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Wavelets: Basic Concepts

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Abstract: - ***Wavelets are mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the ends of mathematics, quantum physics, electrical engineering, and seismic geology. Interchanges between these fields during the last ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar, and earthquake prediction. This paper introduces wavelets to the interested technical person outside of the digital signal processing field. I describe the history of wavelets beginning with Fourier, compare wavelet transforms with Fourier transforms, state properties and other special aspects of wavelets, and finish with some applications.***

Keyword:- Wavelet, Continuous Wavelet Transformation (CWT), Discrete Wavelet Transformation (DWT)

1. **Introduction :**

Wavelet means a "small wave". The smallness refers to the condition that the window function is of finite length (compactly supported).[1] Wave is an oscillating function of time or space and is periodic. In contrast, wavelets are localized waves. They have their energy concentrated in time and are suited to analysis of transient signals. Compare wavelets with sine waves, which are the basis of Fourier analysis. Sinusoids do not have limited duration they extend from minus to plus infinity. And where sinusoids are smooth and predictable, wavelets tend to be irregular and symmetric. Fourier Transform and STFT use waves to analyze signals; the Wavelet Transform uses wavelets of finite energy.

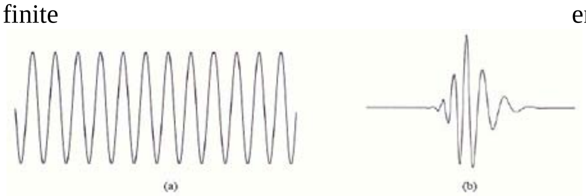


Figure 1: Difference between Wave and Wavelet (a) wave (b) wavelet**.**

2. History:

The first literature that relates to the wavelet transform is Haar wavelet. It was proposed b the mathematician Alfrd Haar in 1909. However, the concept of the wavelet did not exist at that time. Until 1981, the concept was proposed by the geophysicist Jean Morlet. Afterward, Morlet and the physicist Alex Grossman invented the term wavelet in 1984. Before 1985, Haar wavelet was the only orthogonal wavelet people know. A lot of researchers even thought that there was no orthogonal wavelet except Haar wavelet With the appearance of this fast algorithm, the wavelet transform had numerous applications in the signal processing field. Summarize the history.

We have the following table:

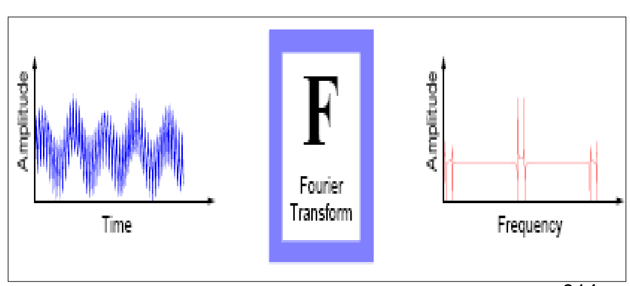
Table I: History for the development of wavelet[2]

S.NO	YEAR	DEVOPMENT
1	1910	Haar families
2	1981	. Morlet, wavelet concept.
3	1984	Morlet and Grossman, "wavelet"
4	1985	Meyer, "orthogonal wavelet
5	1988	Mallat and Meyer, multiresolution
6	1988	Daubechies, compact support orthogonal wavelet
7	1989	Mallat, fast wavelet transform.

3. Fourier Analysis:

3.1 Fourier Analysis:

Signal analysis already have at their disposal an impressive arsenal of tools. Perhaps the most well-known of these is Fourier analysis, which breaks down a signal into constituent sinusoids of different frequencies. Another way to think of Fourier analysis is as a mathematical technique for transforming our view of the signal from time-based to frequency-based.[4]



Fourier analysis

Figure 2: Fourier analysis

3.2 .Short-Time Fourier analysis

In an effort to correct this deficiency, Dennis Gabor (1946) adapted the Fourier transform to analyze only a small section of the signal at a time—a technique called windowing the signal. Gabor's adaptation, called the Short-Time Fourier Transform (STFT), maps a signal into a two-dimensional function of time and frequency.[4]

