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Mass Spectra of Quarkonium in Relativistic Independent Quarks

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Abstract: ***Quarkonium is a flavorless meson comprised of quarks (c,b) and their antiquarks ($\bar{c}$,$\bar{b}$), while Positronium (Ps) is made of electrons and their antiparticles (positrons). Positronium will be composed of two fermionic particles. Like quarkonia, the system's total spin links to a singlet (s = 0) and a triplet (s = 1) state. In this work, we look at the mass spectra of quarkonium in a relativistic square root potential model. We employed two-body Dirac equations to examine relativistic quarks' mass spectra in meson-bound states. At low energies, the spectra of quarkonium are similar to those of Positronium. In both cases, the analogy breaks down either by its parts being destroyed or by the electromagnetic shift. The only thing that makes quarkonium and Positronium different is the basic forces that hold them together. Positronium is only kept together by electromagnetic force, which comprises two leptons with no color. At the same time, quarkonium is attracted to electromagnetic fields and has a color force that is much stronger and governs the potential. Lastly, we compare the spectra of quarkonium and Positronium to show their similarities and differences.***

Keywords: Quarkonium, mass spectra, similarities and differences

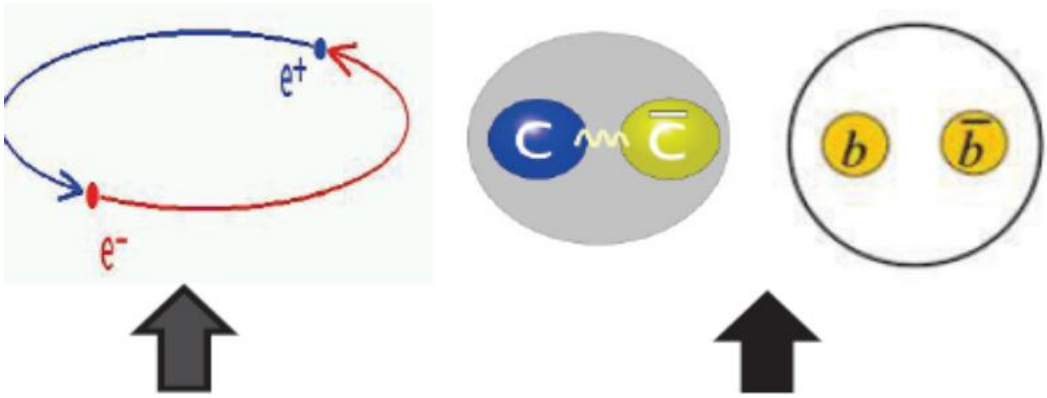
1. Introduction

While heavy quarkonium, $b\underline{b}$ and $c\underline{c}$'are the bound state of quark, andan antiquark is crucial for comprehending hadrons' strong interactions. Quantum chromodynamics (QCD) is commonly considered the only utterly accurate theory in strong interactions.Recently, as a result of the discovery of new states at various high-energy accelerators,including BABAR, Belle, CLEO, and BES-III collaborations [1–4], studies of the mass spectra and decays of heavy quarkonia($b\underline{b}$ and $c\underline{c}$)have become a significant focus. These pique the interest of many researchers who wish to pursue a deeper theoretical understanding of quarkonium. They also serve as a valuable parameter for studying the internal structure of hadrons and the long-distance (nonperturbative) behavior of strong interactions. There have been numerous nonperturbative methods proposed to study these properties, including LQCD [5], chiral perturbation theory[6], QSR (QCD sum rules) [7], heavy-quark effective theory [8], NRQCD [9], dynamical equation-based approaches like the Schwinger-Dyson equation and Bethe-Salpeter equation (BSE) [10-15], and potential models [16]. The Bethe-Salpeter equation (BSE) [11, 15, 17–19] is a well-known method for solving relativistic bound state problems. It provides a great deal of information about the inner structure of quarkonium and is also essential for detecting mesonic decays.

