

Theoretical Analysis of Dynamic Erosion Phenomenon As Applied to Open Channel Flow

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Abstract: *In the classical literature of fluid mechanics, soil mechanics as well as from the books of sediment transportation which are subdivision of geology, does not give any relationship between amount of erosion or sedimentation and fluid flow. With change in velocity of water an unusual pattern observed hence, it becomes very important to analyze quantitative parameters to predict behavior of soil erosion. In this project, by merging concept of fluid mechanics in open channel flow, soil mechanics and sediment transportation it has been observed that between two dimensionless parameters a relationship exist which ultimately yield into a constant and gives a prospect of existence of a third dimensionless number.*

Keywords: Akydual Number (Ak), Relative Dynamic Viscosity (μ_R), Soil erosion, Soilsedimentation,

1. Introduction

The tractive force is that force which is exerted on soil particle on the wetted perimeter of a channel by the water flowing in the channel

The tractive force is actually a shear stress multiplied by an area upon which the stress acts

A component of the force of gravity on side slope material is added to analysis, whereby gravity will tend to cause soil particles to roll or slide down towards the channel invert (bed or bottom)

Whether the tractive + gravity forces are less than the critical tractive force of materials along the wetted perimeter of the channel than the channel should not experience erosion. Thus the critical tractive force is the value at which erosion would be expected to begin.

2. Literature Survey

F.M.WHITE[1]“In natural river systems the bed load stays in equilibrium since the same amount of sediment is eroded as is deposited in a river section”

JAMES A.FAY [2] “The volume change behavior and inter-particle friction depend on the density of the particles, the inter-granular contact forces”

Dr. A. K. JAIN [3], “The uniform flow will be obtained when the resistance force equals the force causing the flow.”

3. Process Parameter

3.1 For Soil

1. **Shape:** Assumed to be spherical throughout.
2. **Size Distribution:** Same size distribution of soil particle throughout the channel.
3. **Effective Stress:** Force that keeps a collection of particles rigid. The term "effective" meant the calculated stress that was effective in moving soil, or causing displacements. It represents the average stress carried by the soil skeleton.
4. **Porosity:** Measure of the void (i.e., "empty") spaces in a material, and is a fraction of the volume of voids over the total volume.
5. **Permeability:** In fluid mechanics and the earth sciences is a measure of the ability of a porous material (often, a rock or unconsolidated material) to allow fluids to pass through it.
6. **Shear Strength:** A term used in soil mechanics to describe the magnitude of the shear stress that a soil can sustain. The shear resistance of soil is a result of friction and interlocking of particles, and possibly cementation or bonding at particle contacts. Due to interlocking, particulate material may expand or contract in volume as it is subject to shear strains. If soil expands its volume, the density of particles will decrease and the strength will decrease; in this case, the peak strength would be followed by a reduction of shear stress.

3.2 For Water

1. **Viscosity:** The viscosity of a fluid is a measure of its resistance to gradual deformation by shear stress or tensile stress. For liquids, it corresponds to the informal concept of "thickness".
2. **Density:** Water is an incompressible fluid so density remain constant throughout the flow channel.
3. **Atmospheric Pressure:** The pressure exerts by atmosphere on water is same everywhere.
4. **Slope of Water Flow Path:** The slope of water path is taken as constant for study purpose.

4. Theoretical Analysis

Cohesion is the attraction of one water molecule to another resulting from hydrogen bonding (water-water bond).

Adhesion is similar to cohesion except with adhesion involves the attraction of a water molecule to a non-water molecule (water-solid bond).

Soil consistence provides a means of describing the degree and kind of cohesion and adhesion between the soil particles as related to the resistance of the soil to deform or rupture.

Since the consistence varies with moisture content, the consistence can be described as dry consistence, moist consistence, and wet consistence.

The rupture resistance is a field measure of the ability of the soil to withstand an applied stress or pressure as applied using the thumb and forefinger.

Soil consistency is defined as the relative ease with which a soil can be deformed use the terms of soft, firm, or hard. Consistency largely depends on soil minerals and the water content.

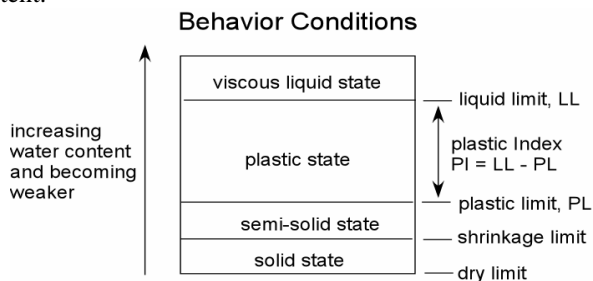


Figure 1: Soil Behavior with Water Contents

Forces on Bed soil particles

The friction forces (resisting particle movement)
 $W_s \tan \theta = \text{Weight of Soil}$

Where θ is the angle of repose of the bed motion and is the weight of a soil particle.

