Rotation of Conics and Quadric Surfaces only using Focal Points and Distance without Using Trigonometry

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Abstract: ***This paper addresses the mathematical equations of Conics and Quadric Surfaces that we can obtain by using two points and distances, using these three are sufficient to specify a rotated Conics and Quadric Surfaces of any shape and orientation without using Trigonometry, This paper has five different equations, which make different geometric shapes in both of Three-Dimensional Coordinate System and Two -Dimensional Coordinate System.***

Keywords: conics, circle, sphere, ellipse, ellipsoid, hyperbola, hyperboloid, line, Quadric Surfaces

1. Introduction

By this functions one can plot the Conics and Quadric Surfaces using any two focal points and a distance, and the rotation of comics is entirely dependent on its focal points

This function takes two focal points and a distance:

$A = (x_0, y_0, z_0)$ First focal point.

$B = (x_1, y_1, z_1)$ Second focal point.

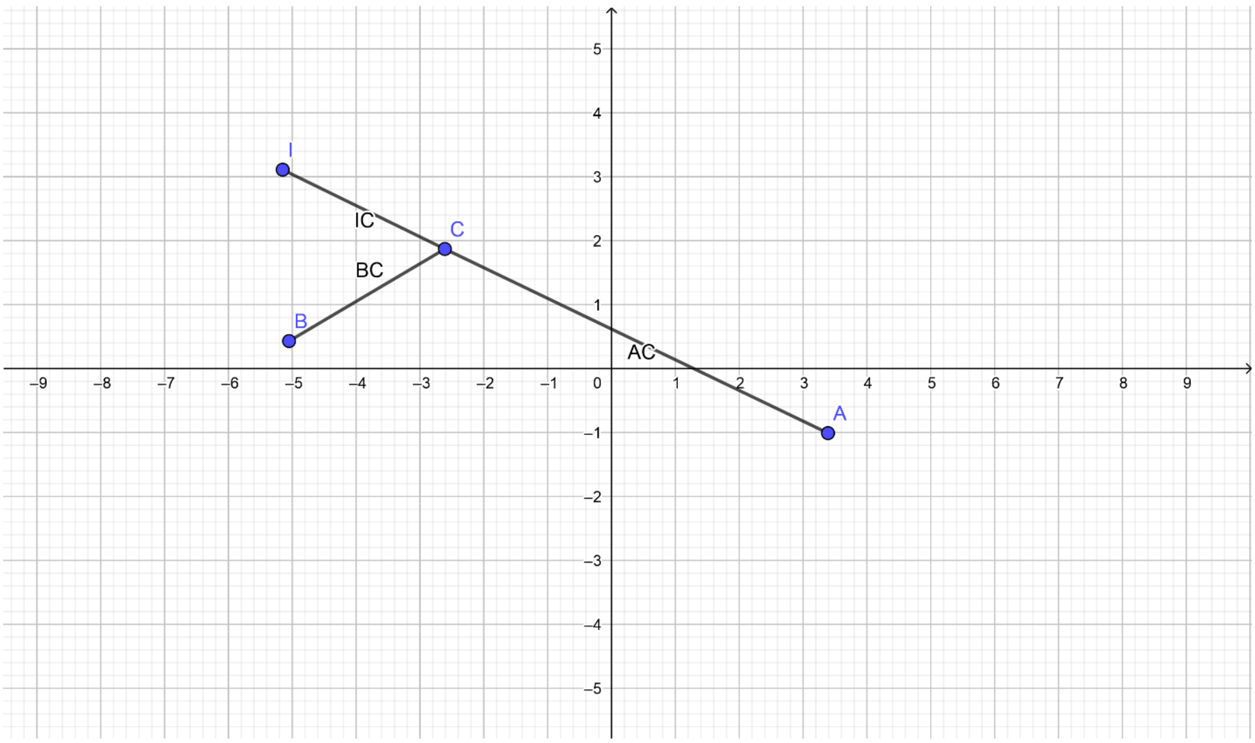
D is a sum of distances of BC and AC if C is any points C on locus.

