The Hybrid projective synchronization of two non-autonomous systems

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Abstract: ***The hybrid projective synchronization between two non-autonomous systems has been studied in this article. 2D non-autonomous Matie and Croquette systems are taken to showing hybrid projective synchronization. The control functions for synchronization are designed in a proper way with the help of active control method. Numerical simulation and graphical presentation are clearly exhibits that the method is reliable and effective for hybrid projective synchronization between two non-autonomous systems after a small time of duration.***

Keywords: Hybrid, Synchronization, Chaos, Non-autonomous systems.

1. Introduction

The presence of nonlinearity in many field of life sciences it is required that we study and know it.In 1990, it was first done by Ott [1] on chaotic system and after that the several methods are investigated on synchronization of chaotic systems. From the theoretical significance and practical aspect, the chaos synchronization has a lot of contributions due to its potential application in many scientific and engineering fields during the recent years. Different kinds of synchronization for the chaotic systems have been proposed by the researchers and engineers, they also contributed in the group of theoretical results due to their great attraction, including complete synchronization [2-3], partial synchronization [4], anti-synchronization [5], generalized synchronization [6], phase synchronization, anti-phase synchronization [7], projective synchronization [8] etc.

These days the chaos synchronization and its application in secure communication have received very increasing attention by researchers and scientists. It is also having a potential application in engineering Section, such as chemical and biological systems, image processing, information science etc [9-13]. Another important application of chaos synchronization is that it can be applied in secure communication simply. The secure communication systems have the development of a signal which contains the information that is to remain undetectable by interceptors within a carrier signal. To ensure the security of this information by we can insert it it into a chaotic signal that is transmitted to a receiver that who will be able to detect and recover the information from the chaotic signal.

Two most popular dynamical system is predator and prey model and Lotka Voltera model. Synchronization of two dynamical system represents the situation when one dynamical system replicates the another dynamical system in which one system is known as drive (master) system and other is known as response (slave) system with different initial conditions In last few years various synchronization schemes, such as linear and nonlinear feedback synchronization, time delay feedback approach, adaptive control, active control [14-17] etc. have been applied for synchronization of chaotic system.

Synchronization of chaotic dynamical system is another area where scientist and researchers are putting much of their effort these days. The chaotic synchronization was done by Yamada and Fujisaka in 1983 [18] and Afraimovich et al. in 1986 [19], however the major interest in the topics of synchronization were developed after introducing a method by Pecora and Carroll (PC) on chaotic synchronization [2] in 1990. The synchronization of non-autonomous chaotic circuits by using a feedback device to correct the phase of the periodic forcing in the response system was studied by Pecora and Carrol in 1993 [20]. In 2012 T. Botmart et al. [21] studied Synchronization of non-autonomous chaotic systems with time-varying delay via delayed feedback control, in the same year Z. Ye and C. Deng [22] studied Adaptive synchronization to a general non-autonomous chaotic system and its applications. Recently, in 2016, Vijay K. Yadav et al. [23] also studied Synchronization between non-autonomous hyperchaotc systems with uncertainties using active control method. In this manuscript, the authors have tried to study the Hybrid synchronization for 2D non-autonomous Matie and Croquette systems using active control method.The article is organized as follows: The section 1 include the introduction of the article, in section 2, the systems description is introduced. Section 3 and 3.1 contains hybrid projective synchronization between 2D non-autonomous Matie and Croquette systems and numerical simulations results and conclusion of the overall research contribution given in Section 4.

2. Systems description

2.1 Non-autonomous Matie system

The non-autonomous 2D Matie system [24] is given as

$$\frac{dx_1}{dt} = y_1$$

$$\frac{dy_1}{dt} = -(d_1 + b_1\cos(\omega_1 t))x_1 - c_1 y_1 - a_1 x_1^3 \qquad (1)$$

For the values of parameters

$a_1 = 1, b_1 = 14, c_1 = 1.05, d_1 = 5, \omega_1 = 2$ and initial condition $(x_1(0), y_1(0)) = (0.5, 1)$, the system (1) shows the chaotic behavior which is depicted through Fig.1.

