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Order, Disorder, Velocity Dispersion and Time-Frequency Analysis

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Abstract: The propagation of wideband pulses in multilayered periodic media shows a variable signal structure depending on the pulse bandwidth B (time resolution). Depending on the ratio between travel time in a layer and the time resolution, the propagated signal can follow different regimes: pulse trains, null (extinction) or dispersive waveform (chirp). The phenomena have been observed both on experimental and simulated data. The simulations have allowed a better understanding the phenomena of velocity dispersion: the presence of a large number of thin layers that could not be separated individually (thickness << time resolution i.e. each individual layer is acoustically "transparent") can affect considerably the propagation and lead to a dispersive "equivalent medium". For such a case, increasing the bandwidth will lead to pulse train structure of propagated signal. Nevertheless, if layer thickness is reduced accordingly, velocity dispersion phenomenon occurs again. Several simulations on random thickness layers have shown that a thickness variation of about 1/8 wavelength (at the central frequency) lead to the destruction of velocity dispersion phenomena. The dispersion is definitely related to a periodicity of the layers. Time-frequency was use to characterize and quantify velocity dispersion. Group velocity was estimated accurately using group delay extracted from Wigner-Ville distribution. An example will be shown for Lamb waves in plates before investigating an industrial application: non destructive evaluation of electrical cables using ultrasounds. The difference between a "good cable" and bad cable'' lies in the insulating material (polyethylene) whose structure can be either amorphous or polymerised (periodical structure at microscopic level). Here again the velocity dispersion is an indicator of ''order''.

1. Introduction

In 1992, Marion and Coudin [1] have published a very interesting paper on experimental investigation of the propagation through a periodic medium. Although the paper was oriented towards geophysical applications, it can lead to several general observations on the propagation of sound in a layered media. The Relevant results are summarized in figure 1 extracted from reference [1]. The samples investigated consist in a sequence of layers of steel and plastic. The stack size is always the same (52 mm). The number of layer in a sample vary from 2 (A) to 64 (F). The same pattern is reproduced periodically for all stacks: 1/3 plastic, 2/3 steel. So, for each sample, the same amount of steel and plastic is present (same total thickness of each material) but it is ordered in a different layered geometry.

The experimental results cover a frequency range from about 50 to about 500 kHz. When a short pulse is transmitted

through a periodic system, the received signal is constituted of several components: direct propagation, single reflexiontransmission on interfaces and multiple reflexions and transmissions.

For thick layers (A), several pulses can be observed and separated that correspond to the cited components. When layer thickness decreases, these echo components are getting closer and closer (B-C) until they cannot be separated any more. When pulses separation is no more possible (D to F), the stack behaves like an "equivalent medium" with two particular properties:

- the minimum propagation time through stacks D-F is about twice the one for A-C
- velocity dispersion occurs that is characterised by the arrival of low frequencies before high frequencies: output is not a succession of pulses but a frequency modulated signal (chirp).



Figure 1: Propagation through multilayered media (time scale: 0-50 µS)

Velocity dispersion occurs when travel time through a layer such a thickness is acoustically transparent, the multiplication of layers leads to quite different properties. It **International Symposium on Ultrasonics-2015, 22-24 January 2015**

Department of Physics, Rashtrasant Tukdoji Maharaj Nagpur University, Nagpur, Maharashtra, India Licensed Under Creative Commons Attribution CC BY is important to note that, thanks to layering, one can build up composite media possessing a sound velocity much lower than the velocity in each constituent.

2. Numerical modelling

It was shown [2] that the propagation through a layered medium can be modelled using propagator matrix method. We will not go into the details of such model but try to use it in order to understand the velocity dispersion observed experimentally.

Several simulations have been run using various signal parameters and stack geometries.

Figures 2 show some representative examples in the case where the travel time is the same in both layer (layer thickness ratio adjusted to velocity ratio in order to have a travel time of $10 \square$ s in each layer).

The reference case consists of a stack of 32 layers of steel and plastic (same properties as [1] and [2]: steel/plastic, velocity: 5535 / 2487 m/s, density: $7.90 / 1.21 \text{ T/m}^3$).

The simulated signal, for the reference case, is a pulse covering the frequency range up to 500 kHz. The reference case (a) corresponds to a stack geometry leading to velocity dispersion: it is clear from observing zero crossings that low frequencies travel faster than high frequencies, in this case.

In the figure 2b, the stack geometry has been kept unchanged while the bandwidth has been increase by a factor of 4 (0-2 MHz), i.e. the time resolution has been improved by a factor of 4. In this case, the dispersive nature of the waves has disappeared and the output signal is a sequence of individual pulses (as in figure 1a).