These three are sufficient to describe a Conics and Quadric Surfaces of any shape and orientation.

Now let $I(x_i, y_i, z_i)$ is an Imaginary point and $C(x, y, z)$ is any point on locus. A point I are the set of points equidistant from one of the focal point of conics.

D is the sum of the distances of focal points from any point on locus

$$D = AC + BC$$



$I(x_i, y_i, z_i)$ Is a point such that it's collinear with point A and C and distance of point BC is same as IC, as in figure [image1] shown.

$$BC = CI$$

Since we will use point $I(x_i, y_i, z_i)$, let find point $I(x_i, y_i, z_i)$

We know line AI that passes through the points $A(x_0, y_0, z_0)$, $C(x, y, z)$ and $I(x_i, y_i, z_i)$ all point are collinear, since the expressions are linear. We can use parametric form of line to find $I(x_i, y_i, z_i)$,we know that t runs from $-\infty$ to ∞, a parametric form gives control over the length of the line, not only the line direction and the distance between initial point $A(x_0, y_0, z_0)$ and point $C(x, y, z)$ is proportional to parameter t. we can find $I(x_i, y_i, z_i)$using parameter t

$$x_i = x + t(x - x_0)$$
$$y_i = y + t(y - y_0)$$
$$z_i = z + t(z - z_0)$$

Because $I(x_i, y_i, z_i)$ is collinear with point $A(x_0, y_0, z_0)$ and $C(x, y, z)$ and $IC = BC$.

And because

$$\frac{IC}{AC} = \frac{BC}{AC} = t$$

by dividing BC with AC we get t

$$t = \frac{((x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2)^{\frac{1}{2}}}{((x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2)^{\frac{1}{2}}}$$
$$(x_i, y_i, z_i) = (x + t(x - x_0), y + t(y - y_0), z + t(z - z_0))$$

Now we have I (x_i, y_i, z_i) $A(x_0, y_0, z_0)$ and $C(x, y, z)$ all points are co-linear

$$AC + CI = AI$$

We already have a point $A(x_0, y_0, z_0)$ and $B(x_1, y_1, z_1)$ and D whereas $C(x, y, z)$ is any point on locus

Now we have to find $D = AC + BC$ and

$$D = AC + CI$$

$$D = AI$$

Distance between $A(x_0, y_0, z_0)$ and $I(x_i, y_i, z_i)$ is D $D = AI$it satisfies the condition required to get all locus points.

Now distance of AI is.

$$D = \sqrt{(x_0 - x_i)^2 + (y_0 + y_i)^2 + (z_0 - z_i)^2}$$

$$D = \sqrt{(x_0 - x - t(x - x_0))^2 + (y_0 + y - t(y - y_0))^2 + (z_0 - z - t(z - z_0))^2}$$

$$\left(x_0 - x - t\,(x - x_0)\right)^2 + \left(y_0 + y - t\,(y - y_0)\right)^2 + \left(z_0 - z - t\,(z - z_0)\right)^2 = D^2$$

Expanded

$$-2x_0x - 4x_0t^{\frac{1}{2}}x + 2t^{\frac{1}{2}}x^2 + tx^2 - 2x_0tx + x^2 + x_0^2 + 2x_0^2t^{\frac{1}{2}} + x_0^2t - 2y_0y - 4y_0t^{\frac{1}{2}}y + 2t^{\frac{1}{2}}y^2 + ty^2 - 2y_0ty + y^2 + y_0^2 + 2y_0^2t^{\frac{1}{2}} + y_0^2t - 2z_0z - 4z_0t^{\frac{1}{2}}z + 2t^{\frac{1}{2}}z^2 + tz^2 - 2z_0tz + z^2 + z_0^2 + 2z_0^2t^{\frac{1}{2}} + z_0^2t = D^2$$