On the other hand, the input model-dependent kernel is the most significant drawback of the BSE approach. Neither QSR (QCD sum rule) nor LQCD can provide more than the ground state or a few exceptionally excited states. In contrast, the potential model helps investigate the excited states. Since the discovery of the charmonium (c,c.) and bottomonium states (b, b) states of heavy quarkonium spectroscopy [4, 5], potential models have played an essential role in understanding heavy quarkonium spectroscopy[5,6,7,8,9]. It has been possible to investigate various properties of both charmonium and bottomonium, such as mass spectra and decay properties, by using an interquark potential in a two-body Schrodinger equation in potential models [8, 9]. To determine interquark potentials, we use a combination of phenomenology and theory.

The possibility with specific parameters in the phenomenological method is that they can be evaluated through fits to the data. Using perturbative QCD and Lattice QCD, one can study the potential form at short and long distances. Frequently used phenomenological potentials include power-law potentials [10-13], harmonic potentials [18-20], linear potentials [13-17], and logarithmic potentials. [21] discusses the characteristics of these potentials in greater detail. The mass spectra estimated in relativistic and non-relativistic potential models agree with the experimental values. They may, however, not be with leptonic decays, hadronic decays, or radiative transitions [22-27].

The results presented in this paper utilize a relativistic potential with a single gluon exchange potential and square root confinement. First, to study the mass spectra of the quarkonia, we solved the Dirac equations using two-body mechanisms. Then, we computed the mass spectra of charmonia and bottomonia states to determine the quark masses and confinement strengths after fitting the spin-averaged ground state masses to experimental data.



(Positronium) **(Charmonium and Bottomonium)**

The paper is structured as follows: after the introduction, we present in section II the fundamental structure for obtaining mass spectra of quarkonium, namely the respective charmonium, and bottomonium. Section III compares the mass spectra of charmonium, bottonium, and Positronium, and section IV analyzes our findings in conjunction with other theoretical models and recent experimental significance.

2. Basic Framework

Suppose the meson is an assembly of a quark and antiquark independently confined by an average potential [2,4],

$$V_q(r) = (1+\gamma^0)\, V(r), \text{where} V(r) = \left(a^{\frac{3}{2}} r^{\frac{1}{2}} + V_0\right) \quad (1)$$

Where 'a' is the potential strength and V_0 is a negative potential depth is a constant. According to the first principle of QCD, the confining part of the interaction is supposed to give the zeroth-order quark dynamics inside the meson through the Lagrangian density

$$\mathcal{L}_q^0(x) = \underline{\psi}_q(x)\left[\frac{i}{2}\gamma^\mu \vec{\partial}_\mu - V(r) - m_q\right]\psi_q(x) \quad (2)$$

The Dirac equations obtain from $\mathcal{L}_q^0(x)$[4]

$$[\gamma^0 E_q - \gamma.P - m_q - (r)]\psi_q(r) = 0 \quad (3)$$

We must solve the two-component (positive and negative energy)Dirac equation to obtain binding energy. Its solution we write as [4,5]

$$\psi_{nlj}(r) = \begin{pmatrix} \psi_{nlj}^{(+)} \\ \psi_{nlj}^{(-)} \end{pmatrix} \quad (4)$$

where the positive and negative energy solutions correspond to the quark and antiquark respectively are written as [4,5],

$$\psi_{nlj}^{(+)}(r) = N_{nlj}\begin{pmatrix} \frac{i\, g(r)}{r} \\ \frac{(\sigma.\hat{r}) f(r)}{r} \end{pmatrix} y_{ljm}(\hat{r}), \quad (5a)$$

$$\psi_{nlj}^{(-)}(r) = N_{nlj}\begin{pmatrix} \frac{i\,(\sigma.\hat{r}) f(r)}{r} \\ \frac{g(r)}{r} \end{pmatrix} (-1)^{j+m_j-l} y_{ljm}(\hat{r}) \quad (5b)$$

Here, overall normalization, N_{nlj}, is constant.The corresponding energy eigenvalue is given by [4, 5],

$$\epsilon = \frac{(E_D - m_q - 2V_0)(m_q + E_D)^{\frac{1}{5}}}{(16\, a^6)^{\frac{1}{5}}} \quad (6)$$

Calculating ∈ from equation 6 and using the formula,

$$E_q = E'_q + \frac{V_o}{2} \quad (7)$$

We calculate the individual quark binding energy E_q Which leads to the energy of the meson core in the zerothorder. The mass of the meson in the zerothorder, we can write a sum of the two quarks' binding energy, i.e.,

$$M^0{}_{Q\underline{Q}} = E_D^Q + E_D^{\underline{Q}} \quad (8)$$

Mass of particular quark-antiquark system, we write as [4,5],

$$M_{Q\underline{Q}} = E_D^Q + E_D^{\underline{Q}} - E_{cm} \quad (7)$$

Here, E_{cm}, in general,it can be state-dependent on which we absorb in our potential parameter V_0,making V_0 as a state-dependent parameter.