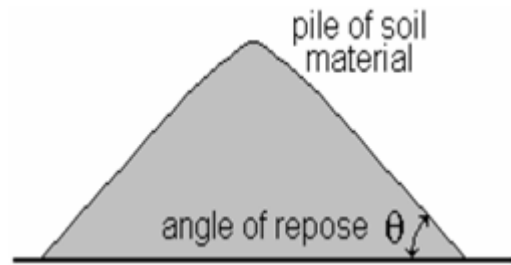


Figure 2: Angle of repose

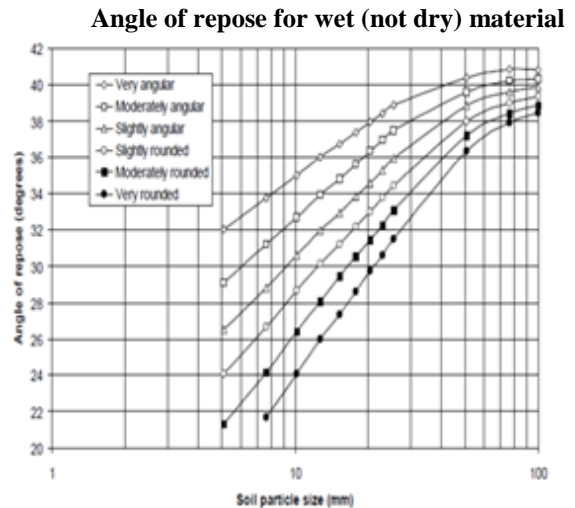


Figure 3: Angle (degrees from horizontal) θ , for non-cohesive earthen materials (adapted from USBR Hyd lab report Hyd-366)

Now the shear on a bed particle

$$T_{shear} = A \cdot \tau_{bed}$$

Where A is the effective particle area and τ_{bed} is the shear stress exerted on the particle by flow of water in the channel. Now when particle movement is impending on the channel bed then

$$W_s \tan \theta = A \cdot \tau_{bed}$$

$$\tau_{bed} = \frac{W_s \tan \theta}{A}$$

Now forces on side-slope particle

The component of gravity down the side slope is

$$W_s \sin \phi$$

Where ϕ is the angle of the side slope

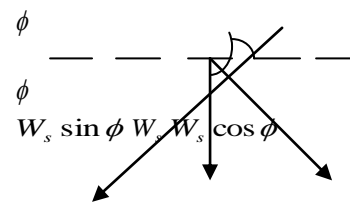


Figure 4: Forces on side slope particle
 The force on the side slope particles in the direction of water flow is

$$A \tau_{side}$$

So the resultant force on the side slope particle is downward and towards the direction of where τ_{side} the shear stress exerted on the side slope particle by the flow of water. Water flow and magnitude is

$$\sqrt{W_s^2 \sin^2 \phi + A^2 \tau_{side}^2}$$

So the resistance to particle movement on the side slope is due to $W_s \cos \phi$ multiplied by coefficient of friction $\tan \theta$

$$(W_s \cos \phi)(\tan \theta)$$

Thus when particle movement is impending on the side slope:

$$(W_s \cos \phi) \tan \theta = \sqrt{W_s^2 \sin^2 \phi + A^2 \tau_{side}^2}$$

$$\tau_{side} = \frac{W_s}{A} \sqrt{\cos^2 \phi \tan^2 \theta - \sin^2 \phi}$$

Or

$$\tau_{side} = \frac{W_s}{A} \tan \theta \sqrt{1 - \frac{\sin^2 \phi}{\sin^2 \theta}}$$

Now

τ_{bed} is the critical shear on bed particles

τ_{side} is the critical shear on side slope particles

So

$$K = \frac{\tau_{side}}{\tau_{bed}}$$

K= tractive force ratio

$$K = \sqrt{1 - \frac{\sin^2 \phi}{\sin^2 \theta}} = \cos \phi \sqrt{1 - \frac{\tan^2 \phi}{\tan^2 \theta}}$$

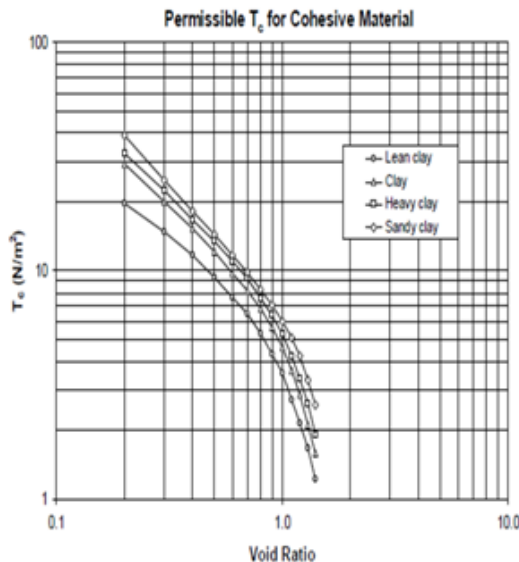


Figure 4: Permissible value of critical shear stress, T_c in N/m^2 , for cohesive earthen material (adapted from USBR Hyd lab report Hyd-366)

