$$\begin{bmatrix} NA_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Q \\ H \end{bmatrix} = \begin{bmatrix} -A_{10}H_0 - A_{11}Q_0 + NA_{11}Q_0 \\ q_0 \end{bmatrix} \quad (11)$$

Therefore,

$$\begin{bmatrix} Q \\ H \end{bmatrix} = \begin{bmatrix} NA_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^{-1} \begin{bmatrix} -A_{10}H_0 - A_{11}Q_0 + NA_{11}Q_0 \\ q_0 \end{bmatrix} \quad (12)$$

or,

$$\begin{bmatrix} Q \\ H \end{bmatrix} = A^{-1} \begin{bmatrix} -A_{10}H_0 - A_{11}Q_0 + NA_{11}Q_0 \\ q_0 \end{bmatrix} \quad (13)$$

Matrix equation, Eq. (13) is simplified for **Q** and **H** bearing in mind the following: (1) nowhere the inverse of non-square matrix is obtained; (2) $\mathbf{NA_{11}}$ = a diagonal matrix of size (X,X) in which diagonal element is $nR_{ox}|Q_{ox}|^{n-1}$; and (3) $\mathbf{A_{21}(NA_{11})^{-1}A_{12}}$ = a symmetrical matrix of size (N,N), whose diagonal element (j,j) is positive summation of $\mathbf{(NA_{11})^{-1}}$ for all pipes connected at the unknown-head nodes *j*, and non-diagonal element *(j,jj)* is negative summation of for all pipes connecting node *j* with other unknown- head node *jj*.

Now obtaining $\mathbf{A^{-1}}$ separately using matrix property $AA^{-1} = I$

$$\begin{bmatrix} NA_{11} & A_{12} \\ A_{21} & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (14)$$

Let us assume $\mathbf{NA_{11} = D^{-1}}$, therefore $\mathbf{DA_{11} = N^{-1}}$. Modifying Eq. (14) by replacing $\mathbf{NA_{11}}$ by $\mathbf{D^{-1}}$

$$\begin{bmatrix} D^{-1} & A_{12} \\ A_{21} & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiplying first row by **D**

$$\begin{bmatrix} 1 & A_{12}D \\ A_{21} & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} D & 0 \\ 0 & 1 \end{bmatrix}$$

Multiplying first row by $\mathbf{A_{21}}$ and subtracting it from the second row

$$\begin{bmatrix} 1 & A_{12}D \\ 0 & -(A_{12}DA_{21}) \end{bmatrix} A^{-1} = \begin{bmatrix} D & 0 \\ -(DA_{21}) & 1 \end{bmatrix}$$

Multiplying second row by $\mathbf{-(A^{12}DA^{21})^{-1}}$

$$\begin{bmatrix} 1 & A_{12}D \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} D & 0 \\ (DA_{21})(A_{12}DA_{21})^{-1} & -(A_{12}DA_{21})^{-1} \end{bmatrix}$$

Multiplying second row by $\mathbf{A_{12}D}$ and subtracting it from the first row

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} D-(DA_{21})(A_{12}DA_{21})^{-1}(A_{12}D) & (A_{12}DA_{21})^{-1}(A_{12}D) \\ (DA_{21})(A_{12}DA_{21})^{-1} & -(A_{12}DA_{21})^{-1} \end{bmatrix}$$

Eliminating identity matrix

$$A^{-1} = \begin{bmatrix} D-(DA_{21})(A_{12}DA_{21})^{-1}(A_{12}D) & (A_{12}DA_{21})^{-1}(A_{12}D) \\ (DA_{21})(A_{12}DA_{21})^{-1} & -(A_{12}DA_{21})^{-1} \end{bmatrix}$$

Substituting A^{-1} in Eq. (13)

$$\begin{bmatrix} Q \\ H \end{bmatrix} A^{-1} = \begin{bmatrix} D-(DA_{21})(A_{12}DA_{21})^{-1}(A_{12}D) & (A_{12}DA_{21})^{-1}(A_{12}D) \\ (DA_{21})(A_{12}DA_{21})^{-1} & -(A_{12}DA_{21})^{-1} \end{bmatrix}$$

$$\begin{bmatrix} -A_{10}H_0 - A_{11}Q_0 + NA_{11}Q_0 \\ q_0 \end{bmatrix} (15)$$

