Synchronization of Time-Delayed Chaotic Systems via Modified Adaptive Function Projective Synchronization Method

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Abstract: ***This article presents a new modified adaptive function projective synchronization method for the synchronization of time-delayed chaotic systems. The adaptive function projective synchronization controller and identification parameter laws are designed on the basis of Lyapunov-Krasovskii functional approach to stabilize the error system which makes the state vector of two chaotic systems asymptotically synchronized. The proposed method is effectively applied to examine the function projective synchronization for the pair of multiple time-delayed Rössler System for three different cases. The striking feature of the article is the successful graphical presentation of numerical simulation results, which are carried out by means of Runge-Kutta Method for delay differential equations and clearly demonstrate that the given modified method will be advantageous for getting faster function projective synchronization of time-delay chaotic systems. In this review article the author has carefully revisited an article [1] by Sudheer K. S. and Sabir M (Physics Letter A. 375 1176 (2011)) and claim that the new proposed method is substantially more effective and reliable as compared to the said existing method for synchronizing time-delayed chaotic systems.***

Keywords: Chaos, Synchronization, Multiple Delay Rössler System, Lyapunov-Krasovskii Functional.

1. Introduction

Synchronization is a vital phenomenon of chaos that may occur when at least two systems are coupled or one system drives the other. It is a difficult phenomenon because of the extreme dependence on initial conditions. During coupling the trajectories of the systems emerging from two different initial conditions will spread exponentially with time caused due to the transition between system variables, which encourages researchers to take challenges for the investigation of synchronization of coupled chaotic systems. There are several types of synchronization phenomena have been demonstrated and identified, such as complete, phase, anti-phase, hybrid, projective synchronizations etc. ([2] – [11]). Function projective synchronization between two systems is a generalization of projective synchronization, which is synchronized upto a scaling factor. This fascinating phenomenon is firstly taken care by Mainieri and Rehacek [12]. Delay differential equations have prospective applications in science and engineering ([13] – [18]) due to the presence of factors like process time existence of some stage structure, modeling via high dimensional compartmental models, estimation of parameters involved in the models etc. Research on synchronization of time delayed chaotic systems ([19]- [21]) has received impressive consideration of the researchers working in population dynamics, laser physics, physiological model, neural networks, control theory etc. ([22]–[26]) because of their characteristic connection to the systems with memory.

In 2011, Sudheer and sabir [1] have investigated adaptive function projective synchronization with some modifications to synchronize the time-delayed chaotic systems and considered estimated parameters in response system. They have used Lyapunov stability theory during stabilization of error system, while in the present article the author has designed controller function with appropriate estimated parameters and used Lyapunov-Krasovskii Functional approach ([27], [28]) to stabilize the error system. The Rössler system having the same parameter values and scaling function factors as in [1] is taken for Function Projective Synchronization throughout the comparison of effectiveness of the methods. It is seen that the time of synchronization through numerical simulation, which are carried out using Raunga-Kutta method for delay differential equation for proposed method, is less as compared to the exiting method [1].To authenticate proposed method another two cases for various initial conditions are accomplished with graphical plots along with the demonstration of graphs obtained using the method described in [1], which also establishes the fact that the proposed method gives the faster synchronization for different considered cases.

2. Proposed Modified Adaptive Function

Projective Synchronization Method (PMAFPS)

Consider the drive system in the form of

$$x'(t) = f(x(t)) + g(x(t))A + h(x(t-\tau))B,\ \ t > 0$$
$$x(t) = \phi(t), \qquad -\tau < t \le 0$$
(1)

and the response system as

$$y'(t) = f(y(t)) + g(y(t))A + h(y(t-\tau))B + U,\ t > 0$$
$$y(t) = \psi(t), \qquad -\tau < t \le 0$$
(2)

where $x, y \in R^n$ are the state vectors of systems (1) and (2) respectively, $A \in R^{m_1}$ and $B \in R^{m_2}$, are the unknown parameter vectors of the systems, $f(x), f(y) \in R^n, g(x), g(y) \in R^{nxm_1}$ and $h(x(t-\tau)), h(y(t-\tau)) \in R^{nxm_2}$, are nonlinear functions, $\phi(t)$ and $\psi(t)$ represents the trajectories of the solutions in the past, τ is the time delay and $U \in R^n$ is the controller.