A. Center-of-mass Correction

In this approach, there seems to be a significant spurious major contributor to the energy,Eq, due to the center of mass motion of the quark-antiquark system. If this part is considered, the theory of quarks moving independently inside this meson core need not arise in a definite physical meson state. Though that topic is still debatable, we use the method proposedby [30,31], which is one method for accounting for center-of-mass motion. Following their prescription, one can obtain a quick estimate of the center-of-mass momentum,$\vec{P}_M$,as.

$$\langle \vec{P}^2{}_M \rangle = \sum_q \langle \vec{P}_q^2 \rangle$$

Where $\langle \vec{P}_q^2 \rangle$ is the average value of the square of the individualquark momentum taken over $1S_{\frac{1}{2}}$single-quark state and definedin this model as

$$\langle \vec{P}_q^2 \rangle = \frac{(E_q + m_q)(E_q - m_q - 2V_o)(5E_q - 3V_o + 2m_q)}{7(3E_q - V_o + 2m_q)} \quad (10)$$

Similarly, the expression for the C.M. corrected mass of the bare meson core was found as

$$E_M = \left(E_M^{O^2} - \sum_q \langle \vec{P}_q^2 \rangle\right)^{\frac{1}{2}} \quad (11)$$

As a result, the energy obtains the required C.M. correction.

$$(\Delta E_M)_{c.m} = E_M - E_M^O = \left(E_M^{O^2} - \sum_q \langle \vec{P}_q^2 \rangle\right)^{\frac{1}{2}} - E_M^O \quad (12)$$

The Lorentz structure of the potential model used in this study has equally mixed scalar and vector parts in square root form. The main benefit of using a Lorentz structure like this is that converting the Dirac equation into a Schrodinger-type equation simplifies the analysis in this paper.

B. One gluon exchange correction:

According to equation (1), the individual quark-antiquark within the meson core is presumed to be undergoing the only force generated by the average potential V q (r) in the eqn. (2). All that should be left within the meson core is the weak one-gluon exchange interaction given by the Lagrangian interaction density,

$$L_1^g(x) = \sum_a J_i^{\mu a}(x) A_\mu^a(x) \quad (13)$$

where $A_\mu^a(x)$ are the 8-vector gluon fields and $J_i^{\mu a}(x)$, is the i[th] quark color current. Since the quarks should be almost free at a small distance, it is reasonable to calculate the energy shift in the mass spectrum arising from the quark interaction energy due to their coupling to the colored gluons using a first-order perturbation theory. Such an approach leads to the color-electric and color-magnetic energy shifts,

$$(\Delta E_M)_g^{e,m} = (\Delta E_M)_g^e + (\Delta E_M)_g^m \quad (14)$$

Which, according to our earlier work [5] on this model, are

$$(\Delta E_M)_g^e = \frac{\alpha_c}{\pi} \sum_{i,j} \sum_a \lambda_i^a \lambda_j^a \rangle N_i^2 N_j^2 I_{ij}^e \quad (15)$$

$$(\Delta E_M)_g^m = -\frac{4\alpha_c}{3\pi} \sum_{i<j} \langle \sum_a \lambda_i^a \lambda_j^a (\vec{\sigma}_i.\vec{\sigma}_j) \rangle \frac{N_i^2 N_j^2}{\lambda_i \lambda_j} I_{ij}^m \quad (16)$$

where λ_i^a, λ_j^a are the usual Gellmann SU(3) matrices and

$$\alpha_c = \frac{g_c^2}{4\pi} \quad (17)$$

$$I_{ij}^e = \int_0^\infty dk\, F_i^e(k) F_j^e(k) \quad (18)$$

$$\text{with} \quad F_i^e = \frac{1}{2\lambda_i^2}\left[(4(E_i - V_0)\lambda_i - k^2) \langle\langle j_0 |\vec{k}| r_i \rangle\rangle - 2\lambda ia\langle\langle ri12j0krj\rangle\rangle \right. \quad (19)$$