$$x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + z^2 - 2z_0z + z_0^2 + tx^2 - 2x_0tx + x_0^2t + ty^2 - 2y_0ty + y_0^2 + tz^2 - 2z_0tz + z_0^2t + 2t^{\frac{1}{2}}x^2 - 4x_0t^{\frac{1}{2}}x + 2x_0^2t^{\frac{1}{2}} + 2t^{\frac{1}{2}}y^2 - 4y_0t^{\frac{1}{2}}y + 2y_0^2t^{\frac{1}{2}} + 2t^{\frac{1}{2}}z^2 - 4z_0t^{\frac{1}{2}}z + 2z_0^2t^{\frac{1}{2}} = D^2$$

Took t common

$$((x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + z^2 - 2z_0z + z_0^2)^{\frac{1}{2}} + (x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2 + z^2 - 2z_1z + z_1^2)^{\frac{1}{2}})^2 = D^2$$

Replacing t with its value

$$(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + z^2 - 2z_0z + z_0^2) + \frac{(x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2 + z^2 - 2z_1z + z_1^2)}{(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + z^2 - 2z_0z + z_0^2)}(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + z^2 - 2z_0z + z_0^2) + 2\frac{(x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2 + z^2 - 2z_1z + z_1^2)^{\frac{1}{2}}}{(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + z^2 - 2z_0z + z_0^2)^{\frac{1}{2}}}(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + z^2 - 2z_0z + z_0^2) = D^2$$

After simplification

$$(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + z^2 - 2z_0z + z_0^2) + 2(x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2 + z^2 - 2z_1z + z_1^2)^{\frac{1}{2}}(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + z^2 - 2z_0z + z_0^2)^{\frac{1}{2}} + (x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2 + z^2 - 2z_1z + z_1^2) = D^2$$

And.

$$((x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + z^2 - 2z_0z + z_0^2)^{\frac{1}{2}} + (x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2 + z^2 - 2z_1z + z_1^2)^{\frac{1}{2}})^2 = D^2$$

Then we have

$$(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + z^2 - 2z_0z + z_0^2)^{\frac{1}{2}} + (x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2 + z^2 - 2z_1z + z_1^2)^{\frac{1}{2}} = D$$

Let assume similar equation is in form

Let remove fraction points from

$$a^{\frac{1}{2}} + b^{\frac{1}{2}} = c$$

$$b^{\frac{1}{2}} = c - a^{\frac{1}{2}}$$

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$$b = c^2 - 2ca^{\frac{1}{2}} + a$$

$$a^{\frac{1}{2}} = \frac{c^2 - b + a}{2c}$$

$$a = \frac{-2c^2b - 2ba + b^2 + c^4 + 2c^2a + a^2}{4c^2}$$

$$a^2 - 2ba + b^2 - 2c^2a - 2c^2b + c^4 = 0$$

Expand everything we got.