Let $e(t) = \lambda(t)x(t) - y(t)$ represents the synchronization error vector, where $\lambda(t)$ is the scaling function matrix. The error dynamical system is

$$\begin{aligned} e'(t) &= \lambda'(t)x(t) + \lambda(t)x'(t) - y'(t) \\ &= \lambda'(t)x(t) + (\lambda(t)f(x(t)) - f(y(t))) + (\lambda(t)g(x(t)) - g(y(t)))A \\ &\quad + (\lambda(t)h(x(t-\tau)) - h(y(t-\tau)))B - U \end{aligned}$$
(3)

Let us design the nonlinear controller function and the adaptive parameter update laws as

$$\begin{aligned} U &= \lambda'(t)x(t) + (\lambda(t)f(x(t)) - f(y(t))) + (\lambda(t)g(x(t)) - g(y(t)))\hat{A}(t) \\ &\quad + (\lambda(t)h(x(t-\tau)) - h(y(t-\tau)))\hat{B}(t) + (1/2 + k)e(t),\ \ k > 0 \end{aligned}$$
(4)

and

$$\hat{A}'(t) = [\lambda(t)g(x(t)) - g(y(t))]^T e(t) + \tilde{A}(t)$$
$$\hat{B}'(t) = [h(x(t-\tau)) - h(y(t-\tau))]^T e(t) + \tilde{B}(t),$$
(5)

where vectors $\hat{A}(t)$ and $\hat{B}(t)$ are the estimated values of unknown parameters A and B respectively, $\tilde{A}(t) = A - \hat{A}(t)$ and $\tilde{B}(t) = B - \hat{B}(t)$ are estimate errors.

Using equation (4), the error dynamical system (3) is reduced to

$$\begin{aligned} e'(t) &= (\lambda(t)g(x(t)) - g(y(t)))(A - \hat{A}(t)) + (\lambda(t)h(x(t-\tau)) - h(y(t-\tau)))(B - \hat{B}(t)) \\ &\quad - (1/2 + k)e(t) \end{aligned}$$
(6)

Let us consider the Lyapunov-Krasovskii Functional [28] to carry out stability analysis as

$$V = \frac{1}{2}e^T(t)e(t) + \frac{1}{2}\int_{-\tau}^{0} e^T(t+\theta)e(t+\theta)d\theta + \frac{1}{2}(\tilde{A}^T(t)\tilde{A}(t) + \tilde{B}^T(t)\tilde{B}(t))$$
(7)

The time derivative of V along the trajectory of error dynamical system is given by

$$\begin{aligned} V' &= e^T(t)e'(t) + \frac{1}{2}(e^T(t)e(t) - e^T(t-\tau)e(t-\tau)) + (\tilde{A}'^T(t)\tilde{A}(t) + \tilde{B}'^T(t)\tilde{B}(t)) \\ &= e^T(t)((\lambda(t)g(x(t)) - g(y(t)))(A - \hat{A}(t)) + (\lambda(t)h(x(t-\tau)) - h(y(t-\tau)))(B - \hat{B}(t)) \\ &\quad - (1/2 + k)e(t)) + \frac{1}{2}(e^T(t)e(t) - e^T(t-\tau)e(t-\tau)) - (\hat{A}'^T(t)\tilde{A}(t) + \hat{B}'^T(t)\tilde{B}(t)), \end{aligned}$$

where, $\tilde{A}'^T(t) = -\hat{A}'^T(t)$ and $\tilde{B}'^T(t) = -\hat{B}'^T(t)$ using adaptive parameters update laws, we get

$$V' = -ke^T(t)e(t) - \frac{1}{2}e^T(t-\tau)e(t-\tau) - \tilde{A}^T(t)\tilde{A}(t) - \tilde{B}^T(t)\tilde{B}(t)) < 0,$$
(8)

where $V \in R$ is positive definite function and $V' \in R$ is negative definite function. Thus $e_i(t) \to 0$ as $t \to \infty$ i = 1, 2, 3. Therefore, the error system is asymptotically stable which means that PMAFPS between the systems (1) and (2) is achieved and it is also seen that the parameters' estimation errors $\tilde{A}(t)$ and $\tilde{B}(t)$ decay to zero as time goes to infinity.