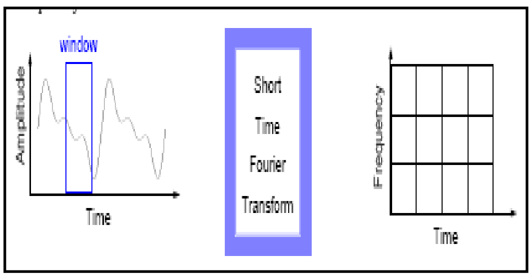


Figure. 3: Short-Time Fourier analysis

4. Need for Wavelet analysis :

Fourier analysis has a serious drawback. In transforming to the frequency domain, time information is lost. When looking at a Fourier transform of a signal, it is impossible to tell when a particular event took place. If the signal properties do not change much over time — that is, if it is what is called a stationary signal—this drawback isn't very important. However, most interesting signals contain numerous non stationary or transitory characteristics: drift, trends, abrupt changes, and beginnings and ends of events. These characteristics are often the most important part of the signal, and Fourier analysis is not suited to detecting them

A wavelet is a waveform of effectively limited duration that has an average value of zero.

Compare wavelets with sine waves, which are the basis of Fourier analysis.Sinusoids do not have limited duration — they extend from minus to plus infinity. And where sinusoids are smooth and predictable, wavelets tend to be irregular and symmetric.

5. Wavelet Transform:

Wavelet Analysis

Wavelet analysis represents the next logical step: a windowing technique with variable-sized regions. Wavelet analysis allows the use of long time intervals where more precise low frequency information is required and shorter regions where high-frequency information.[6]

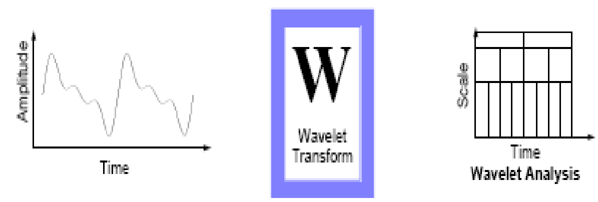


Figure 4: Wavelet Analysis

Wavelet

Wavelet means a "small wave". The smallness refers to the condition that the window function is of finite length (compactly supported). Wave is an oscillating function of time or space and is periodic. In contrast, wavelets are localized waves. They have their energy concentrated in time and are suited to analysis of transient signals. Compare wavelets with sine waves, which are the basis of Fourier analysis. Sinusoids do not have limited duration they extend from minus to plus infinity. And where sinusoids are smooth and predictable, wavelets tend to be irregular and symmetric. Fourier Transform and STFT use waves to analyze signals; the Wavelet Transform uses wavelets of finite energy.

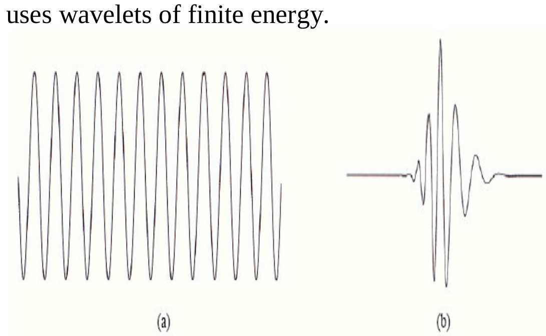


Figure 5: Difference between Wave and Wavelet (a) wave (b) wavelet**.**

6.Mathematical Representation of Wavelet:

Wavelets are functions generated from one single function (basis function) called the prototype or mother wavelet by dilations (scaling) and translations (shifts) in time (frequency) domain[7]. If the mother wavelet is denoted by the $\psi(t)$ there wavelets $\psi_{a,b}(t)$ can be represented as

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right), \qquad a,b \in R, \quad a \neq 0$$

The parameter 'a' is the scaling parameter or scale, and it measures the degree of compression. The parameter 'b' is the translation parameter which determines the time location of the wavelet.

The mother wavelet can be essentially represented at

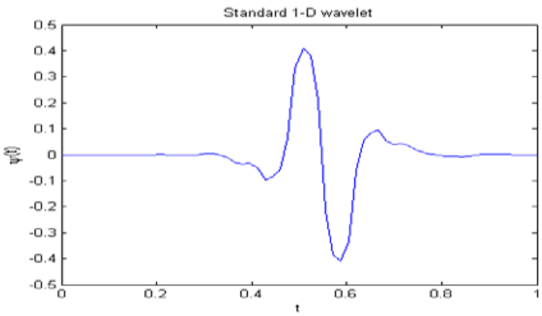


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Figure.6: Waveform of mother wavelet

For any arbitrary a □1 and b = 0, , we can derive that

$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t}{a}\right)$

As shown above, $\psi_{a,b}(t)$ is nothing but a time-scaled (by a) and amplitude-scaled (by a) version of the mother wavelet function , $\psi_{a,b}(t)$ The parameter 'a' causes contraction of□*(t)* n the time axis. If $|a| < 1$, then the wavelet in is the compressed version (smaller support in time- domain) of the mother wavelet and corresponds mainly to higher frequencies. On the other hand, when $|a| > 1$, then $\psi_{a,b}(t)$ has a larger time-width than ψ (t) and corresponds to lower frequencies. The function□□□□□*(t)* is a shift of□□□□□*(t))*in along the time axis by an amount b when b > 0 whereas it is a shift in left along the time axis by an amount b when b < 0.

