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Competition between Acoustical Phonon Induced and Polaron Induced Parametric Interactions in Polar Semiconductors

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Abstract: ***The focus of paper is a comparative study of the performance (amplification and excitation characteristics) of polaron induced and acoustical phonon induced parametric amplifiers. Based on hydrodynamic model and coupled mode theory, an analytical investigation of both kinds of parametric interaction has been made. Explicit expressions for the gain coefficient and threshold pump field for the onset of the acoustic phonon driven and polaron driven parametric instabilities are derived. We have numerically investigated the dependence of the threshold field on external parameters for a typical polar semiconductor (InSb) at 77 K, irradiated with $10.6\mu m$ CO_2 laser. Amplification characteristics are also explored through the estimation of parametric gain. Parametric gain in case of polaron induced parametric interaction is found to be enhanced by a factor of 10 as compared to that of acoustical phonon induced parametric interaction, whereas a close look at threshold pump amplitudes required to incite above said processes reveals that lower pump field is required to achieve polaron induced parametric amplification. External magnetic field and carrier concentration emerged as a control parameter to reduce threshold and improve parametric gain. These results could be important for the clearer understanding and improvement of the performance of parametric amplifiers. It is hoped that a low cost amplifier by using n-InSb- CO_2 system can be fabricated using the outcome of the present work.***

Keywords: Polarons and electron-phonon interactions, Nonlinear phenomena, Collective excitations, High-frequency effects, Plasma effects in solid.

1. Introduction

Parametric interaction is an important mechanism of nonlinear mode conversion from electromagnetic to electrostatic and from high frequency to low frequency waves. Major underlying research was done by Neogi and Ghosh [1], and other workers [2-4]. Recently Dubey and Ghosh [5] reported a significant study of parametric oscillation of polaron modes in magnetized semiconductor plasmas. Considering that the density perturbation arises due to coupling of polaron mode and the pump wave, the polaron induced modulational amplification process has been reported by Ghosh et al. [6].

It seems that uptill now no efforts have been made to study the competition between the amplification and excitation characteristics of polaron induced and acoustical phonon induced parametric interactions. Theoretical understanding of plasmon-LO phonon as well as plasmon-acoustical phonon interactions will be helpful in improving the performance of amplifiers. Hence in this paper we analyze these interactions and compare their influence through absorption coefficient of the amplified wave and threshold electric field required for the onset of parametric amplification process. In light of above, with the help of hydrodynamic model of plasmas and coupled mode theory, we have analytically investigated the parametric amplification characteristics caused due to plasmon-phonon interactions namely plasmon-acoustical phonon and plasmon-longitudinal optical phonon interaction through an n-InSb/CO_2 laser system at 77 K temperature in the presence of transverse magnetostatic field.

2. Theoretical Formulations

We consider a spatially uniform $(|k_0| \approx 0)$ pump electric field $\vec{E}_0 = \hat{x}E_0 \exp(-i\omega_0 t)$ applied in homogeneous polar semiconductor medium immersed in an external magnetostatic field $\vec{B}$ along y-axis. The coupled mode scheme is applied to obtain a simplified expression for the second-order optical susceptibility via nonlinear polarization. The hydrodynamic model of homogeneous semiconductor plasma of infinite extent is considered, which restricts our analysis to be valid only in the regime $kl << 1$ (k is the wave vector and l is the carrier mean free path).

2.1 Threshold and amplification characteristics due to plasmon-acoustical phonon interactions

As far as physics of plasmon–acoustical phonon interaction assisted parametric amplification is concerned, we know that in a semiconductor crystal with dissimilar atoms and partly ionic bonds, the unit cell does not contain a centre of symmetry; carriers may be scattered by longitudinal acoustic waves due to piezoelectric scattering. The density perturbation associated with the phonon mode and the scattered electromagnetic waves arising due to three wave parametric interactions will propagate at the generated frequencies ω_s and $\omega_0 \pm \omega_s$.