3. System Description

A double delayed Rössler System is given by ([1], [29]) as

$$x_1'(t) = -x_2(t) - x_3(t) + a_1x_1(t-\tau_1) + a_2x_1(t-\tau_2),$$
$$x_2'(t) = x_1(t) + b_1x_2(t),$$
$$x_3'(t) = b_2 + x_1(t)x_3(t) - cx_3(t),$$
(9)

where a_1, a_2 are the geometric factors while b_1, b_2 and c are the usual parameters of a classical Rössler system, τ_1 and τ_2 are time delays. The double delayed Rössler system exhibits the chaotic trajectories for the parameter values $a_1 = 0.2, a_2 = 0.5, b_1 = 0.2, b_2 = 0.2, c = 5.7,$ $\tau_1 = 1.0$ and $\tau_2 = 2.0$ with initial condition $(x_1(t), x_2(t), x_3(t)) = (0.5, 1, 1.5)$ as shown in Figure 1, where $-\tau \le t \le 0.$

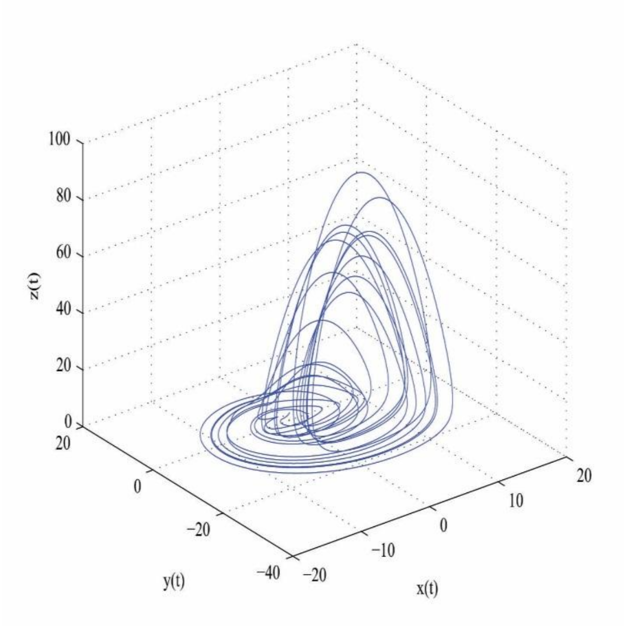

Figure 1: Phase portraits of Rössler system in x1-x2-x3 space.

4. Proposed Modified Adaptive Function Projective Synchronization Between Identical Rössler Systems

In order to achieve PMAFPS behaviour the drive system is taken as (9) and the response system is given by

$$\begin{aligned}
y_1'(t) &= -y_2(t) - y_3(t) + a_1 y_1(t-\tau_1) + a_2 y_1(t-\tau_2) + u_1(t),\\
y_2'(t) &= y_1(t) + b_1 y_2(t) + u_2(t),\\
y_3'(t) &= b_2 + y_1(t) y_3(t) - c y_3(t) + u_3(t),
\end{aligned} \tag{10}$$

where $u_1(t), u_2(t), u_3(t)$ are controllers and parameters (a_1, a_2, b_1, b_2, c) of drive and response systems are unknown. Defining the error states as

$e_i(t) = \lambda_i(t) x_i(t) - y_i(t), i = 1,2,3,$

we get

$$\begin{aligned}
e_1'(t) &= \lambda_1'(t)x_1(t) - \lambda_1(t)(x_2(t) + x_3(t)) + \lambda_1(t)(a_1 x_1(t-\tau_1) + a_2 x_1(t-\tau_2))\\
&\quad + y_2(t) + y_3(t) - a_1 y_1(t-\tau_1) - a_2 y_1(t-\tau_2) - u_1(t),\\
e_2'(t) &= \lambda_2'(t)x_2(t) + \lambda_2(t)(x_1(t) + b_1 x_2(t)) - (y_1(t) + b_1 y_2(t)) - u_2(t),\\
e_3'(t) &= \lambda_3'(t)x_3(t) + \lambda_3(t)(b_2 + x_1(t)x_3(t) - c x_3(t)) - (b_2 + y_1(t)y_3(t) - c y_3(t)) - u_3(t),
\end{aligned} \tag{11}$$