and

$$I_{ij}^m = \int_0^\infty dk\, k^2 \langle\langle j_0 |\vec{k}| r_i \rangle\rangle \langle\langle j_0 (|\vec{k}| r_j) \rangle\rangle \quad (20)$$

where $j_0(|\vec{k}| r_i)$ represents the zeroth-order spherical Bessel functionand the double angular bracket expectation values to the radial angular part $\phi_q(\vec{r})$, of the quark wave function. Now, taking into account the specific quark flavor and spin configuration in various ground-state mesons and using the relations, $\langle \sum_a (\lambda_i^a)^2 \rangle = \frac{16}{3}$and $\langle \sum_a \lambda_i^a \lambda_j^a \rangle_{i \neq j} = -\frac{16}{3}$ for mesons, one can, in general, write the energy correction due to one-gluon exchange as denotedby the

$$(\Delta E_M)_g^e = \alpha_c \sum_{i,j} a_{ij} T_{ij}^e \quad (21)$$

$$(\Delta E_M)_g^m = \alpha_c \sum_{i,j} b_{ij} T_{ij}^m \quad (22)$$

where a_{ij} and b_{ij} are the numerical co-efficient depending upon each meson and the terms $T_{ij}^{e,m}$ are

$$T_{ij}^e = \frac{100}{3\pi} \frac{(E_i - m_i - 2V_0)(E_j + m_j)}{(E_i + 2m_i - V_0)(3E_j - 5V_0 - 2m_j)} I_{ij}^e \quad (23)$$

$$T_{ij}^m = \frac{200}{9\pi} \frac{1}{(3E_i + 2m_i - V_0)(3E_j - V_0 + 2m_j)} I_{ij}^m \quad (24)$$

Observe that the color electric contribution for meson masses is zero when the constituent quark and antiquark masses in a meson core are equal.However, it is non-zero when the constituent quark and antiquark masses in a meson core are not similar. Degeneracy, even amongst the mesons, is eliminated mainly throughout this model due to the high spin-spin interaction energy present only in the color magnetic component of the meson. The coefficients of non-self-conjugate mesons in the Qq system (Q and q denoting heavy and light quarks, respectively) we calculate as

$$a_{QQ} = 1, a_{qq} = 1, a_{Qq} = -2, b_{QQ} = 0 = b_{qq} \quad (25)$$

And

$$b_{Qq} = \{2,\ for\ triplet\ states, -6\ , for\ singlet\ states, \quad (26)$$

The integral expressions for $I_{ij}^{e,m}$, in E.Q.s. (18) and (20), we evaluate with the help of a standard numerical method, and these values would yield the terms $T_{ij}^{e,m}$, from Eqs.(23)and (24), which would ultimately enable one to compute the energy corrections $(\Delta E_M)_g^{e,m}$ due to one-gluon exchange through E.Q.s.(21) and (22).

We are using Eqs.(12),(21), and (22), it is straightforward to calculate the corrections due to spurious center-of-mass motion and residual one-gluon exchange interactions[29-30] in the respective states.

Finally, these corrections are associated with zeroth-order mass, $M^0{}_{Q\underline{Q}}$ of the meso to obtain its physical mass as

$$M_{Q\underline{Q}} == M^0{}_{Q\underline{Q}} + (\Delta E_M)_{c.m} + (\Delta E_M)_g^e + (\Delta E_M)_g^m \quad (27)$$

3. Results and Discussion

In the present model, the calculation of physical mass spectra of quarkonium (c$\underline{c}$) and (b$\underline{b}$), the system depends on the choice of potential parameters (a, V_0) and the quark mass m_q. In our present calculation, we use the parameters in our earlier work [1, 2, 4, 8].

$$(a, V_0) = (0.454, -0.465)GeV \quad (28)$$

and we find that the quark masses as

$$(m_c, m_b) = (1.462, 4.8184) \quad GeV \quad (29)$$

The physical masses of quarkonium (c$\underline{c}$) and (b$\underline{b}$) systems after the center of mass and gluonic correctionsare reasonably well with the corresponding experimental data.

