Linear Programming and Simulation

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Abstract : ***Linear programming is a method to achieve the best outcome in a mathematical model whose requirements are represented by linear relationships. Linear programming determines the optimal use of a resource to maximize or minimize a cost .It is based on a mathematical technique that can be used according to the following three methods: a graphics resolution; an algebraic resolution; and the use of the simplex algorithm .The standard form is a form of linear programming where all of the constraints are inequalities and where the variables are required to be positive .The change into standard form consists of transforming the constraints of inequality into equality by requiring that variables be positive .In the case, one can use mathematical programming, such as linear programming. In this scenario, simulation helps when the parameters contain noise or the evaluation of the problem would demand excessive computer time, due to complexity .One can use mathematical programming, as well as dynamic programming.***

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1. Introduction

Linear programming (LP, also called linear optimization) is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. Linear programming is a special case of mathematical programming (also known as mathematical optimization).By simulation is meant the technique of setting up a stochastic model of a real situation, and then performing sampling experiments upon the model. The feature which distinguishes a simulation from a mere sampling experiment in the classical sense is that of the stochastic model. Whereas a classical sampling experiment in statistics is most often performed directly upon raw data, a simulation entails first of all the construction of an abstract model of the system to be studied.

2. Linear Programming

Linear programming, mathematical modeling technique in which a linear function is maximized or minimized when subjected to various constraints .This technique has been useful for guiding quantitative decisions in business planning, in industrial engineering, and—to a lesser extent—in the social and physical sciences .The solution of a linear programming problem reduces to finding the optimum value (largest or smallest, depending on the problem) of the linear expression (called the objective function). Applications of the method of linear programming were first seriously attempted in the late 1930s by the Soviet mathematician Leonid Kantorovich and by the American economist Wassily Leontief in the areas of manufacturing schedules and of economics, respectively, but their work was ignored for decades. During World War II, linear programming was used extensively to deal with transportation, scheduling, and allocation of resources subject to certain restrictions such as costs and availability. These applications did much to establish the acceptability of this method, which gained further impetus in 1947 with the introduction of the American mathematician George Dantzig's simplex method, which greatly simplified the solution of linear programming problems. However, as increasingly more complex problems involving more variables were attempted, the number of necessary operations expanded exponentially and exceeded the computational capacity of even the most powerful computers. Then, in 1979, the Russian mathematician Leonid Khachiyan discovered a polynomial-time algorithm—in which the number of computational steps grows as a power of the number of variables rather than exponentially—thereby allowing the solution of hitherto inaccessible problems. However, Khachiyan's algorithm (called the ellipsoid method) was slower than the simplex method when practically applied. In 1984 Indian mathematician Narendra Karmarkar discovered another polynomial-time algorithm, the interior point method , that proved competitive with the simplex method [1]

3. History of LP

The problem of solving a system of linear inequalities dates back at least as far as Fourier, who in 1827 published a method for solving them, and after whom the method of Fourier–Motzkin elimination is named.

In 1939 a linear programming formulation of a problem that is equivalent to the general linear programming problem was given by the Soviet mathematician and economist Leonid Kantorovich, who also proposed a method for solving it.It is a way he developed, during World War II, to plan expenditures and returns in order to reduce costs of the army and to increase losses imposed on the enemy. Kantorovich's work was initially neglected in the USSR. About the same time as Kantorovich, the Dutch-American economist T. C. Koopmans formulated classical economic problems as linear programs. Kantorovich and Koopmans later shared the 1975 Nobel prize in economics. In 1941, Frank Lauren Hitchcock also formulated transportation problems as linear programs and gave a solution very similar to the later simplex method. Hitchcock had died in 1957 and the Nobel prize is not awarded posthumously.

During 1946–1947, George B. Dantzig independently developed general linear programming formulation to use for planning problems in the US Air Force. In 1947, Dantzig also invented the simplex method that for the first time efficiently tackled the linear programming problem in most cases. When Dantzig arranged a meeting with John von Neumann to discuss his simplex method, Neumann immediately conjectured the theory of duality by realizing that the problem he had been working in game theory was equivalent. Dantzig provided formal proof in an unpublished report "A Theorem on Linear Inequalities" on January 5, 1948. Dantzig's work was made available to public in 1951. In the post-war years, many industries applied it in their daily planning.