According to our PMAFPS method, we take the synchronization controller as

$$\begin{aligned}
u_1(t) &= \lambda_1'(t)x_1(t) - \lambda_1(t)(x_2(t) + x_3(t)) + y_2(t) + y_3(t) + \hat{a}_1(t)(\lambda_1(t)x_1(t-\tau_1) - y_1(t-\tau_1))\\
&\quad + \hat{a}_2(t)(\lambda_1(t)x_1(t-\tau_2) - y_1(t-\tau_2)) + (1/2 + k_1(t))e_1(t),\\
u_2(t) &= \lambda_2'(t)x_2(t) + \lambda_2(t)x_1(t) - y_1(t) + \hat{b}_1(t)(\lambda_2(t)x_2(t) - y_2(t)) + (1/2 + k_2(t))e_2(t),\\
u_3(t) &= \lambda_3'(t)x_3(t) + \hat{b}_2(t)(\lambda_3(t) - 1) + \lambda_3(t)x_1(t)x_3(t) - y_1(t)y_3(t) - \hat{c}(t)(\lambda_3(t)x_3(t) - y_3(t))\\
&\quad + (1/2 + k_2(t))e_3(t),
\end{aligned} \tag{12}$$

and the estimated parameters as

$$\begin{aligned}
\hat{a}_1'(t) &= (\lambda_1(t)x_1(t-\tau_1) - y_1(t-\tau_1))e_1(t) + (a_1 - \hat{a}_1(t))\\
\hat{a}_2'(t) &= (\lambda_1(t)x_1(t-\tau_2) - y_1(t-\tau_2))e_1(t) + (a_2 - \hat{a}_2(t)),\\
\hat{b}_1'(t) &= (\lambda_2(t)x_2(t) - y_2(t))e_2(t) + (b_1 - \hat{b}_1(t))\\
\hat{b}_2'(t) &= (\lambda_3(t) - 1)e_3(t) + (b_2 - \hat{b}_2(t))\\
\hat{c}'(t) &= -(\lambda_3(t)x_3(t) - y_3(t))e_3(t) + (c - \hat{c}(t)),
\end{aligned} \tag{13}$$

which helps to accomplish the error system as

$$\begin{aligned}
e_1'(t) &= (\lambda_1(t)x_1(t-\tau_1) - y_1(t-\tau_1))(a_1 - \hat{a}_1(t)) + (\lambda_2(t)x_1(t-\tau_2) - y_1(t-\tau_2))(a_2 - \hat{a}_2(t))\\
&\quad - (1/2 + k_1(t))e_1(t),\\
e_2'(t) &= (\lambda_2(t)x_2(t) - y_2(t))(b_1 - \hat{b}_1(t)) - (1/2 + k_2(t))e_2(t),\\
e_3'(t) &= (\lambda_3(t) - 1)(b_2 - \hat{b}_2(t)) - (\lambda_3(t)x_3(t) - y_3(t))(c - \hat{c}(t)) - (1/2 + k_3(t))e_3(t).
\end{aligned} \tag{14}$$

Now proceeding as section 2 with proper choices of controller and estimation of unknown parameters using parameter update laws, we may conclude that the PMAFPS between systems (9) and (10) is achieved.

