Plane Hydromagnetic Shock Wave Through Uniform and Non-Uniform Media

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Abstract: ***The present investigation is concern with the propagation of plane diverging shock wave using CCW[1-3] approach simultaneously for weak and strong shock in an ideal and inviscid gas in presence of a magnetic field having: i) constant axial $(H_{z_0} = constant)$ component only; ii) constant axial and azimuthal $(H_\theta = const = H_{z_0})$ components only and iii) constant axial and variable azimuthal $(H_\theta = r^{1/2} H_{z_0},\ H_{z_0} = constant)$ components only. The context of first situation leads to propagation of plane diverging hydromagnetic shock wave through uniform media; whereas the expressions derived for flow variables corresponds to propagation of plane diverging hydromagnetic shock wave through non-uniform media. The density in the unperturbed state has been assumed to vary as $\rho_0 = \rho' e^{-\lambda r}$ where ρ' is the density at the plane of symmetry and 'λ' is a nondimensionalising constant. Finally the effects of overtaking disturbances behind the flow have also been included. All the three cases have been dealt for weak and strong magnetic field for both the shock conditions viz., weak and strong shock.***

1.Introduction

The present work is related with the propagation of plane shock waves through uniform and non-uniform situations where the pressure and density ahead of the shock waves are variable. It is important for theories of sun spot, magnetic fields in heating solar corona and in stability of stellar atmospheres in magnetic fields. The use of Chisnell[2] Chester[1] Whitham[3] approach (CCW[1-3] method) by including the effects of overtaking disturbances behind the flow on the motion of shocks has shown remarkable agreement with the results obtained by other methods[4-6]. This fact has encouraged accomplishing the present investigations. EOD behind the flow on the adiabatic motion of plane diverging hydromagnetic shock waves have been included in CCW procedure. Assuming an initial density distribution $\rho_0 = \rho' e^{-\lambda r}$, the analytical expressions for flow variables have been deduced in presence of i) constant axial, ii) constant axial and constant azimuthal and iii) constant axial and variable azimuthal magnetic field viz., (i) when the shock is weak and (i) when it is strong.

2.Basic Equations

The equations governing the flow of the gas enclosed by the shock front are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{2\rho}\frac{\partial H_z^2}{\partial r} = 0$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} = 0$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + \gamma p\frac{\partial u}{\partial r} = 0 \quad \text{……….. (1)}$$

$$\frac{\partial H_\theta}{\partial t} + u\frac{\partial H_\theta}{\partial r} + H_\theta\frac{\partial u}{\partial r} = 0$$

$$\frac{\partial H_z}{\partial t} + u\frac{\partial H_z}{\partial r} + H_z\frac{\partial u}{\partial r} = 0$$

Where 'r' is the radial co-ordinate, p, ρ, u, H_θ and H_z are respectively, the pressure, the density, the particle velocity, the azimuthal and axial component of magnetic field.

$a_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$ is local sound speed and $b_0 = \sqrt{\frac{\mu(H_{\theta_0}^2 + H_{z_0}^2)}{\rho_0}}$ is the Alfven speed and 'γ' is the adiabatic index of the gas.

3.Boundary Conditions:-

The magneto hydromagnetic shock conditions can be written in terms of a single parameter 'ξ' as-

$$\rho = \rho_0 \xi,\ H = H_0 \xi,\ u = \frac{(\xi - 1)}{\xi} U,$$

$$U^2 = \frac{2\xi}{(\gamma+1) - (\gamma-1)\xi}\left[a_0^2 + \frac{b_0^2}{2}\{(2-\gamma)\xi + \gamma\}\right]$$

And $p = p_0 + \frac{2\rho_0(\xi-1)}{(\gamma+1)-(\gamma-1)\xi}\left[a_0^2 + \frac{(\gamma-1)}{4} b_0^2 (\xi-1)^2\right]$ .. (2)

Where the subscript '0' stands for the state immediately ahead of the shock front and U is the shock velocity.

For Weak Shock:- For very weak shock the parameter 'ξ' is written as $\rho / \rho_0 = \xi = 1 + \varepsilon$

Where 'ε' is another parameter, ε << 1

Case I:-WSWMF

For very weak magnetic field ie., $b_0^2 << a_0^2$ the above conditions (2) reduce to

$$\rho=\rho_0(1+\varepsilon), H_\theta=H_{\theta_0}(1+\varepsilon), H_z=H_{z_0}(1+\varepsilon),$$

$$U=\left[1+\frac{(\gamma+1)}{4}\varepsilon\right]a_0, p=p_0(1+\gamma\varepsilon)$$

And

$$u=\varepsilon a_0 \quad \text{……. (3)}$$

Case II:- WSSMF

For very strong magnetic field ie., $b_0^2 >> a_0^2$ the above conditions (2) reduce to

$$\rho=\rho_0(1+\varepsilon), H_\theta=H_{\theta_0}(1+\varepsilon), H_z=H_{z_0}(1+\varepsilon),$$

$$U=\left[1+\frac{3}{4}\varepsilon\right]b_0, p=p_0(1+\gamma\varepsilon)$$

And

$$u=\varepsilon b_0 \quad \text{………. (4)}$$

For Strong Shock:- For strong shock the $\rho/\rho_0=\xi$ is very large, for magnetic cases it is achieved in two ways.