With the same time resolution (B = 2 MHz), the layer thickness has been divided by 4 for figure 2c. The signal looks exactly the same with a scale factor of 4 in time. Similarly, for 500 kHz bandwidth, when the thickness is multiplied by 4, the output is the same as in figure 2b.



Figure 2: Illustration of scale factor in multilayered propagation.

These figures clearly illustrate the existence of relationship between the observation bandwidth and the layer thickness that can lead to velocity dispersion in the stack. Like other dispersion equations this relationship is more complex than just a linear one. A close look at output spectra has shown the resonant aspect of the power spectrum of transmitted signals as well as its band-pass band-stop structure. Nevertheless, it is clear that physical phenomena involved possess multiple-scale aspects.

As for other dispersive propagation geometries (waveguides, surface waves...) the dispersion relation is connected to a time repetition of a signal component. It is essential to know how regular this repetition must be and which jitter can lead to the destruction of the dispersion phenomena. For this purpose, a series of simulations have been conducted with random layer thickness. In fact, we started with previous periodic structures (with velocity dispersion) and added random noise on the layer thickness. Hundreds of simulations showed that as long as the standard deviation of the error on thickness is less than $\lambda_0/8$ (λ_0 being the wavelength corresponding to the central frequency), the

dispersion phenomena is present. It starts fading out when the error exceeds this value and disappear totally for large errors (output signal become a series of spikes).

This experiment and the associated simulations have illustrated the complexity of the propagation in a periodic medium. In particular, they have pointed out that, when the travel time in an elementary layer is less than the time resolution, the stack behaves as an "equivalent medium" possessing a slower velocity and velocity dispersion. This dispersion disappears progressively when the "order" in the material is reduced (random thickness).

3. Time-Frequency analysis of dispersive propagation

Introduction

The velocity dispersion expresses a strong relationship between the time and the frequency domain: every frequency is travelling at a different speed and no time measurement can be achieved without prior frequency

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information. Many methods have tried to overcome this strong relationship that operates either the time or the frequency domain. They are commonly using strong approximations (narrow band, small dispersion...) that reduce their potential use in a general case. As the velocity dispersion is a time-frequency relationship, we will describe it explicitly in the time-frequency domain. As physicians, energy distribution is one of our major concerns. For a timefrequency energy representation that is compatible with propagation (time invariance properties), the general expression for finite energy signals (s) is given by the socalled Cohen's class [3], [4]:

$$\rho_{s}(t,f)_{g} = \iiint e^{2i\pi\xi(s-t)} \cdot g(\xi,\tau) \cdot s^{*}(s-\tau/2) \cdot s(s+\tau/2) \cdot e^{-t/2} \cdot g(\xi,\tau) \cdot s^{*}(s-\tau/2) \cdot s(s+\tau/2) \cdot e^{-t/2} \cdot e^{$$

Where g is a smoothing (2D) window while WV_z is the socalled Wigner-Ville distribution:

$$WV_{s}(t,f) = \int s^{*}(t - \frac{\tau}{2}) . s(t + \frac{\tau}{2}) . e^{-2i\pi f \tau} d\tau$$

Aside the time invariant properties, the Wigner-Ville distribution possesses several particular properties that are essential for the interpretation of the propagation of acoustics waves in dispersive media:

- direct access to power spectrum and envelope by ٠ computing function marginals
- conservation of bandwidth and duration
- compatibility with linear filtering and probably the most important property:
- possibility of computing group delay (and thus group velocity) from distribution moment:

$$\tau_g(f) = \frac{1}{2\pi} \frac{d}{dt} \arg \left[Z(f) \right] = \frac{\int W_z(t,f) dt}{\int W_z(t,f) dt}$$

Thank to all these properties, the propagation can simply be ^{2iπ} fdescribed in the time-frequency plane: an impulsive signal is a vertical line while a pure tone is a horizontal one. Figures 3 illustrate the propagation from point a to point b in the time-frequency plane for both dispersive and non dispersive cases: while, for non-dispersive case, the propagation is a simple horizontal translation of the elementary pattern (timefrequency representation of the transmitted signal), the propagation in a dispersive (and absorbent) media lead to a translation and rotation of the pattern as well as to frequency dependant attenuation.

Thanks to the properties of the Wigner-Ville distribution, the pattern amplitude can be directly related to the absorption and the first moment can be used to estimate group delay.