5. Numerical Simulation and Results

To demonstrate the effectiveness of PMAFPS method, during the numerical simulation, the author has taken the initial conditions of state vectors of drive and response systems as (0.5, 1, 1.5) and (2.5, 2, 2.5) respectively. The true values of unknown parameter vectors of drive and response systems are selected as $(a_1 = 0.2, a_2 = 0.5, b_1 = 0.2, b_2 = 0.2, c = 5.7)$. The initial value of estimated parameters of unknown parameter vector is chosen as $(\hat{a}_1(t), \hat{a}_2(t), \hat{b}_1(t), \hat{b}_2(t), \hat{c}(t))$ = (0, 0, 0, 0, 0). To compare the results with the result proposed by Sudheer and Sabir [1], the parametric values of Rössler system are chosen as given in section 3, the scaling function factors are $\lambda_1(t) = 2 + \sin(t + 35)$, $\lambda_2(t) = 1.5 + \sin(t)$ and $\lambda_3(t) = 2 - \cos(t)$ and the control input as (k_1, k_2, k_3) = (2, 2, 2) as considered in [1]. Figures 2(a) and 2(b) represent that the errors $e_1(t), e_2(t), e_3(t) \to 0$ and the convergence of estimated parameters to the original values after small duration of time, which clearly show that in both the occasions it takes much lesser time in comparison with the results as obtained in Figures 3(a) and 3(b) through the method described in [1]. This validates the feasibility and effectiveness of the new proposed method. Figures 4(a), 4(b), 5(a) and 5(b) depict the numerical simulation results of errors and estimated parameters using the proposed method and the existing method respectively for the initial conditions as (1, 1, 1) and (1.5, 1.5, 1.5) and also for scaling function factors $\lambda_1(t) = 2+\sin(t+35)$, $\lambda_2(t) = 1.5+\sin(t)$ and $\lambda_3(t) = 2-\cos(t)$. Figures 6(a), 6(b), 7(a) and 7(b) describe those for initial conditions of drive and response systems as (2.5, 2, 2.5) and (0.5, 1, 1.5) respectively and for scaling function factors $\lambda_1(t) = 1+\cos(0.05t)$, $\lambda_2(t) = 2-\sin(t)$ and $\lambda_3(t) = 3+\cos(t+10)$.

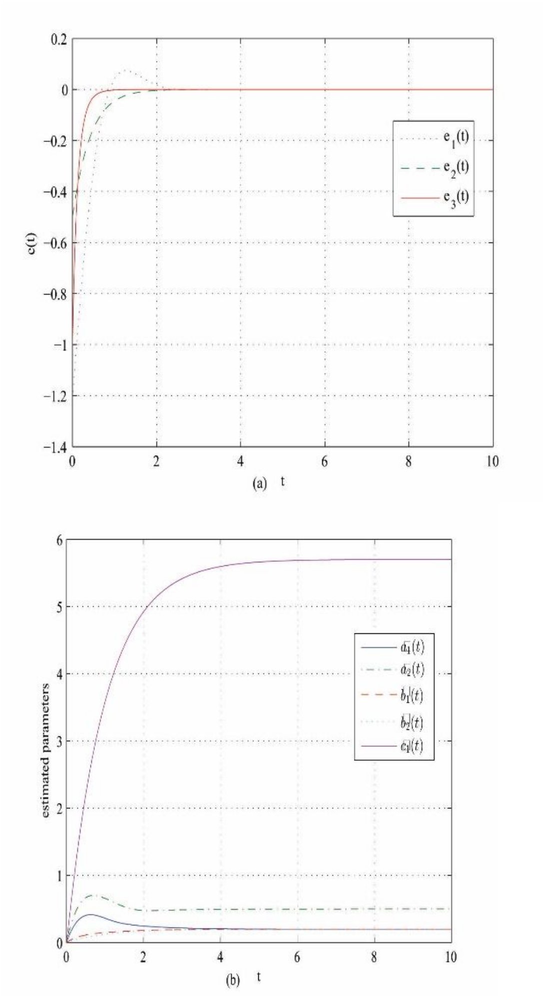

Figure 2: (a) State trajectories of errors system and (b) The estimated parameters using the proposed method for the initial conditions of drive and response systems as (0.5, 1, 1.5) and (2.5, 2, 2.5) respectively.