Froude number (Fr) is a dimensionless number defined as the ratio of the flow inertia to the external field.

$$Fr = \frac{u}{\sqrt{gl}}$$

National Conference on Knowledge, Innovation in Technology and Engineering (NCKITE), 10-11 April 2015

Kruti Institute of Technology & Engineering (KITE), Raipur, Chhattisgarh, India

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Where u is a characteristic flow velocity, g is in general a characteristic external field, and l is a characteristic length. The Froude number has some analogy with the Mach number. In theoretical fluid dynamics the Froude number is not frequently considered since usually the equations are considered in the high Froude limit of negligible external field, leading to homogeneous equations that preserve the mathematical aspects. For example homogeneous Euler equations are conservation equations

The Reynolds Number, the non-dimensional velocity, can be defined as the ratio of

•The inertia force ($\rho u l$), and

•The viscous or friction force (μ) and interpreted as the ratio of

•Twice the dynamic pressure (ρu^2), and the shearing stress ($\mu u / l$) and can be expressed as

$$Re = (\rho u^2) / (\mu u / l)$$

$$= \rho u l / \mu$$

$$= u l / \nu$$

Where

Re = Reynolds Number (non-dimensional)

ρ = density (kg/m^3)

u = velocity based on the actual cross section area of the duct or pipe (m/s)

μ = dynamic viscosity (Ns/m^2)

l = characteristic length (m)

ν = kinematic viscosity (m^2/s)

$$Fr = \frac{u}{\sqrt{gl}} \dots (1)$$

$$Re = \frac{\rho u l}{\mu} \dots (2)$$

$$u = Re \frac{\mu}{\rho l}$$

Substituting the value of u in equation (1)

$$Fr = \frac{Re \frac{\mu}{\rho l}}{\sqrt{gl}}$$

$$\frac{Fr}{Re} = \frac{\mu}{\rho l \sqrt{gl}} \dots (3)$$

Now, considering for per unit length of an open channel flow,

$$l = 1 \text{ meter}$$

$$\rho = 1000 \frac{kg}{m^3}$$

$$g = 9.81 \frac{m}{sec^2}$$

Substituting these values in eq.(3)

$$\frac{Fr}{Re} = 3.192 \times 10^{-4} \mu \dots (4)$$

Now putting $Re=100$ in eq.(4)

$$Fr = 3.192 \times 10^{-2} \mu$$

Putting Re =500 in eq.(4)
 $Fr = 15.95 \times 10^{-2} \mu$
 Putting Re =1000 in eq.(4)
 $Fr = 3.192 \times 10^{-1} \mu$
 Putting Re =2000 in eq.(4)
 $Fr = 6.381 \times 10^{-1} \mu$
 Putting Re =3000 in eq.(4)
 $Fr = 9.57 \times 10^{-1} \mu$
 Putting Re =5000 in eq.(4)
 $Fr = 15.95 \times 10^{-1} \mu$
 Putting Re =7000 in eq.(4)
 $Fr = 22.33 \times 10^{-1} \mu$

$$\Rightarrow Ak = \frac{Fr}{Re} = 3.192 \times 10^{-4} \mu_R$$

5. Results and Discussion

1. The theoretical derivation performed in this work indicates the definite relationship between two important dimensionless quantities viz. Reynolds Number (Re) and Froude Number (Fr).

2. Hence, there must exist a third dimensionless number, mentioned so far in this work as Akydual Number(Ak), which should be able to express the relationship between the gravitational energy and the kinetic energy of the fluid as a function of relative dynamic viscosity, μ_R

3. μ_R is defined as the ratio of localized dynamic viscosity at any point of flow and the dynamic viscosity of pure water. Mathematically,

$$\mu_R = \frac{\mu_L}{\mu_{\text{purewater}}}$$

4. The velocity of flow at any particular time will be a direct function of relative viscosity.

5. The shear stress exerted by the flowing water will be a direct function of the flow velocity.

6. Hence, the rate of erosion shall be a direct function of the shear stress exerted by the fluid and the soil strength.

7. Where, Mk shall be positive, there shall be erosion. Where Mk shall be negative, there shall be sedimentation and for Mk of the order of zero or so, there must exist the point or region of dynamic erosion.

6. Scope of Future Work

By this theory we concluded that prediction of sediment transportation will become much simpler by studying local viscosity of the desired point. In future this theory should be experimentally carried out to check out the extent of agreement of experimental results with respect to this theory.

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