Plasma Dispersion Relations of Weakly and Non-Degenerate States

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Abstract: ***Condition for degeneracy of plasma by using the parameters density and temperature for different kinds of plasma were analysed on the basis of Sommerfield equation. Electrons in a laser produced plasma is considered as a weakly degenerate plasma .Dispersion relations are analyzed and studied.***

Keywords: Degeneracy, Sommerfield equation, coulomb logarithm, dispersion relation

1. Introduction

Plasmas is a quasi neutral gas composed of protons electrons and neutral particles. It is characterized by the density n of the particles. 'n' varies from $10^6$to $10^{34}m^{-3}$ and the temperature which in KT which varies from 0.1 to 10^6 eV. The temperature and density of the gas are measure of the degeneracy of plasma. For that we have considered Sommerfield parameter [1], given by the equation

$$\rho = \frac{nh^3}{\gamma(2\pi mkT)^{3/2}} = \frac{n\lambda^3}{\gamma}$$

where, h *is* is the Planck constant, $\gamma$is the internal degree of degeneracy of the particles considered ($\gamma = 2$ for electrons due to the possible spin states), m *is* the particle mass, k is the Boltzmann constant, and λ is the mean thermal wavelength of the particles. When densities and temperatures are such that the Sommerfield parameter is much smaller than 1, quantum effects are negligible and the gas is said to be non degenerate. Value of ρ much greater than 1 corresponds to degenerate gas. If Sommerfield parameter is in the vicinity of 1, the gas is only weakly degenerate; quantum effects are still non-negligible and they alter the statistical behavior of the gas only mildly, and the gas is considered almost as Maxwellian.

Dispersion relations for weakly and non degenerate plasmas

We can consider a plasma as weakly degenerate if the Sommerfield parameter is in between 0.5 and 1.5. Laser produced plasma is considered as weakly degenerate..For consideration we take plasma with electron density 10^{28} m-3 and electron temperature 1eV has Sommerfield parameter 1.66 and is considered as weakly degenerate. When it is out of thermodynamic equilibrium, as in the case when an electromagnetic wave interacts with it, kinetic theory methods are needed to determine the distribution function of the electrons which is an appropriate formulation of the transport equation.

Basic Equations

Quantum degeneracy introduced in the Boltzmann equation with appropriate collision term gives Boltzmann-Uehling-Uhlenbeck equation[2,3]

$$\frac{\partial f}{\partial t} + \upsilon\nabla f + \frac{F}{m}.\frac{\partial f}{\partial \upsilon} = \left(\frac{\partial f}{\partial t}\right)_{coll} \quad (1)$$

Dispersion relations are obtained by using an expression for electrical current density[4],

$$J=\sigma.E \quad (2)$$

While taking into account the degeneracy of the electron component of plasma as

$$\omega_0^2 = k^2c^2 + \omega_p^2\frac{\omega 0}{\omega 0 - i\omega c}\left[1 - \frac{\rho}{8}\frac{(2i\omega c - \omega c)}{(\omega 0 - i\omega c)}\right] \quad (3)$$

where $\omega_p = \sqrt{\frac{ne^2}{m\varepsilon_0}}$ is the electron plasma frequency.

In equation (3) taking as the limits

$$\omega_c = 0 \rightarrow \omega_0^2 = k^2c^2 + \omega_p^2\left(1 + \frac{\rho}{8}\right) \quad (4)$$

This is valid for collisionless weakly degenerate plasma

For long wavelength limit k=0; then the equation for a collisional non degenerate plasma is

$$\rho = 0 \rightarrow \omega_0^2 = k^2c^2 + \omega_p^2\frac{\omega 0}{(\omega 0 - i\omega c)} \quad (5)$$

The limit for both ρ and ω_c towards zero gives the simple relation

$$\omega_0^2 = k^2c^2 + \omega_p^2$$

equations (4) and (5) are solved numerically by taking degeneracy into account by properly evaluating electron-ion collision frequency. In this work the following form has been adopted

$$\omega_c \approx 2Zn\frac{ln\Lambda}{T^{3/2}}10^{-6}\text{ Hz}$$

where the electron temperature T is expressed ineV, Z is the atomic number of the target and ln Λ is the Coulomblogarithm.