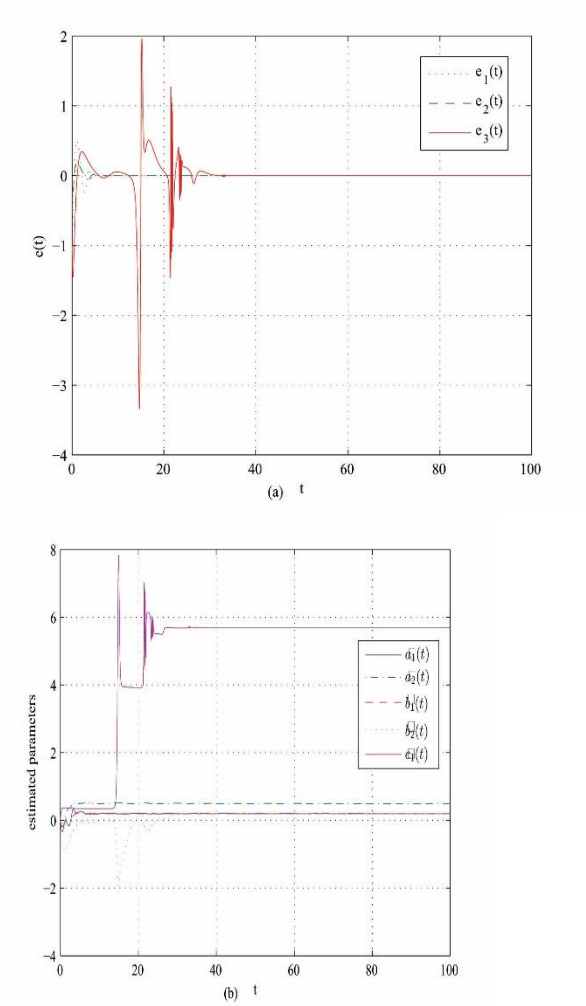

Figure 3: (a) State trajectories of errors system and (b) The estimated parameters obtained using [1] for the initial conditions of drive and response systems as (0.5, 1, 1.5) and (2.5, 2, 2.5) respectively.

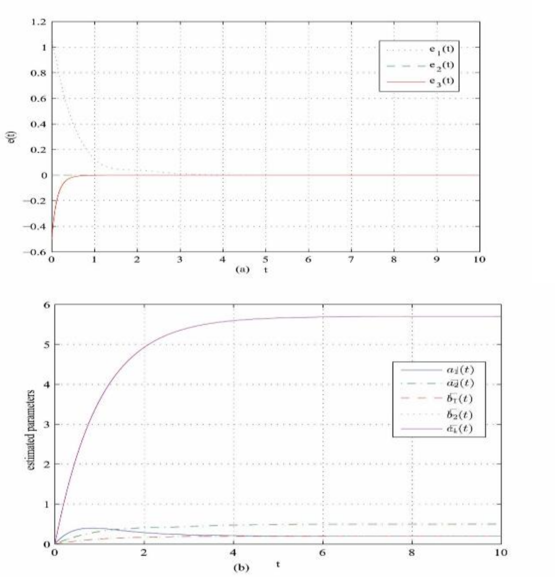

Figure 4: (a) State trajectories of errors system and (b) The estimated parameters using the proposed method for the initial conditions of drive and response systems as (1, 1, 1) and (1.5, 1.5, 1.5) respectively.

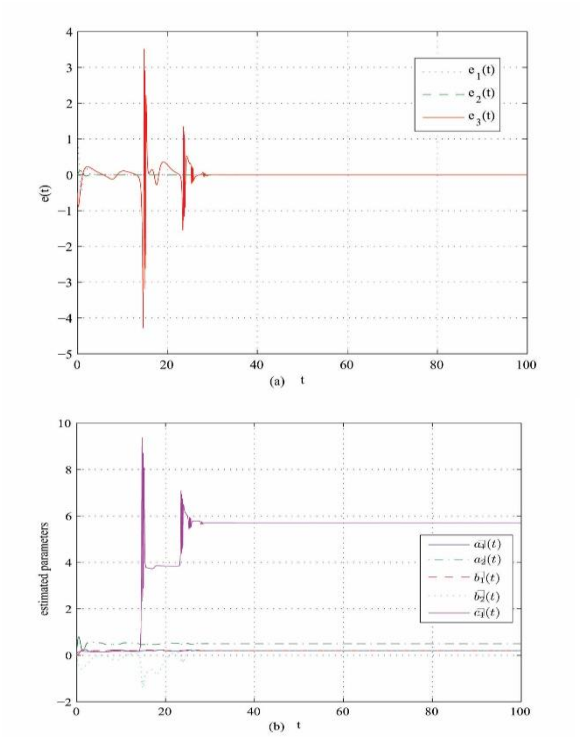

Figure 5: (a) State trajectories of errors system and (b) The estimated parameters obtained using [1] for the initial conditions of drive and response systems as (1, 1, 1) and (1.5, 1.5, 1.5) respectively.