CaseIII:-SSWMF For very weak magnetic field ie., $b_0^2 << a_0^2$ the above conditions (2) reduce to

$$\rho=\rho_0\xi, H_\theta=H_{\theta_0}\xi, H_z=H_{z_0}\xi,$$

$$u=\frac{(\xi-1)}{\xi}U,\ \frac{p}{p_0}=1+\left(\chi' a_0^2+A' b_0^2\right)\frac{U^2}{a_0^4}$$

$$where,\ \chi'=\frac{\gamma(\xi-1)}{\xi}\ and\ A'=\frac{\gamma(\xi-1)}{4\xi}\left[(\gamma-1)(\xi-1)^2-2\{(2-\gamma)\xi+\gamma\}\right]$$

……….. (5)

CaseIV:-SSSMF For very strong magnetic field ie., $b_0^2 >> a_0^2$ the above conditions (2) reduce to

$$\rho=\rho_0\xi, H_\theta=H_{\theta_0}\xi, H_z=H_{z_0}\xi,$$

$$u=\frac{(\xi-1)}{\xi}U,\ \frac{p}{p_0}=1+\chi(b_0^2+Aa_0^2)\frac{U^2}{a_0^2 b_0^2}$$

$$where,\ \chi=\frac{\gamma(\gamma-1)(\xi-1)^3}{2\xi\{(2-\gamma)\xi+\gamma\}}\ and\ A=\left[\frac{4}{(\gamma-1)(\xi-1)^2}-\frac{2}{\{(2-\gamma)\xi+\gamma\}}\right]$$

…………. (6)

Characteristic Equations:- The characteristics form of the system of equations (1) is obtained by forming a linear combination of first and third equation of system of equations (1) in only in (r, t) plane and can be written as

$$dp+\mu H_\theta dH_\theta+\mu H_z dH_z+\rho c du+\mu H_\theta^2\frac{dr}{r}=0 \quad (7)$$

In order to estimate the strength of overtaking disturbances, an independent C_+ characteristic is considered. The differential equation valid across C_+ disturbance is written as

$$dp+\mu H_\theta dH_\theta+\mu H_z dH_z-\rho c du+\mu H_\theta^2\frac{dr}{r}=0 \quad (8)$$

Analytical Relations for Flow Variables: i) { $H_z=H_{z_0}(cons\tan t)$ }

Assuming the medium uniform throughout and in presence of constant axial magnetic field component only.

Weak Shock Weak Magnetic Field (WSWMF):-

Substituting the shock conditions (3) into equation (7), we get

$$\varepsilon_+(r)=k' p_0^{-3(2-\beta^2)/8} \quad \text{…….. (9)}$$

Where k' is a constant of integration and $\beta^2=\frac{\mu H_{z_0}^2}{\gamma p_0}$

Here equation (9), describes the free propagation.

Now Substituting the shock conditions (3) into equation (8), we get

$$\varepsilon\_(r)=k' p_0^{-1/2\beta^2} \quad \text{…… (10)}$$

Now to include the eod behind the flow on the motion of shock, we use the relation

$$u=\varepsilon a_0 \text{ as } du=a_0 d\varepsilon+\varepsilon da_0 \quad \text{….. (11)}$$

In presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be related as $du=du_+ + du_-$, therefore, substituting the respective results, we get

$$\varepsilon_{eod}=\varepsilon_+\varepsilon_-\sqrt{p_0} \quad \text{……. (12)}$$

Here the equation (12) includes the eod behind the flow on the motion of shock

Weak Shock Strong Magnetic Field (WSSMF):-

Substituting the shock conditions (4) into equation (7), we get

$$\varepsilon_+=k' p_0^{-k_1} \quad \text{… (13)}$$

Where k' is the constant of integration and

$$k_1=0.25\left(1-\frac{1}{2\beta^2}\right)\left(1+\frac{2}{\beta^2}\right)$$

Here equation (13) describes the free propagation.

Now Substituting the shock conditions (4) into equation (8), we get

$$\varepsilon\_(r)=k' p_0^{-(1-0.5\beta^2)} \quad \text{….. (14)}$$

Now to include the eod behind the flow on the motion of shock, we use the relation

$$u=\varepsilon b_0 \text{ as } du=b_0 d\varepsilon+\varepsilon db_0 \quad \text{…… (15)}$$

In presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be related as $du = du_+ + du_-$, therefore, substituting the respective results, we get

$$\varepsilon_{eod} = \varepsilon_+ \varepsilon_- \sqrt{p_0} \quad \text{..... (16)}$$

Here the equation (16) includes the eod behind the flow on the motion of shock.

Strong ShockWeak Magnetic Field (SSWMF):-

Substituting the shock conditions (5) into equation (7), we get

$$dU^2 + \frac{dp_0}{\rho_0 K}\left[1 - \frac{A'\beta^2}{\gamma K}\right] = 0 \quad \text{........ (17)}$$

But for initial density distribution $dp_0 = 0$, therefore,
U^2 (free) = k' (constant) .... (18)
Now substituting the shock conditions (5) into equation (8), we get

$$dU^2\left[\left(\frac{\chi'}{\gamma} - \frac{(\xi-1)}{2}\sqrt{\frac{\chi'}{\xi}}\right) + \frac{A'\beta^2}{\gamma}\right] + \frac{dp_0}{\rho_0} = 0 \quad (19)$$

But for initial density distribution $dp_0 = 0$, therefore,
U^2 = k' (constant) .... (20)

In presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be related as $du = du_+ + du_-$, therefore, substituting the respective results, we get

U^2 (eod) = k' (constant) ...... (21)

Similarly using shock conditions (6) into equations (7) and (8), and in presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be
U^2 (eod) = k' (constant) ...... (22)

ii) {
$H_z = H_{z_0}(\text{constant})$ and $H_\theta = H_{z_0}(\text{constant})$
}

Assuming the initial density distribution as $\rho_0 = \rho' e^{-\lambda r}$, the equilibrium state of the gas is assumed to be specified by the condition

$$\frac{\partial}{\partial t}. = 0 = u, H_\theta = H_{z_0} \text{ and } H_z = H_{z_0}(\text{constant}) \quad \text{......}$$
(23)

Using equation (23) in first equation of the system of equations (1), the hydrostatic equilibrium assumed to be specified by the condition prevailing in front of the shock can be written as

$$\frac{1}{\rho_0}\frac{\partial p_0}{\partial r} + \frac{\mu}{2\rho_0}\frac{\partial}{\partial r}(H_{\theta_0}^2 + H_{z_0}^2) + \frac{\mu}{\rho_0}\frac{H_{\theta_0}^2}{r} = 0 \quad \text{... (24)}$$