In order to study this particular interaction, we have used basic equations of Neogi and Ghosh [7] (here after referred as paper I) and following eq. (16b) and employing the relation

$$\alpha_{AP} = \frac{k}{2\varepsilon_1}\left(\chi_{eff}^{(2)}\right)_i E_0 \quad (1)$$

Eq. (16b) of paper I has been used to determine imaginary component of second order susceptibility and hence α_{AP} (effective nonlinear absorption coefficient). The nonlinear growth of the amplified signal is possible only if α_{AP} obtainable from equation (1) is negative. Threshold value of the pump amplitude required to incite the parametric amplification process is determined as

$$E_{AP} = \frac{m}{ek}\omega_s^{\frac{1}{2}}\omega_1^{\frac{3}{2}}\left(1-\frac{\omega_c^2}{\omega_0^2}\right) \quad (2)$$

Here the symbols have their usual meaning as referred in paper I. Phase matching conditions are $k_0 = k_1 \pm k_s$ and $\omega_0 = \omega_1 + \omega_s$. Under spatially uniform laser irradiations $|k_0| \approx 0$ momentum matching condition will be $|k_1| \approx |k_s| = k$ (say).

2.2 Threshold and amplification characteristics due to plasmon- longitudinal optical phonon interactions

Electron-LO phonon interaction leads to apparent increase in electron mass that occurs because the electron drags the heavy ion cores along with it. The combination of the electron and its strain field is known as a polaron [8]. Polaron effect is large in ionic crystals because of the strong Coulomb interaction between ions and electrons. The coherent electron and LO phonon modes i.e. the polaron modes may be excited through ultra fast excitations. The frequencies of the normal modes in the presence of magnetic field with $(k_0 \approx 0)$ and $(k_0 \perp B)$, arising from the coupling of the collective cyclotron excitations with the LO phonons via the macroscopic longitudinal electric field [9], are given by

$$\omega_{0,pl} = \left[\frac{\omega_p^2+\omega_c^2+\omega_L^2+\left[\left(\omega_p^2+\omega_c^2+\omega_L^2\right)^2-4\left(\omega_p^2\omega_T^2+\omega_c^2\omega_L^2\right)\right]^{0.5}}{2}\right]^{0.5} \quad (3)$$

The coupling of collective cyclotron excitations with the LO phonons will give rise to an induced polarization of the medium. This polarization is local in character and is due to the displacements of ions from the equilibrium positions caused by the field produced by the electron density which gives rise to an electron density perturbation at the polaron frequency, which couples nonlinearly with the pump wave and drives the polaron wave at amplified frequency. Using the basic equations of Dubey and Ghosh [5] (here after referred as paper II) and adopting the procedure of paper I, we get

$$\frac{\partial^2 n_1}{\partial t^2} + 2\Gamma_e\frac{\partial n_1}{\partial t} + \bar{\omega}_P^2 A_1 n_1 - \frac{en_o}{m_e}A_1 X_1 C_1\frac{d\vec{R}}{dx} = -ikn_1 A_2\bar{E} \quad (4)$$

where $\bar{\omega}_P^2 = X_1\omega_p^2$ being the electron-plasma frequency modified by magnetic field;

$$X_1 = \frac{4\Gamma_{pl}^2}{4\Gamma_{pl}^2-\omega_c^2}\ .\ A_1 = \frac{\omega_{pl}^2}{\omega_{pl}^2-\omega_p^2-\omega_c^2},$$

$$C_1 = \left(\frac{en_o}{m_e}-\frac{Nq}{\varepsilon_o}\right),\quad A_2 = \frac{\omega_o^2}{\omega_o^2-\omega_p^2-\omega_c^2}$$

and $\bar{E} = -\frac{e}{m_e}E_o\frac{\omega_o^2}{\omega_o^2-\omega_c^2}$. $q = \omega_L\left[MN^{-1}\varepsilon_0\left(\varepsilon^{-1}-\varepsilon_s^{-1}\right)\right]^{0.5}$

is the Callen effective charge of the lattice polarization. Here the meanings of symbols are same as referred in paper II.