In the present phenomenological fit, we calculate the$\vec{P}_M^2\rangle, T_{ij}^{e,m}$, used in the computation of center of mass and gluonic corrections are presented in Table -I.The physical masses obtained in this model, along with the corrections $(\Delta E_M)_{c.m}, (\Delta E_M)_g^{e,m}$ for (c$\underline{c}$) and (b$\underline{b}$) systems are displayed in Table -II,table-III respectively.

Table-1 Relevant quantities (in GeV unit) obtained as solutions for the final calculations. [3]

	s$\underline{s}$ system			c$\underline{c}$ system			b$\underline{b}$ system		
nS	**E'$_{nl}$**	$\langle p^2 \rangle_{nl}$	$I^M{}_{ss}$	**E'$_{nl}$**	$\langle p^2 \rangle_{nl}$	$I^M{}_{cc}$	**E'$_{nl}$**	$\langle p^2 \rangle_{nl}$	$I^M{}_{bb}$
1S	0.5791	0.3230	0.0441	1.6239	0.8665	0.0218	4.8096	2.5427	0.0121
2S	0.9222	0.9032	0.0881	1.9668	2.1673	0.0621	5.1574	6.0833	0.0399
3S	1.2075	1.5966	0.1240	2.2812	3.6085	0.1026	5.4937	9.7897	0.0731

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4S	1.4612	2.3730	0.1555	2.5744	5.1647	0.1415	5.8199	13.6486	0.1089
5S	1.6938	3.2171	0.1842	2.8510	6.8192	0.1778	6.1369	17.6446	0.1459
6S	1.9109	4.1192	0.2109	3.1142	8.5617	0.2125	6.4456	21.7718	0.1834

Table 2: The physical name of the meson of ($c\bar{c}$) system taking corrections due to the center of mass motion and one gluon exchange interaction into account. [3]

N- State	$N^{2S+1}L_j$	E_{nl} (GeV)	$M^0(c\bar{c})$ (GeV)	$(\otimes E)_{c.m}$ (GeV)	$(\otimes E)_g$ (GeV)	$M_{nl}(c\bar{c})$ (GeV)	Experimental Mass (GeV)
1S	1^3S_1 1^1S_0	1.6031	3.2062	-0.1381	0.0292 -0.0878	3.0973 2.9805	3.09693±0.00009 2.97960±0.0016
2S	2^3S_1 2^1S_0	1.9460	3.8920	-0.2892	0.0832 -0.2496	3.666 3.3532	3.68600±0.0001 3.639.2±0.0001
3S	3^3S_1 3^1S_0	2.2604	4.5208	-0.4185	0.1375 -0.4125	4.2398 3.6898	3.76990±0.0025
4S	4^3S_1 4^1S_0	2.5536	5.1072	-0.5335	0.1896 -0.5688	4.7633 4.0049	4.04000±0.01
5S	5^3S_1 5^1S_0	2.8302	5.6604	-0.6384	0.2383 -0.7148	5.2603 4.3072	4.1590±0.02
6S	6^3S_1 6^1S_0	3.0934	6.1868	-0.7353	0.2848 -0.8543	5.7359 4.5968	4.415±0.006

Table 3: The physical name of the meson of ($b\bar{b}$) system taking corrections due to center of mass motion and one gluon exchange interaction into account [3]

N- State	$N^{2S+1}L_j$	E_{nl} (GeV)	$M^0(b\bar{b})$ (GeV)	$(\otimes E)_{c.m}$ (GeV)	$(\otimes E)_g$ (GeV)	$M_{nl}(b\bar{b})$ (GeV)	Experimental Mass (GeV)
1S	1^3S_1 1^1S_0	4.7888	9.5776	-0.1337	0.0162 -0.0486	9.4601 9.3953	9.46032±0.00022 9.339±0.00023
2S	2^3S_1 2^1S_0	5.1366	10.2732	-0.3005	0.0535 -0.1604	10.0262 9.8123	10.02330±0.00031
3S	3^3S_1 3^1S_0	5.4729	10.9458	-0.4567	0.0980 -0.2939	10.5871 10.1952	10.3553±0.0005
4S	4^3S_1 4^1S_0	5.7991	11.5982	0.6041	0.1459 -0.4378	11.1400 10.5563	10.58±0.0035
5S	5^3S_1 5^1S_0	6.1161	12.2322	-0.7439	0.1955 -0.5865	11.6838 10.9018	10.865±0.008
6S	6^3S_1 6^1S_0	6.4248	12.8496	-0.8771	0.2458 -0.7373	12.2183 11.2352	11.019±0.008