Using equations (23) in equation (24) and then integrating, we get

$$\frac{p_0}{p'} = K - \gamma\beta^2 \log r \quad \text{....... (25)}$$

$$\frac{da_0}{a_0} = \frac{1}{2}\left(\frac{dp_0}{p_0} + \lambda dr\right) \text{ and } \frac{db_0}{b_0} = \frac{\lambda}{2}dr$$

Weak Shock Weak Magnetic Field (WSWMF):-

Substituting the shock conditions (3) into equation (7), we get

$$\varepsilon_+(r) = k' p_0^{-k_2} \exp\left\{-\left(k_1 r + \frac{k_3}{p_0}\right)\right\} \quad \text{........ (26)}$$

Where k' is a constant of integration

$$k_1 = \left(\frac{\lambda}{4} - \frac{\lambda\beta^2}{4K}\right),\ k_2 = \left\{\frac{(\gamma-2)}{2\gamma} + \frac{1}{4}\right\} \text{and } k_3 = p'\beta^2 k_2$$

Here equation (26), describes the free propagation.
Now Substituting the shock conditions (3) into equation (8), we get

$$\varepsilon_-(r) = k' r^{(3\gamma-4)/4} \exp\left\{\frac{\lambda\gamma}{4}(r\log r - r) - \frac{\lambda K}{4\beta^2} r\right\} \quad \text{... (27)}$$

In presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be related as $du = du_+ + du_-$, therefore, substituting the respective results, we get

$$\varepsilon_{eod} = \varepsilon_+ \varepsilon_- \sqrt{p_0}\; e^{\frac{\lambda}{2} r} \quad \text{....... (28)}$$

Here the equation (28) includes the eod behind the flow on the motion of shock

Weak Shock Strong Magnetic Field (WSSMF):-

Substituting the shock conditions (4) into equation (7), we get

$$\varepsilon_+(r) = k' r^{k_2} \exp\left\{-\left(k_1 r + \frac{\lambda\gamma}{16}(r\log r - r)\right)\right\} \quad \text{... (29)}$$

Where k' is the constant of integration and

$$k_1 = \left(\frac{\lambda}{4} - \frac{\lambda k}{16\beta^2}\right),\ k_2 = \frac{(\gamma-2)}{4}$$

Here equation (29) describes the free propagation. Now Substituting the shock conditions (4) into equation (8), we get

$$\varepsilon_-(r) = k' p_0^{-\frac{(\gamma-2)}{\gamma}} e^{\left(\frac{\lambda\beta^2}{K}\right) r} \quad \text{..... (30)}$$

In presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be related as $du = du_+ + du_-$, therefore, substituting the respective results, we get

$$\varepsilon_{eod} = \varepsilon_{+}\varepsilon_{-}e^{\frac{\lambda}{2}r} \quad \text{..... (31)}$$

Here the equation (31) includes the eod behind the flow on the motion of shock.

Strong ShockWeak Magnetic Field (SSWMF):-

Substituting the shock conditions (5) into equation (7), we get

$$U^2 = \left[k' - \frac{\gamma p'\beta^2\left(1-\xi^2\right)}{\rho' B}\left\{\log r + \left(\lambda + k_1\right)r + 0.25\left(\lambda + k_1\right)^2 r^2 + \frac{\left(\lambda + k_1\right)^3}{18}r^3\right\}\right]e^{-k_1 r} \quad \text{............... (32)}$$

Where, k' = constant of integration and

$$k_1 = \left(\frac{2\lambda A'\beta^2}{\gamma BK} + \frac{2\lambda\chi' A'\beta^2}{\gamma^2 B^2 K} - \frac{\lambda\chi'}{\gamma B}\right),\ B = \left(\frac{\chi'}{\gamma} + \frac{\left(\xi - 1\right)}{2}\sqrt{\frac{\chi'}{\xi}}\right)$$

Here equation (32) describes the free propagation.

In presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be related as $du = du_+ + du_-$, therefore, substituting the shock conditions (5) into equation (8) and using equation (32), we get

$$U^2 = \left[k' - \frac{\gamma p'\beta^2\left(1-\xi^2\right)}{\rho'}\left(\frac{1}{B} + \frac{1}{B'}\right)\left\{\log r + \left(\lambda + k_1\right)r + 0.25\left(\lambda + k_1\right)^2 r^2 + \frac{\left(\lambda + k_1\right)^3}{18}r^3\right\}\right]e^{-\left(k_1 + k_1'\right)r} \quad \text{....... (33)}$$

Where, k' = constant of integration and $k_1' = \left(\frac{2\lambda A'\beta^2}{\gamma B' K} + \frac{2\lambda\chi' A'\beta^2}{\gamma^2 B'^2 K} - \frac{\lambda\chi'}{\gamma B'}\right),\ B' = \left(\frac{\chi'}{\gamma} - \frac{\left(\xi - 1\right)}{2}\sqrt{\frac{\chi'}{\xi}}\right)$

Here the equation (33) includes the eod behind the flow on the motion of shock.

Strong Shock Strong Magnetic Field (SSSMF):-

Substituting the shock conditions (6) into equation (7), we get

$$U^2 = \left[k' r^{k_1} - \left\{\frac{k_4}{k_1} + \frac{\left(\lambda - k_2 k_4\right)}{\left(1 - k_1\right)}r - \frac{\left(0.5\lambda^2 - \lambda k_2\right)}{\left(2 - k_1\right)}r^2\right\}\right]\exp\left\{-k_2 r + k_3 r\left(\log r - 1\right)\right\} \quad \text{............... (34)}$$

Where, k' = constant of integration and $k_1 = \frac{\chi A}{2C},\ k_2 = -\left\{\frac{\lambda\chi}{\gamma C} + \frac{\lambda\chi^2 AK}{2\gamma^2 C^2\beta^2} + \frac{\lambda\chi AK}{2\gamma C\beta^2}\right\},\ k_3 = \left(\frac{\lambda\chi^2 A}{2\gamma C^2} + \frac{\lambda\chi A}{2C}\right)$

$$k_4 = \frac{\gamma p'\beta^2}{\rho' C}\left(1 - \xi^2\right)\left(1 - \frac{\chi AK}{2\gamma C\beta^2}\right),\ C = \left(\frac{\chi}{\gamma} + \frac{\left(\xi - 1\right)}{2}\sqrt{\frac{\chi}{\xi}}\right)$$

Here equation (34) describes the free propagation.

In presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be related as $du = du_+ + du_-$, therefore, substituting the shock conditions (6) into equation (8) and using equation (34), we get

$$U^2 = \left[k' r^{\left(k_1 + k_1'\right)} - \left\{\frac{k_4 + k_4'}{k_1 + k_1'} + \frac{\left\{\lambda - (k_2 + k_2')(k_4 + k_4')\right\}}{\left(1 - k_1 - k_1'\right)}r - \frac{\left\{0\ \ \lambda^2 - \lambda(k_2 + k_2')\right\}}{\left(2 - k_1 - k_1'\right)}r^2\right\}\right] \quad \text{....... (35)}$$

$$\exp\left\{-(k_2 + k_2')r + (k_3 + k_3')r\left(\log r - 1\right)\right\}$$

Where, k' = constant of integration and $k_1 = \frac{\chi A}{2C'},\ k_2 = -\left\{\frac{\lambda\chi}{\gamma C'} + \frac{\lambda\chi^2 AK}{2\gamma^2 C'^2\beta^2} + \frac{\lambda\chi AK}{2\gamma C'\beta^2}\right\},\ k_3 = \left(\frac{\lambda\chi^2 A}{2\gamma C'^2} + \frac{\lambda\chi A}{2C'}\right)$

$$k_4 = \frac{\gamma p'\beta^2}{\rho' C'}\left(1 - \xi^2\right)\left(1 - \frac{\chi AK}{2\gamma C'\beta^2}\right),\ C' = \left(\frac{\chi}{\gamma} - \frac{\left(\xi - 1\right)}{2}\sqrt{\frac{\chi}{\xi}}\right)$$

Here the equation (35) includes the eod behind the flow on the motion of shock.

iii) { $H_z = H_{z_0} (\text{constant})$ *and* $H_\theta = r^{1/2} H_{z_0}$ **}**

Assuming the initial density distribution as $\rho_0 = \rho' e^{-\lambda r}$, the equilibrium state of the gas is assumed to be specified by the condition

$$\frac{\partial}{\partial t}.=0=u, H_\theta = H_{z_0} \text{ and } H_z = H_{z_0} (\text{constant})$$

Under the above conditions the equation of pressure will be,

$$\frac{p_0}{p'} = K - \frac{3}{2}\gamma\beta^2 r \quad \text{……. (36)}$$

$$\frac{da_0}{a_0} = \frac{1}{2}\left(\frac{dp_0}{p_0} + \lambda dr\right) \text{ and } \frac{db_0}{b_0} = \left(\frac{\lambda}{2}dr + \frac{1}{2(1+r)}dr\right)$$

Weak Shock Weak Magnetic Field (WSWMF):-

Substituting the shock conditions (3) into equation (7), we get

$$\varepsilon_+(r) = k' p_0^{-k_1} \exp\left(k_3 r^2 - k_2 r\right) \quad \text{…….. (37)}$$

Where k' is a constant of integration

$$k_1 = \left(\frac{\gamma-2}{2\gamma} + \frac{1}{4}\right),\ k_2 = \left\{\frac{\lambda}{4} - \frac{\lambda\beta^2}{8K}\right\} \text{and } k_3 = \frac{\lambda\beta^2}{16K}$$

Here equation (37), describes the free propagation. Now Substituting the shock conditions (3) into equation (8), we get

$$\varepsilon_-(r) = k'(1+r)^{-k_1'} e^{-\frac{3\lambda\gamma}{2}r} \quad \text{…… (38)}$$

Where k' is a constant of integration and

$$k_1' = \left(\frac{12-3\gamma}{4p'} - \frac{\lambda K}{2\beta^2} - \frac{3\lambda\gamma}{2}\right)$$

In presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be related as $du = du_+ + du_-$, therefore, substituting the respective results, we get

$$\varepsilon_{eod} = \varepsilon_+ \varepsilon_- \sqrt{p_0}\ e^{\frac{\lambda}{2}r} \quad \text{……. (39)}$$

Here the equation (39) includes the eod behind the flow on the motion of shock

Weak Shock Strong Magnetic Field (WSSMF):-

Substituting the shock conditions (4) into equation (7), we get

$$\varepsilon_+(r) = k'(1+r)^{-k_1} \exp\left\{-\left(k_2 r + \frac{K}{8\beta^2}\frac{1}{(1+r)}\right)\right\} \quad (40)$$

Where k' is the constant of integration and

$$k_1 = \left(-\frac{3(\gamma-2)}{4p'} - \frac{\lambda k}{8\beta^2} - \frac{3\lambda\gamma}{8} + \frac{1}{4}\right),\ k_2 = \left(\frac{3\lambda\gamma}{8} + \frac{\lambda}{4}\right)$$

Here equation (40) describes the free propagation. Now Substituting the shock conditions (4) into equation (8), we get

$$\varepsilon_-(r) = k' p_0^{-\frac{(\gamma-2)}{\gamma}} \exp\left[-\left\{\left(\frac{\lambda\beta^2}{K} + \frac{\beta^2}{2K}\right)r + \frac{\lambda\beta^2}{2K}r^2\right\}\right] \quad (41)$$

In presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be related as $du = du_+ + du_-$, therefore, substituting the respective results, we get

$$\varepsilon_{eod} = \varepsilon_+ \varepsilon_- \sqrt{(1+r)}\ e^{\frac{\lambda}{2}r} \quad \text{….. (42)}$$

Here the equation (42) includes the eod behind the flow on the motion of shock.