Figures 3 [5]: Illustration of the propagation in the time frequency plane

Application to Lamb waves in a plate

Such techniques have been applied to the estimation of group velocity of Lamb waves (measurement of group delay in two points) and validated experimentally [6], [7].

Figures 4 show experimental Lamb waves that have travelled respectively 6 (a) and 12 (b) cm on a plate. For each signal the Wigner-Ville distribution has been computed and displayed. The time dispersion is straightforward when comparing time axes. For each Wigner-Ville distribution, group delay was computed and then group velocity was obtained and compared to theoretical values (figures 5).

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Figures 4 [6]: propagation of lamb (dispersive) waves

top: received signal, bottom: Wigner-Ville representation, Vertical scale: 0-2 MHz, 4dB/grey level.

Figure 5 show clearly the relevance of the method for both qualitative and quantitative description of the propagation in dispersive media: the comparison of theoretical and experimental velocity dispersion shows an error of less of 5% (comparable to experiment accuracy).



Figure 5 [6]: velocity dispersion of Lamb waves, over an aluminium plate of 10 mm

Solid curves correspond to theoretical values; dots correspond to experimental values; dot size gives an order of magnitude of experimental error, discrepancy < 5%.

Application to industrial control, ultrasonic NDE [7], [8]

High voltage cable manufacturing consists in extruding an insulating polymer (around a metallic conductor). In its original form, the polymer is amorphous (monomer molecules are randomly distributed) and is fragile from both mechanical and thermal point of view. The manufacturing process consists then in heating the cable in order to reticulate it (thermo-chemical reaction). The reticulated polymer will possess the desired mechanical and thermal properties. It is essential that the inner part of the polymer (close to metallic conductor) is fully reticulated as, in operation, the heating (that could lead to melting) is generated from inside to outside (while manufacturing heating is from outside to inside).

For industrial control, an ultrasound evaluation of cables and cable materials has been conducted. The size of monomers is about a few hundred of microns. When not reticulated, they form a random arrangement in longer polymer molecules. When heated, they move freely and the electrostatic forces lead them to be arranged periodically. The propagation in reticulated polymer is very similar to the propagation in a periodic stack while the propagation in nonreticulated one is similar to random stack. The results on material samples showed that the propagation in non reticulated polymers is non-dispersive while it becomes dispersive in reticulated ones. After testing polymer samples, several experiments were conducted with real (bad and good) cables (immersed in water for coupling). Figure 6 shows a typical example of a cable echo: it is constituted mainly of two components:

- outer echo component: this first component is due to the (rough) interface between the water and the cable
- inner echo component: this second component is due to the reflexion on the metallic conductor and carries information on two-way travel of ultrasonic waves in the insulator. We have studied this particular component in details.



Figure 6 [7]: Example of cable echo (timescale 0-2 μ s)

International Symposium on Ultrasonics-2015, 22-24 January 2015 Department of Physics, Rashtrasant Tukdoji Maharaj Nagpur University, Nagpur, Maharashtra, India Licensed Under Creative Commons Attribution CC BY The time-frequency analysis (Wigner-Ville) of inner echo component was carried out for tens of cables in different points.

- Time spreading doubled for a good cable
- Additional absorption is associated to this velocity dispersion (frequency spreading reduction of about 40%).

Figures 7 show typical time-frequency plots for both good"" and "bad cables". Figure 7b shows that the dispersive nature of the insulator is visible on inner component:



Figure 7 [7], [8]: Typical time-frequency pattern of inner cable echo

4. Conclusion

The propagation in multilayered media can lead to velocity dispersion when the travel time through a single layer is comparable to time resolution and when the layering is periodic with regularity better than $\Box/8$ the presence of a large number of thin layers (thickness << time resolution, i.e. each individual layer is acoustically transparent) that could not be separated individually, can affect considerably the propagation and lead to a dispersive "equivalent medium". The equivalent composite material obtained by such a construction possesses an equivalent sound velocity much lower that the velocity in each component. The dispersion disappears when the difference in layer thickness exceeds $\Box/8$ The relationship between "geometrical order" and "velocity dispersion" has been observed in both macroscopic and microscopic (polymer) cases.

Velocity dispersion can be fully characterized and evaluated quantitatively (group velocity) without any approximation by using Wigner-Ville time-frequency analysis of dispersed signal even in complex cases (several propagation modes present). An illustration has been given for Lamb waves.

Finally an industrial control application has been shown: control of electrical cables. The order properties and the associated velocity dispersion observed on reticulated polymers sampled were also present in cable echo and were successfully used for characterizing good cables.

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