The typical values of ln Λ is calculated for different values of KT_e and density n as follows [5]

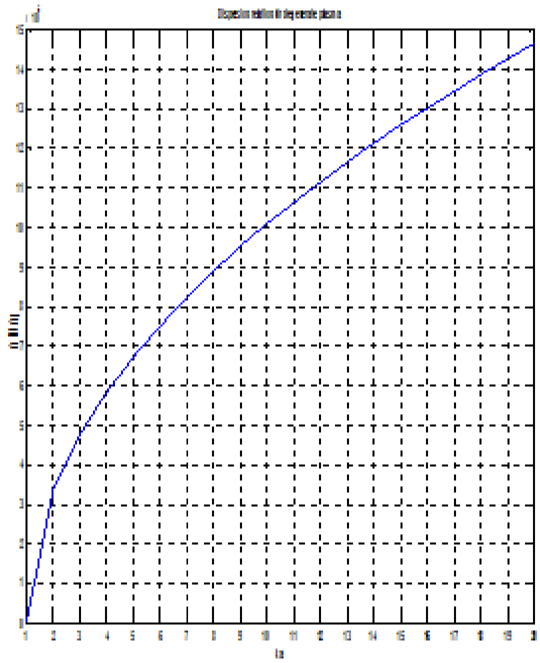
KT_e(in KeV)	*n*	*n*	*n*
1	10^{12}	10^{14}	10^{16}
1	10^{12}	10^{14}	10^{16}
10	10^{12}	10^{14}	10^{16}
100	10^{12}	10^{14}	10^{16}
	ln Λ	ln Λ	ln Λ
1	16.5	14.2	11.9
1	20	17.7	15.4
10	23.4	21.1	18.8
100	26.9	24.6	22.3

For classical non-degenerate plasmas the Coulomb logarithm, which can be defined as the natural logarithm of the ratio between the maximum and the minimum impact parameters for a binary collision, varies smoothly between 3and 10. However, in the present case the maximum and the minimum impact parameters have very different values from the that of anon-degenerate plasma[6] not only the distribution function, which is also different from a Maxwell an one, but also the high values of the electron densities render the impact parameters much lower than the classical ones. From the reduced average distance between electrons, the difference between the maximum impact parameter and the minimum will be very small, so that the Coulomb logarithm results significantly lower than the classical plasma. The corresponding binary collision frequency is therefore lower than the classical one. This is justified by the fact that at high density values the plasma dissipates through collective mechanisms and binary collisions. Additionally the collision potentials are screened more efficiently than in the classical case: consequently the sphere of direct influence is smaller than the classical Debye sphere. The minimum impact parameter is limited by quantum in determinant [7,8] to values of the de Broglie wavelength. According to Heisenberg principle, the minimum impact parameter can be set equal to $b_{\min} \approx \frac{\hbar}{mv}$; the maximum impactparameter $b_{\max}$ is not, as in classical plasmas, by the Debye length λ_D and can be more significantly by the Fermi screening length λ_F defined in the CGS-ESU system as

$$\lambda_F = \sqrt{\frac{E_F}{6\pi n e^2}}$$

where E_F is the Fermi energy[10] of the electron component.

For 1eV and n= 28m^{-3} electron plasma Fermi energy is approximately E_F= 1.7 eV and the respective Fermi length is approximately λ_F = 7.9 $\times 10^{-11}$ m. The problems in evaluating the minimum impact parameter are bigger since the velocity of particles is distribution and therefore does not have a precise unique value. The most coherent solution to this is to make use of an average speed; however this quantity depends strongly on the intensity of the applied fields. As a consequence, the collision frequency will depend logarithmically on the fields themselves.



2. Result

Compared the dispersion relations of electromagnetic waves in weakly degenerate collisional plasma and non degenerate plasma. The maximum and the minimum impact parameters have very different values from the non-degenerate plasma. While considering weakly degenerate plasma[9] quantum effects are considered and quantum statistics are used for study.

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