begin{bmatrix} b_x^2 & b_x b_y & b_x b_z \\ b_x b_y & b_y^2 & b_y b_z \\ b_x b_z & b_y b_z & b_z^2 \end{bmatrix}$$

(8.)

$$\mathbf{b}^{*2} = \begin{bmatrix} -b_z^2 - b_y^2 & b_x b_y & b_x b_z \\ b_x b_y & -b_x^2 - b_z^2 & b_y b_z \\ b_x b_z & b_y b_z & -b_x^2 - b_y^2 \end{bmatrix} = \mathbf{b} \otimes \mathbf{b} - \mathbf{b} \mathbf{b}^{\sim} \mathbf{1} = \mathbf{b}^{\sim} \mathbf{b} - \mathbf{b} \mathbf{b}^{\sim} \mathbf{1} = \mathbf{b} \otimes \mathbf{b} - b^2 \mathbf{1}$$

The latter equation is frequently utilized. Matrix $\mathbf{1}$ is the 3x3 identity matrix.

The following operators are essential for the definition of the rigid body rotational motion. If matrix $\mathbf{R}$ is rotation matrix [3] then the following two operators are defined:

$$(9.) \quad \mathbf{X}' \mathbf{R} = \mathbf{X} \quad \text{- Forward frame transform operator}$$

$$(10.) \quad \mathbf{X} \mathbf{R}^{\sim} = \mathbf{X}' \quad \text{- Inverse frame transform operator}$$

These operators are defined using matrix multiplication. The argument $\mathbf{X}$ or $\mathbf{X}'$ is a 3-dimensional vector presented in row-matrix form. Prime-variables are defined in the body reference frame while non-prime variables are defined in the space reference frame. Another operator and its inverse is the well-known *similarity transformation operator* or *matrix rotation operator* [4]:

$$(11.) \quad \mathbf{R}^{\sim} \mathbf{A}' \mathbf{R} = \mathbf{A} \quad \text{- similarity transformation operator or matrix rotation operator}$$

$$(12.) \quad \mathbf{R} \mathbf{A} \mathbf{R}^{\sim} = \mathbf{A}' \quad \text{- inverse similarity transformation operator or inverse matrix rotation operator}$$

The variables, describing the rigid body linear and rotational motion systematized in a table follow:

Table 1: Rigid body dynamic parameters for linear and rotational motion

Parameter	Linear motion	Angular (rotational) motion
Position	Linear position is presented by the radius-vector of the center of mass $\mathbf{r}_c$. All linear variables are given with respect to the space reference frame, while the origin of the body reference frame coincides with the center of mass $\mathbf{r}_c = \overrightarrow{OO'}$.	Angular position or orientation is expressed by the rotation matrix $\mathbf{R}$ or any of its reduction derivatives, such as Euler angles, rotation quaternion, etc. The rotation matrix connects the space and the body reference frames by the <i>forward frame transform operator</i> $\mathbf{X}'\mathbf{R} = \mathbf{X}$ and the <i>reverse frame transform operator</i> $\mathbf{X}\mathbf{R}^{\sim} = \mathbf{X}'$.
Velocity	Linear velocity vector of the center of mass $\mathbf{v}_c = \dot{\mathbf{r}}_c$.	Angular velocity vector $\boldsymbol{\omega}$, where $\boldsymbol{\omega}^* = \dot{\mathbf{R}}^{\sim}\mathbf{R}$ [3] and $\boldsymbol{\omega} = \begin{bmatrix} \omega_{32}^* & \omega_{13}^* & \omega_{21}^* \end{bmatrix}$. In the body reference frame the angular velocity is $\boldsymbol{\omega}' = \boldsymbol{\omega}\mathbf{R}^{\sim}$.
Acceleration	Linear acceleration vector of the center of mass $\mathbf{a}_c = \ddot{\mathbf{r}}_c$.	Angular acceleration vector $\boldsymbol{\varepsilon} = \dot{\boldsymbol{\omega}}$ or $\boldsymbol{\varepsilon}^* = \ddot{\mathbf{R}}^{\sim}\mathbf{R} + \dot{\mathbf{R}}^{\sim}\dot{\mathbf{R}}$ [3] and $\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{32}^* & \varepsilon_{13}^* & \varepsilon_{21}^* \end{bmatrix}$. In the body reference frame the angular acceleration is $\boldsymbol{\varepsilon}' = \boldsymbol{\varepsilon}\mathbf{R}^{\sim}$.
Inertia	Total mass of the rigid body $m = \iiint_V \rho(\mathbf{r})dV = \iiint_V \rho(x, y, z)dx dy dz$ Note: $\mathbf{r} = \begin{bmatrix} x & y & z \end{bmatrix}$.	Moment of inertia tensor in the body reference frame $\mathbf{I}' = \begin{bmatrix} I'_{xx} & I'_{xy} & I'_{xz} \\ I'_{xy} & I'_{yy} & I'_{yz} \\ I'_{xz} & I'_{yz} & I'_{zz} \end{bmatrix} = \iiint_{V'} (r'^2 \mathbf{1} - \mathbf{r}' \otimes \mathbf{r}') dm =$ $= -\iiint_{V'} \mathbf{r}'^{*2} \rho(\mathbf{r}') dV' = -\iiint_{V'} \mathbf{r}'^{*2} \rho(x', y', z') dx' dy' dz'$ $\mathbf{r}'^{*2}$ is a symmetric matrix and its integral is also a symmetric matrix. Hence, tensor $\mathbf{I}'$ is a symmetric matrix and has only six degrees of freedom. In the space reference frame, the moment of inertia tensor is $\mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} = \mathbf{R}^{\sim} \mathbf{I}' \mathbf{R}$. Here $\mathbf{I} = \mathbf{R}^{\sim} \mathbf{I}' \mathbf{R}$ and $\mathbf{I}' = \mathbf{R} \mathbf{I} \mathbf{R}^{\sim}$ are <i>similarity transformations</i> . Note that, while $\mathbf{I}'$ is constant, $\mathbf{I}$ depends on the current body orientation. It is also obvious that $\mathbf{I} = \mathbf{R}^{\sim} \mathbf{I}' \mathbf{R} = -\iiint_{V'} \mathbf{R}^{\sim} \mathbf{r}'^* \mathbf{R} \mathbf{R}^{\sim} \mathbf{r}'^* \mathbf{R} \rho(\mathbf{r}') dV' =$ $= -\iiint_V \mathbf{r}^{*2} \rho(\mathbf{r}) dV$ Matrix $\mathbf{I}$ is also symmetric, because rotation preserves symmetry and anti-symmetry. The moment of inertia tensor $\mathbf{I}$ is a tensor of second rank that relates vector $\mathbf{L}$ to vector $\boldsymbol{\omega}$ (see below). $\mathbf{I}$ is equivalent to a 3x3 matrix.
Momentum	Linear momentum of the center of mass $\mathbf{p}_c = m\mathbf{v}_c$.	Angular momentum. Its differential form is $d\mathbf{L}(\mathbf{r}) = \mathbf{r} d(\mathbf{p}(\mathbf{r}))^* = \mathbf{r}(\mathbf{v}(\mathbf{r}) dm)^* = \mathbf{r}(\boldsymbol{\omega} \mathbf{r}^* dm)^* = -\boldsymbol{\omega} \mathbf{r}^{*2} dm$ and its integral form is

		$\mathbf{L} = \iiint_V d\mathbf{L}(\mathbf{r}) = -\iiint_V \boldsymbol{\omega} \mathbf{r}^{*2} dm = -\boldsymbol{\omega} \iiint_V \mathbf{r}^{*2} \rho(\mathbf{r}) dV = \boldsymbol{\omega} \mathbf{I}.$ In the body reference frame the angular momentum is $\mathbf{L}' = \mathbf{L} \mathbf{R}^{\sim} = \boldsymbol{\omega} \mathbf{I} \mathbf{R}^{\sim} = \boldsymbol{\omega} \mathbf{R}^{\sim} \mathbf{R} \mathbf{I} \mathbf{R}^{\sim} = \boldsymbol{\omega}' \mathbf{I}'.$
Force	Force $\sum \mathbf{F}_{ext} = m \mathbf{a}_c = m \dot{\mathbf{v}}_c = \dot{\mathbf{p}}_c.$ The sum of all external forces, applied to the rigid body, changes the linear momentum of the center of mass in respect to time as follows: $\dot{\mathbf{p}}_c = \sum \mathbf{F}_{ext} \text{ or }$ $\Delta \mathbf{p}_c = \int_0^{\Delta t} \sum \mathbf{F}_{ext} dt.$	Torque (moment of force) $\boldsymbol{\tau} = \mathbf{r} \mathbf{F}^*$. The sum of all external torques, applied to the rigid body, changes the angular momentum in respect to time as follows: $\dot{\mathbf{L}} = \sum \boldsymbol{\tau}_{ext} = \sum \mathbf{r} \mathbf{F}_{ext}^* \text{ or }$ $\Delta \mathbf{L} = \int_0^{\Delta t} \sum \boldsymbol{\tau}_{ext} dt = \int_0^{\Delta t} \sum \mathbf{r} \mathbf{F}_{ext}^* dt.$ In the body reference frame the torque is $\boldsymbol{\tau}' = \boldsymbol{\tau} \mathbf{R}^{\sim}$.
Kinetic energy	Kinetic energy of the linear motion $E_K = \frac{m v_c^2}{2}.$	Kinetic energy of the rotational motion $E_{Krot} = \iiint_V \frac{(dm)(v_{rot}(\mathbf{r}))^2}{2} = \frac{1}{2} \iiint_V \boldsymbol{\omega} \mathbf{r}^* (\boldsymbol{\omega} \mathbf{r}^*)^{\sim} \rho(\mathbf{r}) dV =$ $= -\frac{1}{2} \iiint_V \boldsymbol{\omega} \mathbf{r}^{*2} \boldsymbol{\omega}^{\sim} \rho(\mathbf{r}) dV = \frac{\boldsymbol{\omega} \mathbf{I} \boldsymbol{\omega}^{\sim}}{2} = \frac{\mathbf{n} \mathbf{I} \mathbf{n}^{\sim} \omega^2}{2} = \frac{I \omega^2}{2}$ Note: vector $\mathbf{n} = \frac{\boldsymbol{\omega}}{\omega}$ and $I = \mathbf{n} \mathbf{I} \mathbf{n}^{\sim}$.