Simplifying Eq. (15) for unknowns, **Q** and **H**, substituting $\mathbf{D = NA_{11}^{-1}}$ and reintroducing prefixing subscript,

$$_{t+1}\mathbf{H} = -[\mathbf{A_{21}(NA_{11})^{-1}A_{12}}]^{-1}.[\mathbf{A_{21}(NA)^{-1}}](\mathbf{A^{11}}\,_{t}\mathbf{Q} + \mathbf{A_{10}H_0}) - (\mathbf{A_{21}}\,_{t}\mathbf{Q} - \mathbf{q_0})] \quad (16)$$

$$_{t+1}\mathbf{Q} = (\mathbf{I}-\mathbf{N^{-1}})_{t}\mathbf{Q} - [\mathbf{N^{-1}A_{11}^{-1}}(\mathbf{A_{12}}\,_{t+1}\mathbf{H} + \mathbf{A_{10}H_0})] \quad (17)$$

For the assumed or known values of pipe discharges, Eq. (16) provides improved nodal heads, and corresponding improved pipe discharges for the improved nodal heads are obtained by Eq. (17). The maximum size of matrix involved is (X, X) against $(X+N, X+N)$ required in the solution of matrix equation, Eq. (1).

4. Results and Discussion

Present Water Supply Condition: The method is illustrated with an application to study the various design constraints. Aurangabad City is situated in central part of Maharashtra State. However, a part of water supply network of area from Aurangabad city is considered for optimal design purpose. The source for the study area is in the form of ESR which is located at N-8, Aurangabad with an average ground level of 595.43 m, staging height of 15 m and with a fixed capacity. Various pipe materials such as R.C.C., C.I., and Asbestos cement are laid in the city for the supply system and for feeding ESR.

Data collection:
The following data need to collect while finding out optimal design of water supply system;
Capacity and Location of ESR
Existing water supply network information
All Controlling levels of ESR
All Reduced levels of all components in supply network

Table 1: Daily Water Demand

Particulars	Present Stage Year 2015		Ultimate Stage Year 2045	
	Popul-ation	Qty MLD	Popul - ation	Qty MLD
Domestic Demand	7296	0.985	22645	3.057
Institutional Demand		0.148		0.459
Public Use Demand		0.099		0.306
Total Net Demand		1.231		3.821
Total Gross Demand with 15% Losses		1.449		4.496
Max.Design Demand (2.7 x Total Gross)		3.911		12.139

Rate of Lpcd is assumed as 135, Institutional Demand as 15%, Public Use Demand as 10%. Therefore, total per capita demand calculated comes equal to 200 lpcd.
Hazen William's formula for calculating head loss;
$V = 0.85C_H R^{0.63} S^{0.64}$
where,
C_H = Dimensionless Hazen Williams Coefficient
R = Hydraulic radius of the pipe in m
S = Slope of section
V = Flow velocity through pipe in m/sec

The WaterGEMS software also gives how the chlorine concentrations changes throughout the network. Water quality results are represented either using colour coding, tables or graphs and check the minimum chlorine criteria at

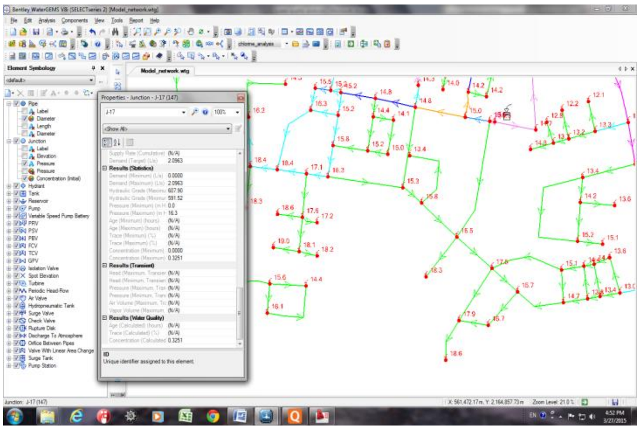

Figure 9: Calculated Chlorine (water quality) concentration at a Node

5. Conclusion

In this paper WaterGEMS software is used for obtaining optimal design of water supply network of a part of Aurangabad city. With the help of WaterGEMS software design of optimal water supply network and its water quality analysis (chlorine analysis) is done with achieving objective of minimizing the overall cost while meeting the water demand requirements at sufficient pressures for specified maximum discharge over a long period of time.
The software also gives different alternative optimal design solution considering pipe diameters, pipe material and roughness coefficient based on head dependent analysis. Mainly in water quality analysis, this software calculates the chlorine concentration with respect to the given initial concentration at all the nodes in water supply network and helps to discover what better initial values for chlorine concentration might be set in the network.
The WaterGEMS software provide required standard and economical environment for design, analysis and troubleshooting of new and existing supply network with accuracy and minimum time duration . The software is also used for solving problems in existing network and also in expansion of existing water supply network.

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