7. Types of Wavelet Transforms:

There are mainly two types of Wavelet Transforms-

- Continuous Wavelet Transformation (CWT)
- Discrete Wavelet Transformation (DWT)

CWT stands for continuous wavelet transformation is an implemention of the wavelet transform using an arbitrary scales and almost arbitrary wavelet.[4] Where **DWT** stands for Discrete Wavelet transform is an implemention of the wavelet transform using a discrete set of the wavelet scales and translations obeying some define rules.[4]

Two different kinds of wavelet transform can be distinguished, a continuous and a discrete Wavelets transform. The continuous wavelet transform is calculated by the convolution of the Signal and a wavelet function. A wavelet function is a small oscillatory wave which contains both the analysis and the window function. The discrete wavelet transform uses filter banks for the analysis and synthesis of a signal. The filter banks contain wavelet filters and extract the frequency content of the signal in various sub bands.[12]

8. Properties of Wavelet:

- 'Regularity' defined as: if r is an integer and a function is r-time continuously differentiable at x_0, then the regularity is r. If r is not an integer, let n be the integer such that $n < r < n+1$, then function has a regularity of r in x_0 if its derivative of order n resembles $(x - x_0)$ r− n locally around x_0.
- This property is useful for getting nice features, such as smoothness, of the reconstructed signals.
- The support of a function is the smallest space-set (or time-set) outside of which function is identically zero.
- The number of vanishing moments of wavelets determines the order of the polynomial that can be approximated and is useful for compression purposes.
- The wavelet symmetry relates to the symmetry of the filters and helps to avoid dephasing in image processing. Among the orthogonal families, the Haar wavelet is the only symmetric wavelet. For biorthogonal wavelets it is possible to synthesize wavelet functions and scaling functions that are symmetric or antisymmetric.

9. Advantages of Wavelet Theory:

- One of the main advantages of wavelets is that they offer a simultaneous localization in time and frequency domain.
- The second main advantage of wavelets is that, using fast wavelet transform, it is computationally very fast.
- Wavelets have the great advantage of being able to separate the fine details in a signal. Very small wavelets can be used to isolate very fine details in a signal, while very large wavelets can identify coarse details.
- A wavelet transform can be used to decompose a signal into component wavelets.
- In wavelet theory, it is often possible to obtain a good approximation of the given function f by using only a few coefficients which is the great achievement in compare to Fourier transform.
- Wavelet theory is capable of revealing aspects of data that other signal analysis techniques miss the aspects like trends, breakdown points, and discontinuities in higher derivatives and self-similarity.
- It can often compress or de-noise a signal without appreciable degradation.
- Wavelet-based compression provides multi-resolution hierarchical characteristics. Hence an image can be compressed at different levels of resolution and can be sequentially processed from low resolution to high resolution.
- High robustness to common signal processing.

10. Wavelet Application:

The following applications show just a small sample of what researchers can do with wavelets.

- Computer and Human Vision
- FBI Fingerprint compression
- De-noising Noisy Data
- Detecting self similar behavior in a time series
- Musical tone generation

11.Conclusion

This report discusses the main issues regarding the wavelet transform and provides a general introduction of the wavelet theory. The various wavelet analysis methods are described in comparison to the widely known Fourier transform. The Fourier transform only retrieves the global frequency content of a signal, all time information is lost. To overcome this problem the short time Fourier transform is developed, however this method suffers from a limitation due to a fixed resolution in both time and frequency. Wavelet analysis has a wide range of applications. In this report the applications which are of most interest for mechanical engineers have been mentioned. Wavelet analysis can be applied for numerical analysis, i.e. solving ordinary and partial differential equations. Furthermore the Wavelet transform is used in signal analysis, e.g. for compression, denoising and feature extraction. For control applications wavelets are used in motion tracking, robot positioning, identification and both linear and nonlinear control purposes. Finally, wavelets are a powerful tool for the analysis and adjustment of audio signals.

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