4. Benefits

The student benefits from the rigid body motion table in matrix form in a number of situations. Here, some of these situations are disclosed in order to unveil the erroneous approach of presenting the rigid body motion table using only vector notation.

Example 1. Particle Kinematics in Inertial and Non-inertial Reference Frames

We shall examine the equations describing particle kinematics using matrix formalism instead of vector formalism. Let O be the inertial and not moving reference frame and O' be a non-inertial, moving and rotating reference frame (Figure 2.). All quantities in regard to the inertial reference frame are non-primed, while all quantities in regard to the non-inertial reference frame are primed.

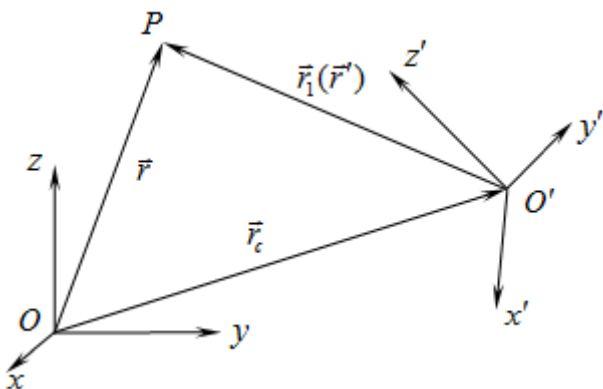


Figure 2: Transformation between frames O and O' .

We want to describe how the kinematic quantities of a particle in regard to the inertial reference frame depend on the kinematic quantities of this particle in regard to the non-inertial reference frame. I.e. we want to make the relation between non-primed and primed quantities. This relation is based on the kinematic quantities of the non-inertial reference frame in regard to the inertial reference frame. The non-inertial reference frame has position and orientation ($\mathbf{r}_c$, $\mathbf{R}$) in regard to the inertial reference frame. The latter quantities may be called linear and angular positions of the non-inertial reference frame. For vector $\mathbf{r}$ and its derivatives we have that:

$$(13.) \quad \mathbf{r} = \mathbf{r}_c + \mathbf{r}_1 = \mathbf{r}_c + \mathbf{r}' \mathbf{R}$$

$$(14.) \quad \dot{\mathbf{r}} = \dot{\mathbf{r}}_c + \dot{\mathbf{r}}_1 = \mathbf{v}_c + \mathbf{v}' \mathbf{R} + \mathbf{r}' \dot{\mathbf{R}}$$

$$(15.) \quad \ddot{\mathbf{r}} = \ddot{\mathbf{r}}_c + \ddot{\mathbf{r}}_1 = \mathbf{a}_c + \dot{\mathbf{v}}' \mathbf{R} + \mathbf{v}' \dot{\mathbf{R}} + \dot{\mathbf{r}}' \dot{\mathbf{R}} + \mathbf{r}' \ddot{\mathbf{R}} = \mathbf{a}_c + \mathbf{a}' \mathbf{R} + 2 \mathbf{v}' \dot{\mathbf{R}} + \mathbf{r}' \ddot{\mathbf{R}}$$

The derivative and double derivative of the rotation matrix are found in [3]. Using (14.) and (15.) we obtain:

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{\mathbf{r}}_c + \dot{\mathbf{r}}_1 = \mathbf{v}_c + \mathbf{v}' \mathbf{R} + \mathbf{r}' \dot{\mathbf{R}} = \mathbf{v}_c + \mathbf{v}' \mathbf{R} + \mathbf{r}' \mathbf{R} \boldsymbol{\omega}^{\sim}$$

$$(17.) \quad \mathbf{a} = \ddot{\mathbf{r}} = \mathbf{a}_c + \mathbf{a}' \mathbf{R} + 2 \mathbf{v}' \mathbf{R} \boldsymbol{\omega}^{\sim} + \mathbf{r}' \ddot{\mathbf{R}} = \mathbf{a}_c + \mathbf{a}' \mathbf{R} + 2 \mathbf{v}' \mathbf{R} \boldsymbol{\omega}^{\sim} + \mathbf{r}' \mathbf{R} \boldsymbol{\omega}^{\sim} \boldsymbol{\omega}^{\sim} + \mathbf{r}' \mathbf{R} \boldsymbol{\varepsilon}^{\sim}$$

The terms in (17.) are the different accelerations that appear when a particle is moving in a non-inertial reference frame. Let the student recognize how these accelerations are called.

Example 2. Free Rigid Body Motion.

The free rigid body motion will be observed as an example. Notably, the free rigid body motion is comprehended with difficulty by students. Here we show that the most difficulties come from the inadequate presentation of variables using only vector mathematical notation. The difficulties are easily overcome using Table 1. variables in matrix form.

The first invariant of free rigid body motion is the angular momentum vector $\mathbf{L}$. The student can easily find the relation between the angular momentum vector and the angular velocity vector $\boldsymbol{\omega}$ using Table 1.:

$$(18.) \quad \mathbf{L} = \boldsymbol{\omega} \mathbf{I} \quad \text{and} \quad \mathbf{L}' = \boldsymbol{\omega}' \mathbf{I}'$$

$$(19.) \quad \boldsymbol{\omega} = \mathbf{L} \mathbf{I}^{-1} \quad \text{and} \quad \boldsymbol{\omega}' = \mathbf{L}' \mathbf{I}'^{-1}$$

If the body reference frame is chosen along the principal axis of inertia, the moment of inertia tensor transforms to a diagonal matrix

$$(20.) \quad \mathbf{I}' = \begin{bmatrix} I'_{xx} & 0 & 0 \\ 0 & I'_{yy} & 0 \\ 0 & 0 & I'_{zz} \end{bmatrix} = \text{const}$$

Its inverse is also diagonal and has the simple form of

$$(21.) \quad \mathbf{I}'^{-1} = \begin{bmatrix} I'_{xx}^{-1} & 0 & 0 \\ 0 & I'_{yy}^{-1} & 0 \\ 0 & 0 & I'_{zz}^{-1} \end{bmatrix} = \begin{bmatrix} 1/I'_{xx} & 0 & 0 \\ 0 & 1/I'_{yy} & 0 \\ 0 & 0 & 1/I'_{zz} \end{bmatrix} = \text{const}$$

The second invariant in free rigid body motion is the kinetic energy of rotational motion E_{Krot} . This conservation parameter leads to the following constraint of movement (Table 1.):

(22.)

$$E_{Krot} = \frac{\boldsymbol{\omega} \mathbf{I} \boldsymbol{\omega}}{2} = \frac{\boldsymbol{\omega} \mathbf{R}^{-1} \mathbf{R} \mathbf{I} \mathbf{R}^{-1} \mathbf{R} \boldsymbol{\omega}}{2} = \frac{\boldsymbol{\omega}' \mathbf{I}' (\boldsymbol{\omega} \mathbf{R})}{2} = \frac{\boldsymbol{\omega}' \mathbf{I}' \boldsymbol{\omega}'}{2} = \text{const}$$

Analogously, the above equation can be transformed in respect to the angular momentum:

Here, vector $\mathbf{n}_L = \frac{\mathbf{L}}{L} = \mathbf{n}_\gamma$ is the normal vector of an invariable plane γ .

$$(23.) \quad E_{Krot} = \frac{\boldsymbol{\omega} \mathbf{I} \boldsymbol{\omega}}{2} = \frac{\mathbf{L} \mathbf{I}^{-1} \mathbf{I} (\mathbf{L} \mathbf{I}^{-1})}{2} = \frac{\mathbf{L} \mathbf{I}^{-1} \mathbf{I} \mathbf{I}^{-1} \mathbf{L}}{2} = \frac{\mathbf{L} \mathbf{I}^{-1} \mathbf{L}}{2} = \frac{\mathbf{L}' \mathbf{I}'^{-1} \mathbf{L}'}{2} = \text{const}$$