$$\begin{aligned}
x_0^4 &- 4x_0^3x + 2x_0^2y_0^2 - 4x_0^2y_0y + 2x_0^2z_0^2 - 4x_0^2z_0z + 6x_0^2x^2 + 2x_0^2y^2 + 2x_0^2z^2 - 4x_0y_0^2x + 8x_0y_0xy - 4x_0z_0^2x + 8x_0z_0xz \\
&- 4x_0x^3 - 4x_0xy^2 - 4x_0xz^2 + y_0^4 - 4y_0^3y + 2y_0^2z_0^2 - 4y_0^2z_0z + 2y_0^2x^2 + 6y_0^2y^2 + 2y_0^2z^2 - 4y_0z_0^2y \\
&+ 8y_0z_0yz - 4y_0x^2y - 4y_0y^3 - 4y_0yz^2 + z_0^4 - 4z_0^3z + 2z_0^2x^2 + 2z_0^2y^2 + 6z_0^2z^2 - 4z_0x^2z - 4z_0y^2z \\
&- 4z_0z^3 + x^4 + 2x^2y^2 + 2x^2z^2 + y^4 + 2y^2z^2 + z^4 - 2x_0^2x_1^2 + 4x_0^2x_1x - 2x_0^2y_1^2 + 4x_0^2y_1y - 2x_0^2z_1^2 \\
&+ 4x_0^2z_1z - 2x_0^2x^2 - 2x_0^2y^2 - 2x_0^2z^2 + 4x_0x_1^2x - 8x_0x_1x^2 + 4x_0y_1^2x - 8x_0y_1xy + 4x_0z_1^2x - 8x_0z_1xz \\
&+ 4x_0x^3 + 4x_0xy^2 + 4x_0xz^2 - 2y_0^2x_1^2 + 4y_0^2x_1x - 2y_0^2y_1^2 + 4y_0^2y_1y - 2y_0^2z_1^2 + 4y_0^2z_1z - 2y_0^2x^2 \\
&- 2y_0^2y^2 - 2y_0^2z^2 + 4y_0x_1^2y - 8y_0x_1xy + 4y_0y_1^2y - 8y_0y_1y^2 + 4y_0z_1^2y - 8y_0z_1yz + 4y_0x^2y + 4y_0y^3 \\
&+ 4y_0yz^2 - 2z_0^2x_1^2 + 4z_0^2x_1x - 2z_0^2y_1^2 + 4z_0^2y_1y - 2z_0^2z_1^2 + 4z_0^2z_1z - 2z_0^2x^2 - 2z_0^2y^2 - 2z_0^2z^2 \\
&+ 4z_0x_1^2z - 8z_0x_1xz + 4z_0y_1^2z - 8z_0y_1yz + 4z_0z_1^2z - 8z_0z_1z^2 + 4z_0x^2z + 4z_0y^2z + 4z_0z^3 - 2x_1^2x^2 \\
&- 2x_1^2y^2 - 2x_1^2z^2 + 4x_1x^3 + 4x_1xy^2 + 4x_1xz^2 - 2y_1^2x^2 - 2y_1^2y^2 - 2y_1^2z^2 + 4y_1x^2y + 4y_1y^3 \\
&+ 4y_1yz^2 - 2z_1^2x^2 - 2z_1^2y^2 - 2z_1^2z^2 + 4z_1x^2z + 4z_1y^2z + 4z_1z^3 - 2x^4 - 4x^2y^2 - 4x^2z^2 - 2y^4 \\
&- 4y^2z^2 - 2z^4 + x_1^4 - 4x_1^3x + 2x_1^2y_1^2 - 4x_1^2y_1y + 2x_1^2z_1^2 - 4x_1^2z_1z + 6x_1^2x^2 + 2x_1^2y^2 + 2x_1^2z^2 \\
&- 4x_1y_1^2x + 8x_1y_1xy - 4x_1z_1^2x + 8x_1z_1xz - 4x_1x^3 - 4x_1xy^2 - 4x_1xz^2 + y_1^4 - 4y_1^3y + 2y_1^2z_1^2 \\
&- 4y_1^2z_1z + 2y_1^2x^2 + 6y_1^2y^2 + 2y_1^2z^2 - 4y_1z_1^2y + 8y_1z_1yz - 4y_1x^2y - 4y_1y^3 - 4y_1yz^2 + z_1^4 - 4z_1^3z \\
&+ 2z_1^2x^2 + 2z_1^2y^2 + 6z_1^2z^2 - 4z_1x^2z - 4z_1y^2z - 4z_1z^3 + x^4 + 2x^2y^2 + 2x^2z^2 + y^4 + 2y^2z^2 + z^4 \\
&- 2x_0^2D^2 + 4x_0D^2x - 2y_0^2D^2 + 4y_0D^2y - 2z_0^2D^2 + 4z_0D^2z - 2D^2x^2 - 2D^2y^2 - 2D^2z^2 - 2D^2x_1^2 \\
&+ 4D^2x_1x - 2D^2y_1^2 + 4D^2y_1y - 2D^2z_1^2 + 4D^2z_1z - 2D^2x^2 - 2D^2y^2 - 2D^2z^2 + D^4 = 0
\end{aligned}$$