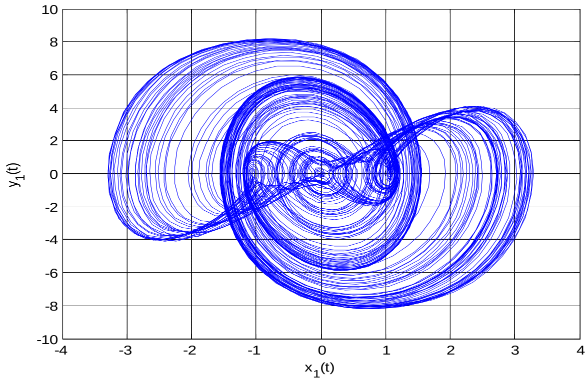


Fig. 1. Phase portrait of Matie system in $x_1 - y_1$ plane.

2.2 Non-autonomous Croquette system

The non-autonomous 2D Croquette system [24] is taken as

$$\frac{dx_2}{dt} = y_2$$

$$\frac{dy_2}{dt} = -c_2 y_2 - a_2 \sin x_2 - b_2 \sin(x_2 - \omega_2 t) \qquad (2),$$

where a_2, b_2, c_2 and ω_2 are the systems parameters? And the values of the parameters

$a_2 = 1.15, b_2 = 1, c_2 = 0.54,$ and $\omega_2 = 1$ and initial condition $(x_2(0), y_2(0)) = (2, 1.5)$, the 2D Croquette system shows chaotic attractor which is described through Fig. 2.

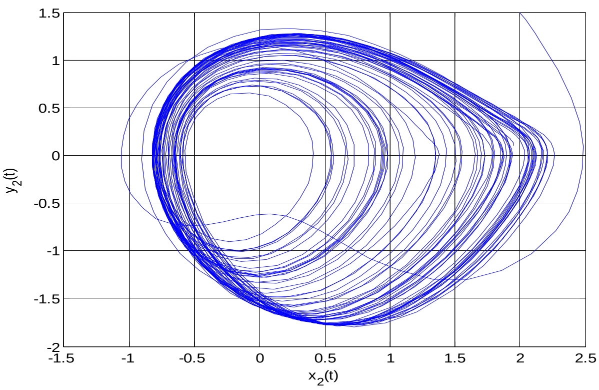


Fig. 2. Phase portrait of Croquette system in $x_2 - y_2$ plane.

3. Hybrid projective synchronization between 2D non-autonomous Matie and Croquette systems

In this section, we study the hybrid projective synchronization between non-autonomous Matie and Croquette systems using active control method. Matie system (1) considered as a master system and Croquette system (2) taken as response system and defined as

$$\frac{dx_2}{dt} = y_2 + u_1(t)$$
$$\frac{dy_2}{dt} = -c_2 y_2 - a_2 \sin x_2 - b_2 \sin(x_2 - \omega_2 t) + u_2(t) \qquad (3)$$

,

where $u_1(t)$ and $u_2(t)$ are control functions. Defining the error function as $e_1 = x_2 - m_1 x_1, \ \ e_2 = y_2 - m_2 y_1$, where m_1, m_2 are the scaling factors. We get the following error systems as

$$\frac{de_1}{dt} = e_2 + (m_2 - m_1) y_1 + u_1(t)$$
$$\frac{de_2}{dt} = -c_2 e_2 - a_2 \sin x_2 - b_2 \sin(x_2 - \omega_2 t) - m_2[-(d_1 + b_1 \cos(\omega_1 t)) x_1$$
$$+ (c_2 - c_1) y_1 - a_1 x_1^3] + u_2(t). \qquad (4)$$

Here our goal is to design the control functions $u_1(t)$, $u_2(t)$ as

$$u_1(t) = -(m_2 - m_1) y_1 + V_1(t)$$
$$\begin{aligned} u_2(t) = {} & a_2 \sin x_2 + b_2 \sin(x_2 - \omega_2 t) \\ & + m_2[-(d_1 + b_1 \cos(\omega_1 t)) x_1 \\ & + (c_2 - c_1) y_1 - a_1 x_1^3] + V_2(t) \end{aligned}.$$

Then the error systems will be

$$\frac{de_1}{dt} = e_2 + V_1(t)$$
$$\frac{de_2}{dt} = -c_2 e_2 + V_2(t) \qquad (5)$$

The error systems (5) considered as control problem, which is a linear system with control inputs $V_1(t)$, $V_2(t)$. Now we design control inputs to stabilize the above system so that e_1 and e_2 converge to zero as time t become large which implies that non-autonomous Matie and Croquette systems are synchronized. There are many choices for control inputs. Let us choose

$$\begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \end{bmatrix},$$

where A is the 2×2 matrix. In order to make the closed loop system stable, matrix A should be selected in such a way that the feedback system will have the eigenvalues $\lambda_i, \ i = 1, 2$ with negative real parts.