Strong ShockWeak Magnetic Field (SSWMF):-

Substituting the shock conditions (5) into equation (7), we get

$$U^2 = \left[k' - k_3\left\{r + 0.5(\lambda + k_1)r^2 + \frac{k_2}{6}r^3 + ...\right\}\right] e^{-(k_1 r + 0.5 k_2 r^2)} \quad \text{…………… (43)}$$

Where, k' = constant of integration and

$$k_1 = \left(\frac{\lambda\chi' A'\beta^2}{\gamma^2 B^2 K} - \frac{\lambda A'\beta^2}{\gamma BK} - \frac{\lambda\chi'}{\gamma B} + \frac{A'\beta^2}{\gamma BK}\right),\ k_2 = \left(\frac{\lambda\chi' A'\beta^2}{\gamma^2 B^2 K} - \frac{\lambda A'\beta^2}{\gamma BK}\right),\ k_3 = \frac{3\gamma p'\beta^2(1-\xi^2)}{2\rho' B}$$

$$\text{and } B = \left(\frac{\chi'}{\gamma} + \frac{(\xi-1)}{2}\sqrt{\frac{\chi'}{\xi}}\right)$$

Here equation (43) describes the free propagation.

In presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be related as $du = du_+ + du_-$, therefore, substituting the shock conditions (5) into equation (8) and using equation (43), we get

$$U^2 = \left[k' - (k_3 + k_3') \left\{ r + 0.5\left(\lambda + k_1 + k_1'\right) r^2 + \frac{k_2 + k_2'}{6} r^3 + ... \right\} \right] e^{\left[-\left\{ (k_1 + k_1') r + 0.5 (k_2 + k_2') r^2 \right\} \right]} \quad .... (44)$$

Where, k' = constant of integration and

$$k_1 = \left(\frac{\lambda\chi' A' \beta^2}{\gamma^2 B'^2 K} - \frac{\lambda A' \beta^2}{\gamma B' K} - \frac{\lambda\chi'}{\gamma B'} + \frac{A'\beta^2}{\gamma B' K} \right),\ k_2 = \left(\frac{\lambda\chi' A' \beta^2}{\gamma^2 B'^2 K} - \frac{\lambda A' \beta^2}{\gamma B' K} \right),\ k_3 = \frac{3\gamma p' \beta^2 \left(1 - \xi^2\right)}{2\rho' B'}$$

and $B' = \left(\frac{\chi'}{\gamma} - \frac{(\xi - 1)}{2} \sqrt{\frac{\chi'}{\xi}} \right)$

Here the equation (44) includes the eod behind the flow on the motion of shock.

Strong Shock Strong Magnetic Field (SSSMF):- Substituting the shock conditions (6) into equation (7), we get

$$U^2 = \left[k'(1+r)^{-k_1} - \left\{ k_6 (1+r) + k_7 r (1+r) - k_8 (1+r)^2 + k_9 \right\} \right] \exp\left\{ -\frac{k_3}{(1+r)} + k_2 r \right\} \quad .... (45)$$

Where, k' = constant of integration and $k_1 = \left(\frac{\lambda\chi^2 A}{2C} + \frac{\lambda\chi^2 AK}{C\gamma^2\beta^2} - \frac{\lambda\chi}{\gamma C} - \frac{3\chi A}{2C} \right),\ k_2 = \frac{\lambda\chi^2 A}{2C},\ k_3 = \frac{\chi\ AK}{C\gamma\ \beta^2}$

$$k_4 = \left(\frac{9\chi A\gamma p' \beta^2 \left(\xi^2 - 1\right)}{4\rho' C^2} - \frac{3\chi AKp' \left(1 - \xi^2\right)}{2\rho' C^2} \right),\ k_5 = \left(\frac{3\gamma p' \beta^2 \left(\xi^2 - 1\right)}{2\rho' C} + \frac{9\chi A\gamma p' \beta^2 \left(\xi^2 - 1\right)}{4\rho' C^2} \right),$$

$$C = \left(\frac{\chi}{\gamma} + \frac{(\xi - 1)}{2} \sqrt{\frac{\chi}{\xi}} \right)$$

$$k_6 = \left(k_2 k_4 - \lambda k_4 + \lambda k_3 k_5 - k_5 \right) / \left(1 + k_1\right),\ k_7 = \frac{\left(k_2 - \lambda\right) k_5}{\left(1 + k_1\right)},$$

$$k_8 = k_7 / \left(2 + k_1\right),\ k_9 = \left(k_4 - k_3 k_5 + k_2 k_4 - \lambda k_4 + \lambda k_3 k_5 \right) / k_1$$

Here equation (45) describes the free propagation.

In presence of both C_+ and C_- characteristics the fluid velocity increment behind the shock will be related as $du = du_+ + du_-$, therefore, substituting the shock conditions (6) into equation (8) and using equation (45), we get

$$U^2 = \left[k'(1+r)^{-D_1} - \left\{ D_6 (1+r) + D_7 r (1+r) - D_8 (1+r)^2 + D_9 \right\} \right] \exp\left\{ -\frac{D_3}{(1+r)} + D_2 r \right\}$$

.... (46)

Where, k' = constant of integration and $D_1 = (k_1 + k_1'),\ D_2 = (k_2 + k_2'),\ D_3 = (k_3 + k_3'),\ D_4 = (k_4 + k_4'),\ D_5 = (k_5 + k_5')$

$$D_6 = \left(D_2 D_4 - \lambda D_4 + \lambda D_3 D_5 - D_5 \right) / \left(1 + D_1\right),\ D_7 = \frac{\left(D_2 - \lambda\right) D_5}{\left(1 + D_1\right)},$$

$$D_8 = D_7 / \left(2 + D_1\right),\ D_9 = \left(D_4 - D_3 D_5 + D_2 D_4 - \lambda D_4 + \lambda D_3 D_5 \right) / D_1$$

$$k_1' = \left(\frac{\lambda\chi^2 A}{2C'} + \frac{\lambda\chi^2 AK}{C'\gamma^2\beta^2} - \frac{\lambda\chi}{\gamma C'} - \frac{3\chi A}{2C'} \right),\ k_2' = \frac{\lambda\chi^2 A}{2C'},\ k_3' = \frac{\chi\ AK}{C'\gamma\ \beta^2}$$

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$$k_4' = \left(\frac{9\chi A\gamma p'\beta^2\left(\xi^2-1\right)}{4\rho' C'^2} - \frac{3\chi AKp'\left(1-\xi^2\right)}{2\rho' C'^2} \right),\ k_5' = \left(\frac{3\gamma p'\beta^2\left(\xi^2-1\right)}{2\rho' C'} + \frac{9\chi A\gamma p'\beta^2\left(\xi^2-1\right)}{4\rho' C'^2} \right),$$

$$C' = \left(\frac{\chi}{\gamma} - \frac{(\xi-1)}{2}\sqrt{\frac{\chi}{\xi}} \right)$$

Here the equation (46) includes the eod behind the flow on the motion of shock.