After simplifying equations we got our final result for Quadric Surfaces

$$\begin{gathered}
4x^2((x_0 - x_1)^2 - D^2) + 4y^2((y_0 - y_1)^2 - D^2) + 4z^2((z_0 - z_1)^2 - D^2) + \\
8xy(x_0 - x_1)(y_0 - y_1) + 8xz(x_0 - x_1)(z_0 - z_1) + 8yz(y_0 - y_1)(z_0 - z_1) + \\
4x((x_0 - x_1)(x_1^2 + y_1^2 + z_1^2 - x_0^2 - y_0^2 - z_0^2) + x_0D^2 + x_1D^2) + \\
4y((y_0 - y_1)(x_1^2 + y_1^2 + z_1^2 - x_0^2 - y_0^2 - z_0^2) + y_0D^2 + y_1D^2) + \\
4z((z_0 - z_1)(x_1^2 + y_1^2 + z_1^2 - x_0^2 - y_0^2 - z_0^2) + z_0D^2 + z_1D^2) + \\
(x_0^2 + y_0^2 + z_0^2)(x_0^2 + y_0^2 + z_0^2 - D^2 - x_1^2 - y_1^2 - z_1^2) - \\
D^2(x_0^2 + y_0^2 + z_0^2 - D^2 + x_1^2 + y_1^2 + z_1^2) - \\
(x_1^2 + y_1^2 + z_1^2)(x_0^2 + y_0^2 + z_0^2 + D^2 - x_1^2 - y_1^2 - z_1^2) = 0
\end{gathered}$$

This above equation takes two points as foci's and distance.
It results different in different cases.

Sphere

Equation results circle when the distance between focal points is zero.

Point $A = B$

$D = 2r$ twice of radius
$D = AC + BC$

Prolate Ellipsoid

Equation results prolate Ellipsoid when the distance between focal points is less than the sum of the distances of both focal points from any point on the curve.

$A \neq B$
AC and BC always positive

Hyperboloid of two sheet

Equation results Hyperboloid of two sheet when the distance between focal points is greater than the sum of the distances of both focal points from any point on the curve.

$A \neq B$
Either one of AC or BC will be negative, the other will be greater than D, so that sum of both equal to D.

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Line

Equation results line when the distance between focal points is equals to D.

$A \neq B$

$D = AC + BC$

This only happens when the distance between focal points is equals to sum of the distances.

Similarly for two dimensional

By using similar method we get equation for two dimensional is.

$$D = \sqrt{(x_0 - x_i)^2 + (y_0 + y_i)^2}$$

Because $I(x_i, y_i, z_i)$ is collinear with point $A(x_0, y_0, z_0)$ and $C(x, y, z)$ and $IC = BC$, by dividing BC with AC we get t

$$t = \frac{((x_1 - x)^2 + (y_1 - y)^2)^{\frac{1}{2}}}{((x - x_0)^2 + (y - y_0)^2)^{\frac{1}{2}}}$$

$$D = \sqrt{(x_0 - x - t(x - x_0))^2 + (y_0 + y - t(y - y_0))^2}$$

After expanding

$$-2x_0x - 4x_0t^{\frac{1}{2}}x + 2t^{\frac{1}{2}}x^2 + tx^2 - 2x_0tx + x^2 + x_0^2 + 2x_0^2t^{\frac{1}{2}} + x_0^2t - 2y_0y - 4y_0t^{\frac{1}{2}}y + 2t^{\frac{1}{2}}y^2 + ty^2 - 2y_0ty + y^2 + y_0^2 + 2y_0^2t^{\frac{1}{2}} + y_0^2t = D^2$$