Equation (4) describes a three wave parametric interaction process involving polaron mode, the signal wave and intense pump wave. Resolving eq. (4) and using rotating wave approximation (RWA), we obtain, the slow component (n_s) associated with polaron mode that produces density perturbation at signal frequency and the fast component (n_f) as

$$n_f = \frac{-ikA_2\bar{E}}{\delta_1^2-2i\Gamma_e\omega_1}n_s^* \text{ and } n_s = ik\sqrt{NM}\frac{en_o}{m_e\varepsilon_o}QG\vec{E}_{pl} \quad (5)$$

Where, $\delta_1^2 = A_1\bar{\omega}_P^2-\omega_1^2$,

$$G = \left[\delta_2^2-2i\Gamma_{pl}\omega_{pl}-\frac{k^2A_2^2|\bar{E}|^2}{\delta_1^2+2i\Gamma_e\omega_1}\right]^{-1},$$

$$Q = \frac{A_1TX_1Y_1}{F_1} \text{ in which } T = \frac{n_1eq}{M}+\frac{Neq}{m_e}-\frac{n_oe^2}{m_e}-\frac{Nq^2}{M},$$

$$F_1 = \omega_{o,pl}^2-\omega_{pl}^2-2i\Gamma_{pl}\omega_{pl},\ \delta_2^2 = A_1\bar{\omega}_P^2-\omega_{pl}^2,$$

$$Y_1 = \frac{4\Gamma_{pl}^2}{4\Gamma_{pl}^2-\omega_{c,pl}^2} \text{ in which } \omega_{c,pl} = \left(\frac{-e}{m_e}+\frac{q}{M}\right)\vec{B}.$$

On using eqs. (9) and (10) of paper I, total effective polarization will be

$$P_{eff}^{(2)}(\omega_1) = \frac{-iek\sqrt{NM}}{m_e\omega_1}\left(\frac{\omega_o\omega_p^2}{\omega_o^2-\omega_c^2}\right)QG^*\vec{E}_o\vec{E}_{pl}^* \quad (6)$$

It is essential that for the resonant amplification, the energy transfer between the pump and polaron mode must satisfy phase matching conditions ($\omega_0 = \omega_{pl} \pm \omega_1$ and $k_0 = k_{pl} \pm k_1$) under spatially uniform laser irradiations $|k_o| \approx 0$ it reduces to $|k_1| \approx |k_o \pm k_{pl}| = k$ (say).

We may obtain second order susceptibility via nonlinear polarization as

$$P_{eff}^{(2)} = \varepsilon_o\chi_{eff}^{(2)}\vec{E}_o\vec{E}_{pl}^*$$

$$\chi_{eff}^{(2)} = \frac{-ike\sqrt{NM}}{\varepsilon_o m_e\omega_1}\left(\frac{\omega_o\omega_p^2}{\left(\omega_o^2-\omega_c^2\right)^2}\right)QG^* = \left[\chi_{eff}^{(2)}\right]_r + i\left[\chi_{eff}^{(2)}\right]_i \quad (7)$$

Equation (7) characterizes the steady state optical response of the medium and reveals that the total crystal susceptibility is influenced by nonzero plasma frequency and cyclotron frequency. It can be observed from eq. (7) that there is an intensity dependent refractive index [via $\left(\chi_{eff}^{(2)}\right)_r$] leading to the possibility of a focusing or defocusing effect of the propagating beam. The positive dispersive characteristics of the dissipative medium is possible at $\omega_p >> \Gamma_e$. As $\left(\chi_{eff}^{(2)}\right)_r$ becomes more positive, one may expect more effective self-defocusing of the amplified polaron mode. In order to express the possibility of parametric amplification in semiconductor plasma, we employ the relation

$$\alpha_{LOP} = \frac{k}{2\varepsilon_1}\left(\chi_{eff}^{(2)}\right)_i E_0 \quad (8)$$

Here, α_{LOP} is the effective nonlinear absorption coefficient. The nonlinear growth of the amplified signal is possible only if α_{LOP} obtainable from eq. (8) is negative. The threshold value of the pump amplitude required for the onset of the parametric interaction is obtained via setting

$$P_{eff}^{(2)} = 0 \text{ as } E_{LOP} = \left| \frac{m_e}{ekA_2}\left[\frac{\omega_o^2 - \omega_c^2}{\omega_o^2}\right]\delta_1\delta_2 \right| \quad (9)$$

It is observed from eq. (9) that the parametric instability of the signal wave has a nonzero intensity threshold, even in absence of the damping. The threshold field E_{LOP} is found to have complex characteristics and is strongly dependent on the external magnetostatic field

3. Results and Discussion

This section addresses detailed numerical investigations based on the analysis made in the previous section to appreciate the possibility of parametric amplification resulting from the transfer of power from the pump wave to the product wave. Main focus of this paper is on the comparative study of threshold and amplification characteristics of the modulated wave due to plasmon-acoustic phonon and plasmon-longitudinal optical phonon interactions, thus an n-InSb crystal is assumed to be irradiated by a pulsed $10.6\,\mu m$ CO_2 laser at 77 K and the relevant parameters are given in papers I and II.