Analytical expression for Flow variables:-

The flow variables expressions in three cases can be written as

i) {$H_z = H_{z_0}(cons\tan t)$}, by substituting the equations (9, 12), (13, 16) in boundary conditions (3), (4) for WSWMF and WSSMF;

ii) {$H_z = H_{z_0}(cons\tan t)\ and\ H_\theta = H_{z_0}(cons\tan t)$}, by substituting the equations (26, 28), (29, 31), (32, 33) and (34, 35) respectively, in boundary conditions (3), (4), (5) and (6) for WSWMF, WSSMF, SSWMF and SSSMF and

iii) {$H_z = H_{z_0}(cons\tan t)\ and\ H_\theta = r^{1/2}H_{z_0}$}, by substituting the equations (37, 39), (40, 42), (43, 44) and (45, 46) respectively, in boundary conditions (3), (4), (5) and (6) for WSWMF, WSSMF, SSWMF and SSSMF as below-

i) {$H_z = H_{z_0}(cons\tan t)$}

WSWMF (FP):-,

$$\frac{U}{a_0} = \left[1 + \frac{(\gamma+1)}{4}\varepsilon_+\right],\ p = p_0(1+\gamma\varepsilon_+)$$

$$\frac{U}{a'} = \left[1 + \frac{(\gamma+1)}{4}\varepsilon_+\right]\sqrt{\frac{p_0}{p'}} \text{ And } \frac{u}{a'} = \varepsilon_+\sqrt{\frac{p_0}{p'}}$$

……. (47)

EOD:-

$$\frac{U}{a_0} = \left[1 + \frac{(\gamma+1)}{4}\varepsilon_{eod}\right],\ p = p_0(1+\gamma\varepsilon_{eod})$$

$$\frac{U}{a'} = \left[1 + \frac{(\gamma+1)}{4}\varepsilon_{eod}\right]\sqrt{\frac{p_0}{p'}} \text{ And } \frac{u}{a'} = \varepsilon_{eod}\sqrt{\frac{p_0}{p'}}$$

…….. (48)

WSSMF (FP):-

$$\frac{U}{a_0} = \left[1 + \frac{3}{4}\varepsilon_+\right]\sqrt{\frac{\mu H_{z_0}^2}{\gamma p_0}},\ p = p_0(1+\gamma\varepsilon_+)$$

$$\frac{U}{a'} = \left[1 + \frac{3}{4}\varepsilon_+\right]\sqrt{\frac{\mu H_{z_0}^2}{\gamma p'}} \text{ And } \frac{u}{a'} = \varepsilon_+\sqrt{\frac{\mu H_{z_0}^2}{\gamma p'}}$$

….. (49)

EOD:-

$$\frac{U}{a_0} = \left[1 + \frac{3}{4}\varepsilon_{eod}\right]\sqrt{\frac{\mu H_{z_0}^2}{\gamma p_0}},\ p = p_0(1+\gamma\varepsilon_{eod})$$

$$\frac{U}{a'} = \left[1 + \frac{3}{4}\varepsilon_{eod}\right]\sqrt{\frac{\mu H_{z_0}^2}{\gamma p'}} \text{ And}$$

$$\frac{u}{a'} = \varepsilon_{eod}\sqrt{\frac{\mu H_{z_0}^2}{\gamma p'}}$$

….. (50)

ii) {$H_z = H_{z_0}(cons\tan t)\ and\ H_\theta = H_{z_0}(cons\tan t)$}

WSWMF (FP):-,

$$\frac{U}{a_0} = \left[1 + \frac{(\gamma+1)}{4}\varepsilon_+\right],\ p = p_0(1+\gamma\varepsilon_+)$$

$$\frac{U}{a'} = \left[1 + \frac{(\gamma+1)}{4}\varepsilon_+\right]\sqrt{\frac{p_0}{p'}e^{\lambda r}} \text{ And}$$

$$\frac{u}{a'} = \varepsilon_+\sqrt{\frac{p_0}{p'}e^{\lambda r}}$$

……. (51)

EOD:-

$$\frac{U}{a_0} = \left[1 + \frac{(\gamma+1)}{4}\varepsilon_{eod}\right],\ p = p_0(1+\gamma\varepsilon_{eod})$$

$$\frac{U}{a'} = \left[1 + \frac{(\gamma+1)}{4}\varepsilon_{eod}\right]\sqrt{\frac{p_0}{p'}e^{\lambda r}} \text{ And}$$

$$\frac{u}{a'} = \varepsilon_{eod}\sqrt{\frac{p_0}{p'}e^{\lambda r}}$$

…….. (52)

WSSMF (FP):-

$$\frac{U}{a_0}=\left[1+\frac{3}{4}\varepsilon_+\right]\sqrt{\frac{2\mu H_{z_0}^2}{\gamma p_0}},\ p=p_0(1+\gamma\varepsilon_+)$$

$$\frac{U}{a'}=\left[1+\frac{3}{4}\varepsilon_+\right]\sqrt{\frac{2\mu H_{z_0}^2 e^{\lambda r}}{\gamma p'}}\ \text{And}$$

$$\frac{u}{a'}=\varepsilon_+\sqrt{\frac{2\mu H_{z_0}^2 e^{\lambda r}}{\gamma p'}}\ \ldots..\ (53)$$