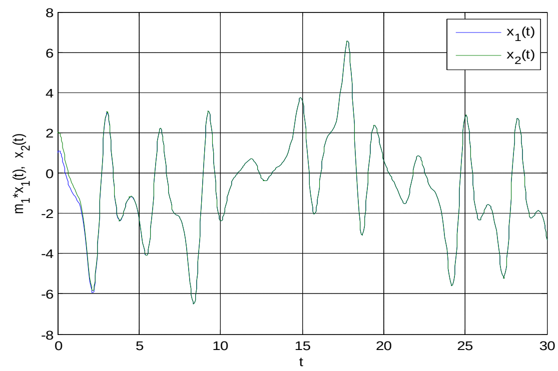
Choosing the matrix A as

$$A = \begin{bmatrix} -1 & -1 \\ 0 & -1 + c_2 \end{bmatrix}$$

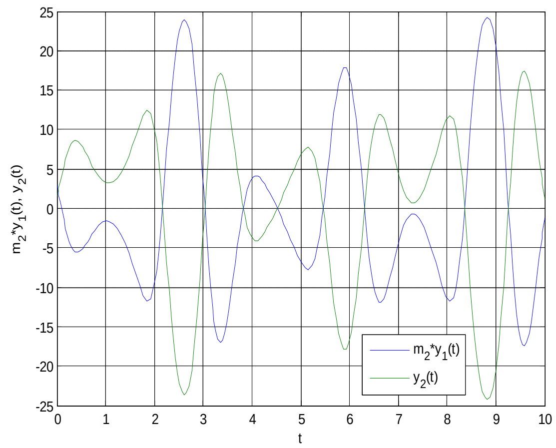
where

$$\frac{de_1}{dt} = -e_1$$
$$\frac{de_2}{dt} = -e_2 \qquad (6).$$

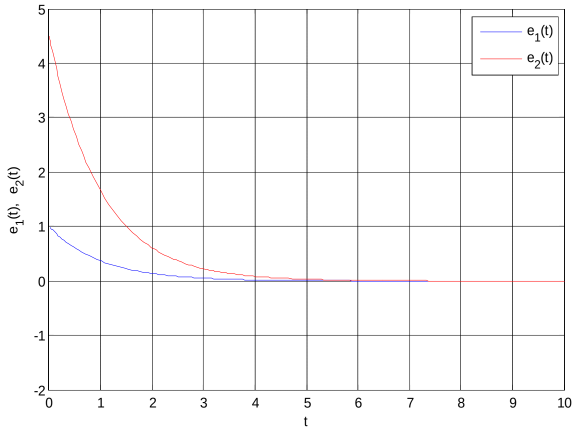
All the eigenvalues of the error systems (6) are negative and hence the systems are stable and required hybrid synchronization between non-autonomous systems is obtained.



(a)



(b)



(c)

Fig. 3. State trajectories of master system (1) and response system (3): (a) between $m_1 * x_1(t)$ and $x_2(t)$; (b) between $m_2 * y_1(t)$ and $y_2(t)$; (c) The evolution of the error functions $e_1(t), e_2(t)$.

3.1 Numerical simulation and result

In this section, we take the earlier considered values of the parameters of both the systems. During synchronization, the initial conditions of drive and response systems are taken as $(x_1(0), x_2(0)) = (0.5, 1)$ and $(y_1(0), y_2(0)) = (2, 1.5)$ respectively. The scaling factors are taken as $m_1 = 2, m_2 = -3$, hence the initial condition of error system will be $(e_1(0), e_2(0)) = (2.5, -2)$.

The hybrid projective synchronization between non-autonomous Matie and Croquette systemsare displayed through Fig. 3(a)-(b). Fig. 3(c) shows that error states converge to zero after a small time of duration.

4. Conclusion

In this article, the hybrid projective synchronization between two 2D non-autonomous Matie and Croquette systems is successfully demonstrated using active control method. The graphical results and numerical simulation, which is clearly exhibits the reliability and potential of the method for hybrid projective synchronization of two non-autonomous systems and the procedure of this article is also effective.

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