3.1 Threshold Characteristics

Equations (2) and (9) reflect the strong dependence of external magnetic field and carrier concentration of the medium on the threshold characteristics of the parametric interactions. The numerical estimations dealing with these external parameters influencing the threshold field required to incite parametric amplification are plotted in Figures 1 and 2.

Figures 1 and 2 depict the variation of threshold electric field required to incite the parametric amplification induced by plasmon–acoustical phonon and plasmon–longitudinal optical phonon interactions with respect to wave vector k and magnetic field B. In these figures E_{AP} and E_{LOP} represent the threshold fields to incite parametric amplification due to plasmon–acoustic phonon and plasmon–longitudinal optical phonon interaction respectively. Figure 1 depicts that in both the cases threshold electric field decreases on increasing wave vector k. This behavior reflects the fact that threshold fields are inversely proportional to k, in confirmity with eqs. (2) and (9).

Figure 2 displays that initially threshold electric field E_{LOP} slightly increases and then decreases with further increase in magnetic field whereas E_{AP} decreases throughout the magnetic field regime. It can be inferred from both the figures that polaron induced parametric amplification occurs at lower threshold than acoustical phonon induced parametric amplification throughout the wave number and magnetic field regime studied.

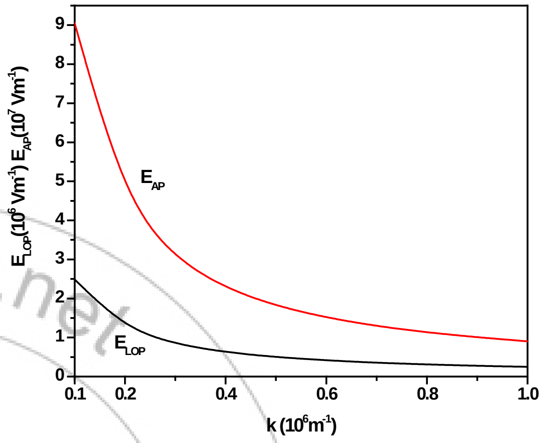

Figure 1: Variation of threshold electric fields with wave vector

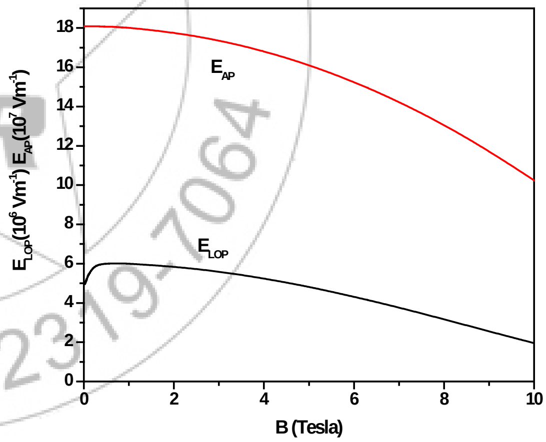

Figure 2: Variation of threshold electric fields with magnetic field

3.2 Amplification Characteristics

Next aim is to explore amplification characteristics of polaron mode in semiconductor medium. Equations (1) and (8) can be used to estimate the ratio between the gain coefficients in terms of material parameters. Thus for a fixed input pump intensity greater than threshold intensities in both the cases

$$\frac{g_{LOP}}{g_{AP}} = 8.59 \text{ at } k = 2\times10^7\, m^{-1} \text{ with } \omega_c = 0.001\omega_0$$

The numerical estimations dealing with the external parameters (such as magnetic field, carrier concentration etc) influencing the threshold and amplification characteristics are plotted in Figures 3 to 6. α_{LOP} and α_{AP} represent absorption coefficients of polaron driven and acoustical phonon driven parametric process.