EOD:-

$$\frac{U}{a_0}=\left[1+\frac{3}{4}\varepsilon_{eod}\right]\sqrt{\frac{2\mu H_{z_0}^2}{\gamma p_0}},\ p=p_0(1+\gamma\varepsilon_{eod})$$

$$\frac{U}{a'}=\left[1+\frac{3}{4}\varepsilon_{eod}\right]\sqrt{\frac{2\mu H_{z_0}^2 e^{\lambda r}}{\gamma p'}}\ \text{And}$$

$$\frac{u}{a'}=\varepsilon_{eod}\sqrt{\frac{2\mu H_{z_0}^2 e^{\lambda r}}{\gamma p'}}\ \ldots..\ (54)$$

SSWMF (FP):-

$$\frac{U}{a_0}=\sqrt{\frac{k'}{a'^2}-\frac{\beta^2\left(1-\xi^2\right)}{B}\left\{\log r+\left(\lambda+k_1\right)r+0.25\left(\lambda+k_1\right)^2 r^2+\frac{\left(\lambda+k_1\right)^3}{18}r^3\right\}e^{-k_1 r/2}}$$

$$\frac{U}{a'}=\frac{U}{a_0}\sqrt{\frac{p_0}{p'}e^{\lambda r}},\qquad \frac{u}{a'}=\frac{(\xi-1)}{\xi}\frac{U}{a'},$$

$$\frac{p}{p'}=\frac{p_0}{p'}\left[1+\left\{\chi'+\frac{2A'\beta^2}{p_0/p'}\right\}\left(\frac{U}{a_0}\right)^2\right]\ \ldots\ldots.\ (55)$$

and $\rho/\rho'=e^{\lambda r}\xi$

SSWMF (EOD):-

$$\frac{U}{a_0}=\sqrt{\frac{k'}{a'^2}-\beta^2\left(1-\xi^2\right)\left(\frac{1}{B}+\frac{1}{B'}\right)\left\{\log r+\left(\lambda+k_1\right)r+0.25\left(\lambda+k_1\right)^2 r^2+\frac{\left(\lambda+k_1\right)^3}{18}r^3\right\}e^{-\left(k_1+k_1'\right)r/2}}$$

$$\frac{U}{a'}=\frac{U}{a_0}\sqrt{\frac{p_0}{p'}e^{\lambda r}},\qquad \frac{u}{a'}=\frac{(\xi-1)}{\xi}\frac{U}{a'},$$

$$\frac{p}{p'}=\frac{p_0}{p'}\left[1+\left\{\chi'+\frac{2A'\beta^2}{p_0/p'}\right\}\left(\frac{U}{a_0}\right)^2\right]\ \ldots\ldots.\ (56)$$

and $\rho/\rho'=e^{\lambda r}\xi$

SSSMF (FP):-

$$\frac{U}{a_0}=\sqrt{\frac{k'}{a'^2}r^{k_1}-\frac{\rho'}{\gamma p'}\left\{\frac{k_4}{k_1}+\frac{\left(\lambda-k_2k_4\right)}{\left(1-k_1\right)}r-\frac{\left(0.5\lambda^2-\lambda k_2\right)}{\left(2-k_1\right)}r^2\right\}\text{ex}}$$

$$\frac{U}{a'}=\frac{U}{a_0}\sqrt{\frac{p_0}{p'}e^{\lambda r}},\qquad \frac{u}{a'}=\frac{(\xi-1)}{\xi}\frac{U}{a'},$$

$$\frac{p}{p'}=\frac{p_0}{p'}\left[1+\chi\left\{1+\frac{Ap_0}{2p'\beta^2}\right\}\left(\frac{U}{a_0}\right)^2\right]\ \ldots..\ (57)$$

and $\rho/\rho'=e^{\lambda r}\xi$

SSSMF (EOD):-

$$\frac{U}{a_0}=\sqrt{\frac{k'}{a'^2}r^{(k_1+k_1')}-\frac{\rho'}{\gamma p'}\left\{\frac{k_4+k_4'}{k_1+k_1'}+\frac{\{\lambda-(k_2+k_2')(k_4+k_4')\}}{\left(1-k_1-k_1'\right)}r-\right.}$$

$$\exp\sqrt{\left\{-(k_2+k_2')r+(k_3+k_3')r\left(\log r-1\right)\right\}}$$

$$\frac{U}{a'}=\frac{U}{a_0}\sqrt{\frac{p_0}{p'}e^{\lambda r}},\qquad \frac{u}{a'}=\frac{(\xi-1)}{\xi}\frac{U}{a'},$$

$$\frac{p}{p'}=\frac{p_0}{p'}\left[1+\chi_3\left\{1+\frac{Ap_0}{2p'\beta^2}\right\}\left(\frac{U}{a_0}\right)^2\right]\ \ldots\ldots\ (58)$$

iii) { $H_z=H_{z_0}\,(cons\tan t)\ and\ H_\theta=r^{1/2}H_{z_0}$ }

WSWMF (FP):-,

$$\frac{U}{a_0}=\left[1+\frac{(\gamma+1)}{4}\varepsilon_+\right],\ p=p_0(1+\gamma\varepsilon_+)$$

$$\frac{U}{a'}=\left[1+\frac{(\gamma+1)}{4}\varepsilon_+\right]\sqrt{\frac{p_0}{p'}e^{\lambda r}}\ \text{And}$$

$$\frac{u}{a'}=\varepsilon_+\sqrt{\frac{p_0}{p'}e^{\lambda r}}\ \ldots..\ (59)$$

EOD:-

$$\frac{U}{a_0}=\left[1+\frac{(\gamma+1)}{4}\varepsilon_{eod}\right],\ p=p_0(1+\gamma\varepsilon_{eod})$$

$$\frac{U}{a'}=\left[1+\frac{(\gamma+1)}{4}\varepsilon_{eod}\right]\sqrt{\frac{p_0}{p'}e^{\lambda r}}\ \text{And}$$

$$\frac{u}{a'}=\varepsilon_{eod}\sqrt{\frac{p_0}{p'}e^{\lambda r}}\ \ldots\ldots..\ (60)$$