Evolving these two equations by the vector components and the principal moments of inertia gives:

(24.)

$$\boldsymbol{\omega}' \mathbf{I}' \boldsymbol{\omega}' = \omega_x'^2 I'_{xx} + \omega_y'^2 I'_{yy} + \omega_z'^2 I'_{zz} = 2E_{Krot} = \text{const}$$

(25.)

$$\mathbf{L}' \mathbf{I}'^{-1} \mathbf{L}' = L_x'^2 I'_{xx}^{-1} + L_y'^2 I'_{yy}^{-1} + L_z'^2 I'_{zz}^{-1} = 2E_{Krot} = \text{const}$$

These are the equations of two ellipsoids which are static (invariant) in the body reference frame. These two ellipsoids depend solely on the inertial properties of the rigid body and will be denoted with $\varepsilon_{\boldsymbol{\omega}}$ and $\varepsilon_{\mathbf{L}}$ respectively. If the rotational kinetic energy is to stay constant, both vectors $\boldsymbol{\omega}'$ and $\mathbf{L}'$ are constrained to point on the surface of the two ellipsoids defined by equations (24.) and (25.) respectively. If either of vectors $\boldsymbol{\omega}'$ or $\mathbf{L}'$ points outside of its constraint ellipsoid, E_{Krot} will increase. Analogously, if either vector points inside its constraint ellipsoid, E_{Krot} will decrease.

By implementing the $\mathbf{L}$ constraint in the E_{Krot} constraint, one observes that:

$$(26.) \quad E_{Krot} = \frac{\boldsymbol{\omega} \mathbf{I} \boldsymbol{\omega}}{2} = \frac{\mathbf{L} \mathbf{I}^{-1} \mathbf{I} \boldsymbol{\omega}}{2} = \frac{\mathbf{L} \boldsymbol{\omega}}{2} = \frac{\boldsymbol{\omega} \mathbf{L}}{2} = \text{const} \Rightarrow \boldsymbol{\omega} \mathbf{L} = 2E_{Krot} = \text{const}$$

In other words, $\boldsymbol{\omega} \mathbf{L} = \omega L \cos \alpha = 2E_{Krot}$, but

$\mathbf{L} = \text{const} \Rightarrow \omega \cos \alpha = \frac{2E_{Krot}}{L} = \text{const}$. The projection

vector $\boldsymbol{\omega}_L$ of vector $\boldsymbol{\omega}$ in the direction of vector $\mathbf{L}$ is always the same (Figure 3.):

$$\boldsymbol{\omega}_L = \frac{\boldsymbol{\omega} \mathbf{L} \mathbf{L}}{|\mathbf{L}|^2} = \frac{\omega L \cos \alpha}{L^2} \mathbf{L} = \frac{\omega \cos \alpha}{L} \mathbf{L} = \omega \cos \alpha \mathbf{n}_L = \frac{2E_{Krot}}{L} \mathbf{n}_L$$

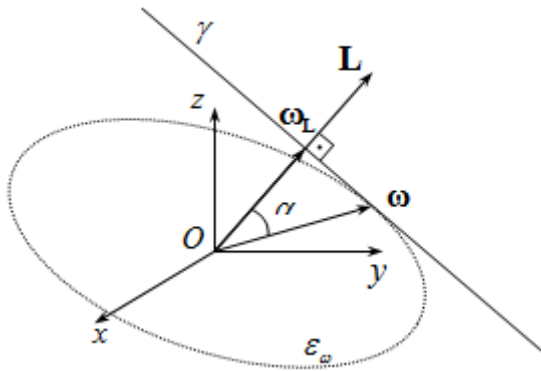


Figure 3: Relation between angular velocity and angular momentum in free rigid body motion.

the institute. He has a large experience in unmanned aircraft design, spacecraft design, analog electronics, analog computers, etc.

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5. Future Scope

Authors intend to conduct experiments with students in General physics, Analytical mechanics and Theoretical mechanics at different universities. The experiments would prove the effectiveness of the proposed systematic formalization for learning the rigid body dynamics. The understanding of the mathematical formalism and the physical and real phenomena by students using this table is expected to improve. This supposition should be proven by the planned experimental tests.

6. Conclusions

Understanding rigid body motion and manipulating the variables describing it may be harmed severely if only vector mathematical formalism is used. By applying matrix formalism the student is liberated from such burdens and is allowed to easily and clearly formalize and describe the studied phenomena. Solving problems becomes a systematic and pleasant pursuit. Furthermore, physical laws become consistently formalized, and formulas: easily derived from one another.

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