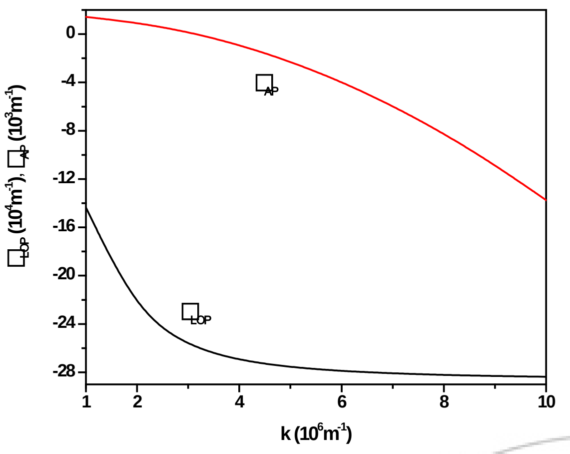

Figure 3: Variation of absorption coefficients with wave vector

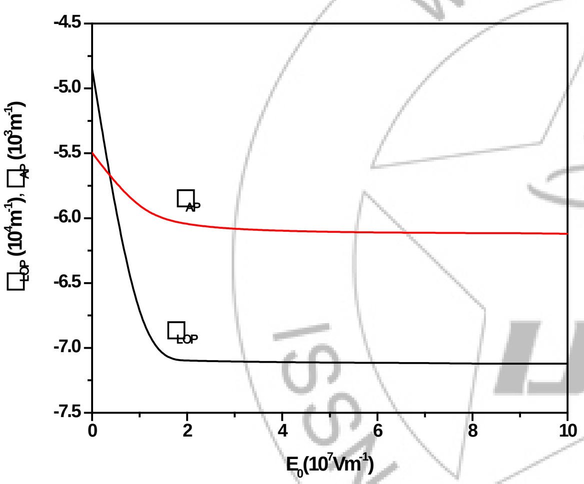

Figure 4: Variation of the absorption coefficients with pump field

Figure 3 illustrates the variation of α_{LOP} and α_{AP} with respect to the wave vector k. As wave vector increases α_{AP} first increases slowly and then enhances rapidly with the increase in wave vector while α_{LOP} behaves conversely. It is clear from the figure that in both the cases gain coefficients increase in the whole wave vector regime.

Figure 4 depicts the variation of α_{LOP} and α_{AP} with respect to the pump electric field E_0. Both the curves are nearly identical in nature for the range of pump field considered. It is clear from the figure that in both the cases increment in pump field increases both the gain coefficients. Gain coefficients achieve maximum values $\left(\alpha_{LOP} \approx 7.1\times10^4 m^{-1} and\ \alpha_{AP} \approx 6.1\times10^3 m^{-1}\right)$ at $E_0 \approx 1.5\times10^7 Vm^{-1}$. On further increasing pump field, gain saturates. It can be further inferred from Figure 4 that polaron induced parametric amplification results into a larger gain coefficient as compared to the acoustical phonon induced parametric amplification process at particular values of carrier density and magnetic field strength.