Figure 1 represents the mass spectra of bottomonium (in MeV)

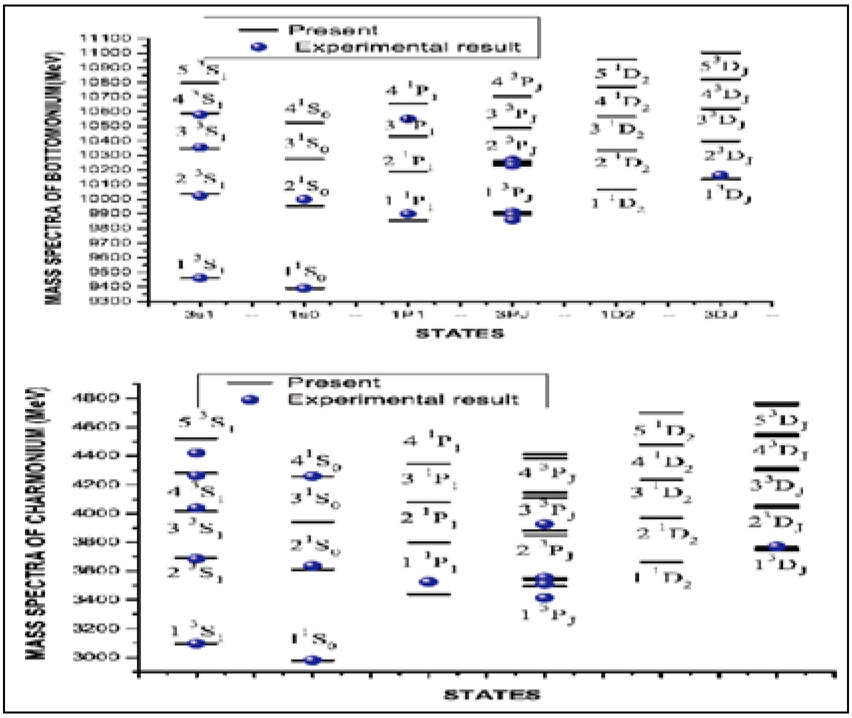
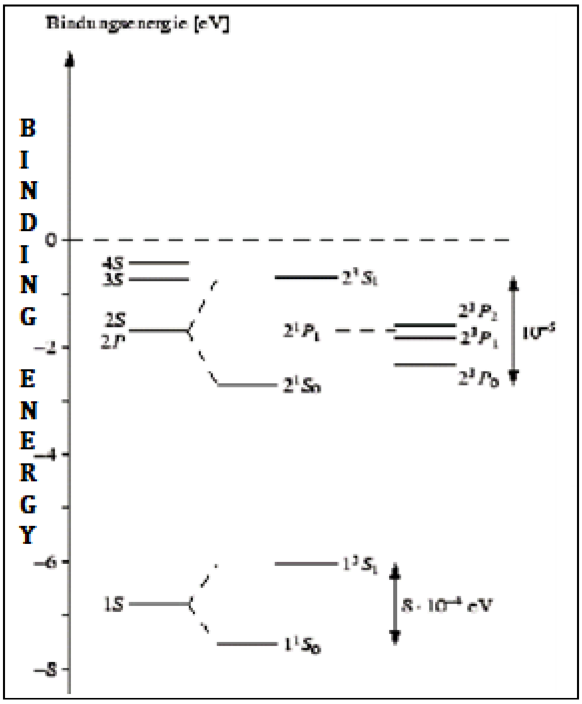


Figure 3: Represents the mass spectra of Positronium (in MeV)



4. Results and Discussion

- Our computed mass spectra for quarkonium are in good agreement with experimental results.
- From our observations, it has been noticed that at low energies, the quarkonium spectrum isvery similar to that of Positronium, and deviations from the spectra of the Positronium are only obtained with higher excitations. Another similarity is their binding energies arerelatively small.

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