Dantzig's original example was to find the best assignment of 70 people to 70 jobs. The computing power required to test all the permutations to select the best assignment is vast; the number of possible configurations exceeds the number of particles in the observable universe. However, it takes only a moment to find the optimum solution by posing the problem as a linear program and applying the simplex algorithm. The theory behind linear programming drastically reduces the number of possible solutions that must be checked.

The linear programming problem was first shown to be solvable in polynomial time by Leonid Khachiyan in 1979, but a larger theoretical and practical breakthrough in the field came in 1984 when Narendra Karmarkar introduced a new interior-point method for solving linear-programming problems.

4. Uses of LP

Linear programming is a widely used field of optimization for several reasons. Many practical problems in operations research can be expressed as linear programming problems. Certain special cases of linear programming, such as network flow problems and multicommodity flow problems are considered important enough to have generated much research on specialized algorithms for their solution. A number of algorithms for other types of optimization problems work by solving LP problems as sub-problems. Historically, ideas from linear programming have inspired many of the central concepts of optimization theory, such as duality, decomposition, and the importance of convexity and its generalizations. Likewise, linear programming was heavily used in the early formation of microeconomics and it is currently utilized in company management, such as planning, production, transportation, technology and other issues. Although the modern management issues are ever-changing, most companies would like to maximize profits and minimize costs with limited resources. Therefore, many issues can be characterized as linear programming problems.

5. Applications of LP

Linear programming and Optimization are used in various industries. The manufacturing and service industry uses linear programming on a regular basis.

Manufacturing industries use linear programming for analyzing their supply chain operations. Their motive is to maximize efficiency with minimum operation cost. As per the recommendations from the linear programming model, the manufacturer can reconfigure their storage layout, adjust their workforce and reduce the bottlenecks.

Linear programming is also used in organized retail for shelf space optimization. Since the number of products in the market has increased in leaps and bounds, it is important to understand what does the customer want. Optimization is aggressively used in stores like Walmart, Hypercity, Reliance, Big Bazaar, etc. The products in the store are placed strategically keeping in mind the customer shopping pattern. The objective is to make it easy for a customer to locate & select the right products. This is subject to constraints like limited shelf space, a variety of products, etc.

Optimization is also used for optimizing Delivery Routes. This is an extension of the popular traveling salesman problem. The service industry uses optimization for finding the best route for multiple salesmen traveling to multiple cities. With the help of clustering and greedy algorithm, the delivery routes are decided by companies like FedEx, Amazon, etc. The objective is to minimize the operation cost and time.

Optimizations are also used in Machine Learning. Supervised Learning works on the fundamental of linear programming. A system is trained to fit on a mathematical model of a function from the labeled input data that can predict values from an unknown test data.[2]

6. Limitations of LP

Linear programming has turned out to be a highly useful tool of analysis for the business executive. It is being increasingly made use of in theory of the firm, in managerial economics, in inter-regional trade, in general equilibrium analysis, in welfare economics and in development planning.
But it has its limitations:
1. It is not easy to define a specific objective function.
2. Even if a specific objective function is laid down, it may not be so easy to find out various technological, financial and other constraints which may be operative in pursuing the given objective.
3. Given a specific objective and a set of constraints, it is possible that the constraints may not be directly expressible as linear inequalities.
4. Even if the above problems are surmounted, a major problem is one of estimating relevant values of the various constant coefficients that enter into a linear programming mode, i.e., prices, etc.

5. This technique is based on the assumption of linear relations between inputs and outputs. This means that inputs and outputs can be added, multiplied and divided. But the relations between inputs and outputs are not always linear. In real life, most of the relations are non-linear.
6. This technique assumes perfect competition in product and factor markets. But perfect competition is not a reality.
7. The LP technique is based on the assumption of constant returns. In reality, there are either diminishing or increasing returns which a firm experiences in production.
8. It is a highly mathematical and complicated technique. The solution of a problem with linear programming requires the maximization or minimization of a clearly specified variable. The solution of a linear programming problem is also arrived at with such complicated method as the 'simplex method' which involves a large number of mathematical calculations.
9. Mostly, linear programming models present trial-and-error solutions and it is difficult to find out really optimal solutions to the various economic problems.