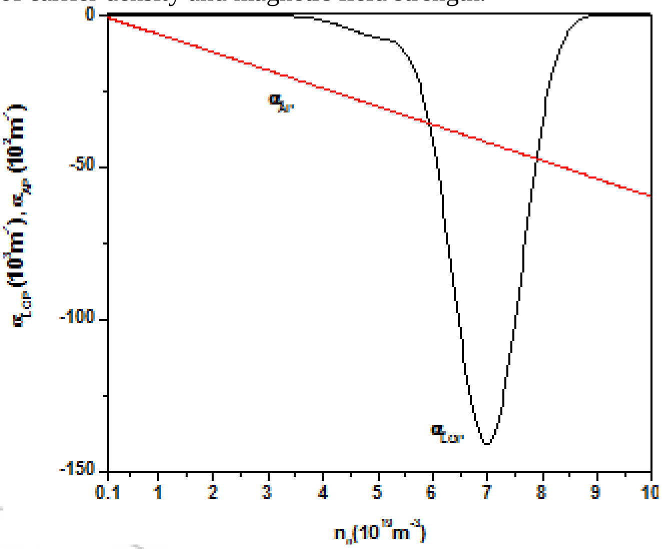

Figure 5: Variation of the absorption coefficients with carrier concentration

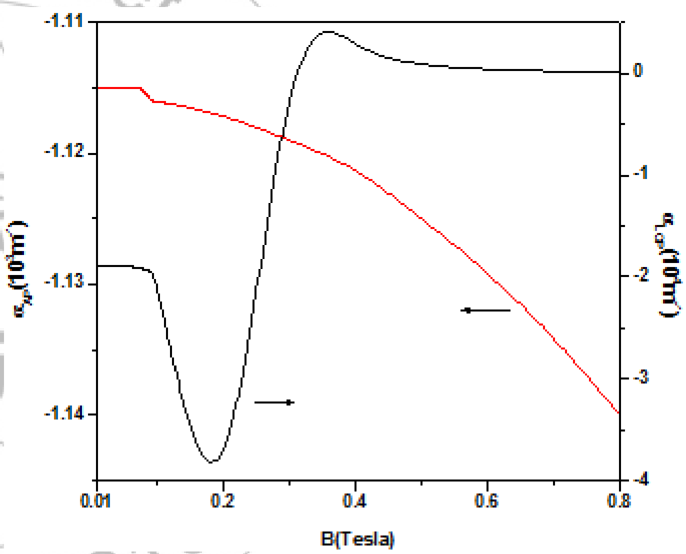

Figure 6: Variation of the absorption coefficients with magnetic field

Behavior of absorption coefficient as a function of carrier concentration is shown in Figure 5. It may be inferred from Figure 5 that in the whole carrier concentration regime α_{AP} decreases or gain increases. α_{LOP} decreases or corresponding gain increases with increasing n_0; at resonance between plasma frequency and signal frequency it touches its maximum $\left(\approx 1.4\times10^5 m^{-1}\right)$ at $n_0 = 7\times10^{19} m^{-3}$. The maximum gain for polaron induced parametric amplification enhances by a factor of 10^2 than acoustical phonon induced parametric amplification.

Figure 6 shows the dependence of α_{LOP} and α_{AP} on the external magnetic field B. It is found that α_{LOP} have both positive and negative values under the anomalous regime.

Resonance condition: $\omega_c^2 \approx 4\Gamma_{pl}^2\left[\frac{A_1\omega_p^2 - \omega_1^2}{\omega_1^2}\right]$ gives rise to maximum parametric amplification. Further slight variation in magnetic field, α_{LOP} immediately increases ensuing into absorption or loss of signal frequency owing to resonance between magnetostatic field dependent polaron cyclotron frequency (polaron cyclotron resonance) and $4\Gamma_{pl}^2$. This behavior could be utilized for optical switching in various ultrafast optoelectronic devices. Whereas in case of acoustical phonon assisted parametric amplification α_{AP} decreases or gain increases for all values of magnetic field. Thus, it is clear from present study that the applied magnetic field can be used as a control parameter to reduce threshold and improve gain. Polaron induced parametric amplification causes more gain (10 times) in comparison to that of acoustical phonon induced parametric amplification when the magnetic field is varied.

4. Conclusions

A comparative study of polaron induced and acoustical phonon induced parametric interactions has been undertaken in the present paper. Concerning three wave interactions, we have used equations of motion and then solved them within the parametric approximation, taking into account phase matching both in modulus and direction. This analysis predicts many of the features that optimize sensitivity of parametric amplifiers, advanced detectors and optoelectronic devices. We now make few final remarks:

1. In view of cost effectiveness of the interactions, it is worthwhile to compare threshold characteristics of parametric interactions due to plasmon-LO phonon coupling with those due to plasmon-acoustical phonon. Polaron induced parametric interaction proves to be more significant than acoustical phonon induced parametric interaction owing to its lower threshold pump requirement.
2. Enhancement of gain coefficient by a factor of 10 is found in polaron induced parametric amplification as compared to acoustical phonon induced parametric amplification.
3. The large growth rate suggests that Fröhlich interaction significantly affects the low temperature performance of InSb based amplifiers.
4. The present study probably for the first time establishes the potential of Fröhlich interaction while comparing it with plasmon-acoustic phonon interactions in a lightly doped polar semiconductor.
5. Present study confirms that due to the strong ionic nature of the III-V semiconductors the Fröhlich interaction is stronger than acoustical phonon induced interaction.
6. It is hoped that a low cost amplifier and ultrafast optical switch by using n-InSb-CO_2 system can be fabricated by utilizing the outcome of the present work.

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