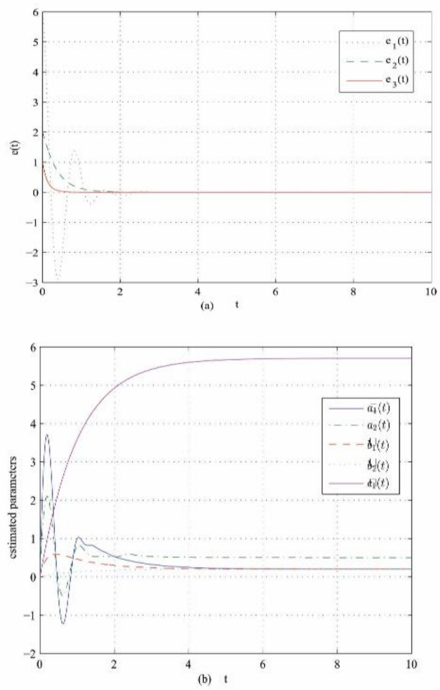

Figure 6: (a) State trajectories of errors system and (b) The estimated parameters using the proposed method for scaling functions _1(t) = 1 + cos(0.05t), _2(t) = 2−sin(t), _3(t) = 3+cos(t+10) and the initial conditions of drive and response systems as (2.5, 2, 2.5) and (0.5, 1, 1.5) respectively.

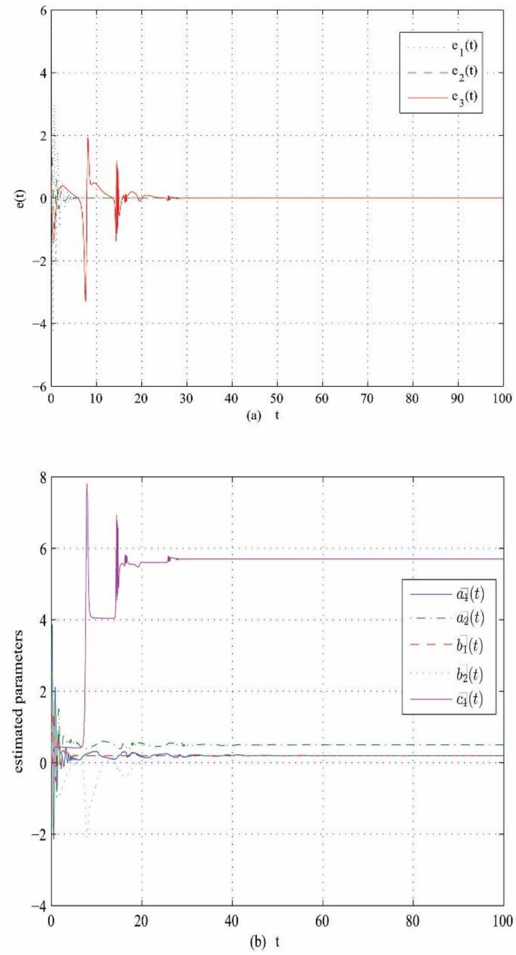

Figure 7: (a) State trajectories of errors system and (b) The estimated parameters obtained using [1] for scaling functions _1(t) = 1+cos(0.05t), _2(t) = 2−sin(t), _3(t) = 3+cos(t+10) and the initial conditions of drive and response systems as (2.5, 2, 2.5) and (0.5, 1, 1.5) respectively.

6. Conclusion

In the present article the author has proposed a new method for function projective synchronization of time-delayed chaotic systems through proper design of controller functions with corresponding parameter identification laws developed on the basis of Lyapunov-Krasovskii stability theory. The technique is applied for function projective synchronization of identical Rössler system during numerical simulation to compare the results with the outcomes described in [1]. The principle highlight of the article is the demonstration of minimum time requirement for synchronization by applying the new method as compared to the earlier results for three different cases. The author is optimist that new proposed method will be valuable to the scientists and engineers working in the field of dynamical system especially those involved in synchronization of time-delayed chaotic systems.

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