7. Simulation

Simulation in general is pretending that one deals with a real thing while really working with an imitation. In operations research the imitation is a computer model of the simulated reality. A flight simulator on a PC is a computer model simulation of some aspects of a flight: it shows on the screen, the controls, and what the "pilot" (the person who operates it) is supposed to see from the "cockpit" (his armchair).

To fly a simulator is safer and cheaper than the real airplane. For precisely this reason models are used in industry, commerce, and the military. It is very costly, dangerous, or often impossible to make experiments with real systems (nuclear explosions). Simulation may be an effective alternative provided that the models are adequate descriptions of reality, and experimenting with them can save money, suffering and even time.

Systems which change with time such as a gas station where cars come and go (called dynamic systems) and involve randomness (nobody can guess at exactly which time the next car should arrive at the station) are good candidates to simulation. Modeling some complex dynamic systems mathematically may require too many simplifications and the emerging models may not be valid. Simulation of the system may not require that many simplifying assumptions, making it one of the few tools available.

Simulations may be performed manually. More often, however, the system model is written either as a computer code or as some kind of input into a simulator software.

8. Types of Simulation

Discrete event simulations. These attempt to describe systems that are assumed to change instantaneously in response to certain discrete occurrences. Arrivals and departures of cars at a toll booth is an example of discrete events.

Continuous. In some systems the state changes all the time, not just at time of some discrete events. For example water level in a reservoir with a given inflow and outflow may change all the time. In such cases continuous simulation is more appropriate, although discrete event simulation can serve as an approximation.

9. Software for simulation

Simulation may be carried out on a spreadsheet, by running a computer program written in some general language (such as C++), by running a computer program written in some special language for simulations, or by running specially built simulators with appropriate input.

Simulation with spreadsheets is the most primitive form. Spreadsheets can be used for small simulations quite effectively. They are laborious and slow however. Their usefulness can be improved considerably by including macros and Visual Basic code.

Simulation written in some general language. This is the most flexible method and according to some authors the most often chosen, too. This is probably the best method if the simulation is just one component of a larger system or when there is a considerable amount of data processing involved. The problem here is that writing a simulation from scratch may be a lengthy undertaking. However, with a good simulation library and a skeleton program the necessary investment can be reduced to minimal.

Simulation written in some special language for simulations. There are many special languages for simulations. The disadvantage is that if the problem does not fall exactly in the category that these languages were designed for, it is very difficult, if not impossible, to write a suitable program.

Special purpose simulators are the most inflexible programs: They work only for a narrow field and even there they cannot deal with all tasks, but only those for which they are destined.

10. Monte Carlo Method

The expression "Monte Carlo method" is actually very general. Monte Carlo (MC) methods are stochastic techniques--meaning they are based on the use of random numbers and probability statistics to investigate problems. You can find MC methods used in everything from economics to nuclear physics to regulating the flow of traffic. Of course the way they are applied varies widely from field to field, and there are dozens of subsets of MC even within a single field. But, strictly speaking, to call something a "Monte Carlo" experiment, all you need to do is use random numbers to examine some problem.

The use of MC methods to model physical problems allows us to examine more complex systems than we could otherwise. Solving equations which describe the interactions between two atoms is fairly simple; solving the same equations for hundreds or thousands of atoms is impossible.

With MC methods, a large system can be sampled in a number of random configurations, and that data can be used to describe the system as a whole.

The first real use of a Monte Carlo calculation was probably during the Manhattan Project where very complex equations that could not be attacked with traditional methods had to be solved. The Monte Carlo technique is now used in virtually all areas of science and engineering where problems are too difficult or complex to be addressed with other methods. [4]

11. Conclusion

Linear programming is one of the widely used programming technique .Linear programming consists of an objective function (also known as cost function) which have to be minimized or maximized subject to a certain number of constraints. The objective function consists of a certain number of variables. The constraints are linear inequalities of the variables used in the objective function. This technique is closely related to Linear algebra and uses inequality problem statement rather than equality. But simulation is meant to be the technique of setting up a stochastic model of a real situation, and then performing sampling experiments upon the model. The feature which distinguishes a simulation from a mere sampling experiment in the classical sense is that of the stochastic model.

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