WSSMF (FP):-

$$\frac{U}{a_0}=\left[1+\frac{3}{4}\varepsilon_+\right]\sqrt{\frac{\mu H_{z0}^2(1+r)}{\gamma p_0}},\ p=p_0(1+\gamma\varepsilon_+)$$

$$\frac{U}{a'}=\left[1+\frac{3}{4}\varepsilon_+\right]\sqrt{\frac{\mu H_{z0}^2(1+r)e^{\lambda r}}{\gamma p'}}\ \text{And}\ \frac{u}{a'}=\varepsilon_+\sqrt{\frac{\mu H_{z0}^2(1+r)e^{\lambda r}}{\gamma p'}} \quad \text{..... (61)}$$

EOD:-

$$\frac{U}{a_0}=\left[1+\frac{3}{4}\varepsilon_{eod}\right]\sqrt{\frac{\mu H_{z0}^2(1+r)}{\gamma p_0}},\ p=p_0(1+\gamma\varepsilon_{eod})$$

$$\frac{U}{a'}=\left[1+\frac{3}{4}\varepsilon_{eod}\right]\sqrt{\frac{\mu H_{z0}^2(1+r)e^{\lambda r}}{\gamma p'}}\ \text{And}\ \frac{u}{a'}=\varepsilon_{eod}\sqrt{\frac{\mu H_{z0}^2(1+r)e^{\lambda r}}{\gamma p'}} \quad \text{..... (62)}$$

SSWMF (FP):-

$$\frac{U}{a_0}=\sqrt{\frac{k'}{a'^2}-\frac{\rho'}{\gamma p'}k_3\left\{r+0.5(\lambda+k_1)r^2+\frac{k_2}{6}r^3+....\right\}}\,e^{-(k_1 r+0.5k_2 r^2)/2}$$

$$\frac{U}{a'}=\frac{U}{a_0}\sqrt{\frac{p_0}{p'}e^{\lambda r}}\ ,\ \frac{u}{a'}=\frac{(\xi-1)}{\xi}\frac{U}{a'}\ ,\ \frac{p}{p'}=\frac{p_0}{p'}\left[1+\left\{\chi'+\frac{2A'\beta^2}{p_0/p'}\right\}\left(\frac{U}{a_0}\right)^2\right] \quad \text{....... (63)}$$

and $\rho/\rho'=e^{\lambda r}\xi$

SSWMF (EOD):-

$$\frac{U}{a_0}=\sqrt{\frac{k'}{a'^2}-\frac{\rho'}{\gamma p'}(k_3+k_3')\left\{r+0.5(\lambda+k_1+k_1')r^2+\frac{k_2+k_2'}{6}r^3+....\right\}}\,e^{\sqrt{\left[-\left\{(k_1+k_1')r+0.5(k_2+k_2')r^2\right\}\right]}}$$

$$\frac{U}{a'}=\frac{U}{a_0}\sqrt{\frac{p_0}{p'}e^{\lambda r}}\ ,\ \frac{u}{a'}=\frac{(\xi-1)}{\xi}\frac{U}{a'}\ ,\ \frac{p}{p'}=\frac{p_0}{p'}\left[1+\left\{\chi'+\frac{2A'\beta^2}{p_0/p'}\right\}\left(\frac{U}{a_0}\right)^2\right] \quad \text{....... (64)}$$

and $\rho/\rho'=e^{\lambda r}\xi$

SSSMF (FP):-

$$\frac{U}{a_0}=\sqrt{\frac{k'}{a'^2}(1+r)^{-k_1}-\frac{\rho'}{\gamma p'}\left\{k_6(1+r)+k_7 r(1+r)-k_8(1+r)^2+k_9\right\}}\ \exp\sqrt{\left\{-\frac{k_3}{(1+r)}+k_2 r\right\}}$$

$$\frac{U}{a'}=\frac{U}{a_0}\sqrt{\frac{p_0}{p'}e^{\lambda r}}\ ,\ \frac{u}{a'}=\frac{(\xi-1)}{\xi}\frac{U}{a'}\ ,\ \frac{p}{p'}=\frac{p_0}{p'}\left[1+\chi\left\{1+\frac{Ap_0}{2p'\beta^2}\right\}\left(\frac{U}{a_0}\right)^2\right] \quad \text{..... (65)}$$

and $\rho/\rho'=e^{\lambda r}\xi$

SSSMF (EOD):-

$$\frac{U}{a_0}=\sqrt{\frac{k'}{a'^2}(1+r)^{-D_1}-\frac{\rho'}{\gamma p'}\left\{D_6(1+r)+D_7 r(1+r)-D_8(1+r)^2+D_9\right\}}\ \exp\sqrt{\left\{-\frac{D_3}{(1+r)}+D_2 r\right\}}$$

$$\frac{U}{a'}=\frac{U}{a_0}\sqrt{\frac{p_0}{p'}e^{\lambda r}}\ ,\ \frac{u}{a'}=\frac{(\xi-1)}{\xi}\frac{U}{a'}\ ,\ \frac{p}{p'}=\frac{p_0}{p'}\left[1+\chi\left\{1+\frac{Ap_0}{2p'\beta^2}\right\}\left(\frac{U}{a_0}\right)^2\right] \quad \text{...... (66)}$$

4.Results and Discussion

The expressions of flow variables for both weak and strong shocks representing CCW description for the adiabatic motion of hydromagnetic plane diverging shock waves assuming an initial density distribution $\rho_0 = \rho' e^{-\lambda r}$ are given by equations (47-66). It is found analytically that the inclusion of effects of overtaking disturbances behind the flow on the motion of plane diverging shock in first situation {$H_z = H_{z_0} (cons\tan t)$} is insignificant when the shock is strong.

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