Arranged in way so that we can get t common

$$x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 + tx^2 - 2x_0tx + x_0^2t + ty^2 - 2y_0ty + y_0^2t + 2t\frac{1}{2}x^2 - 4x_0t\frac{1}{2}x + 2x_0^2t\frac{1}{2} + 2t\frac{1}{2}y^2 - 4y_0t\frac{1}{2}y + 2y_0^2t\frac{1}{2} = D^2$$

Took t common

$$(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2) + t(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2) + 2t^{\frac{1}{2}}(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2) = D^2$$

Replacing t with its value

$$((x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2) + \frac{(x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2)}{(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2)}(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2) + 2\frac{(x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2)^{\frac{1}{2}}}{(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2)}(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2) = D^2$$

When simplified.

$$(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2) + 2(x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2)^{\frac{1}{2}}(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2)^{\frac{1}{2}} + (x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2) = D^2$$

Again

$$(x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2)^{\frac{1}{2}} + (x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2)^{\frac{1}{2}} = D$$

By using point and after simplifying equations we got our final result for two dimension.

$$\begin{gathered} 4x^2((x_0 - x_1)^2 - D^2) + 4y^2((y_0 - y_1)^2 - D^2) + \\ 8 * x * y(x_0 - x_1)(y_0 - y_1) + 4x((x_0 - x_1)(x_1^2 + y_1^2 - x_0^2 - y_0^2) + x_0D^2 + x_1D^2) + \\ 4y((y_0 - y_1)(x_1^2 + y_1^2 - x_0^2 - y_0^2) + y_0D^2 + y_1D^2) + \\ (x_0^2 + y_0^2)(x_0^2 + y_0^2 - D^2 - x_1^2 - y_1^2) - D^2(x_0^2 + y_0^2 - D^2 + x_1^2 + y_1^2) \\ -(x_1^2 + y_1^2)(x_0^2 + y_0^2 + D^2 - x_1^2 - y_1^2) = 0 \end{gathered}$$

Circle

Equation results circle when the distance between focal points is zero.

Point $A = B$

$D = 2r$ twice of radius

$D = AC + BC$

Ellipse

Equation results ellipse when the distance between focal points is less than the sum of the distances of both focal points from any point on the curve.

$A \neq B$

AC and BC always positive

Hyperbola

Equation results hyperbola when the distance between focal points is greater than the sum of the distances of both focal points from any point on the curve.

$$A \neq B$$
$$D = AC + BC$$

Either one of AC or BC will be negative, the other will be greater than D, so that sum of both equal to D.

Line

Equation results line when the distance between focal points is equals to sum of the distances of both focal points from any point on the line.

$$A \neq B$$
$$D = AC + BC$$

This only happens when the distance between focal points is equals to sum of the distances so we can find new equation that gives equation of line all time

We know, if D is get replaced with distance $D = ((x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2)^{\frac{1}{2}}$ calculated formula always return line.

$D = ((x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2)^{\frac{1}{2}}$

Replacing D in equation to find equation of line.

By expanding and simplifying the equation we got equation of line.

$$\begin{gathered}
x^2(-(y_0 - y_1)^2 - (z_0 - z_1)^2) + \\
y^2(-(x_0 - x_1)^2 - (z_0 - z_1)^2) + \\
z^2(-(x_0 - x_1)^2 - (y_0 - y_1)^2) + \\
2 * x * y(x_0 - x_1)(y_0 - y_1) + 2 * x * z(x_0 - x_1)(z_0 - z_1) + 2 * y * z(y_0 - y_1)(z_0 - z_1) + \\
2x(x_0(y_1(y_1 - y_0) + z_1(z_1 - z_0)) + x_1(y_0(y_0 - y_1) + z_0(z_0 - z_1))) + \\
2y(y_1(x_0(x_0 - x_1) + z_0(z_0 - z_1)) + y_0(x_1(x_1 - x_0) + z_1(z_1 - z_0))) + \\
2z(z_1(x_0(x_0 - x_1) + y_0(y_0 - y_1)) + z_0(x_1(x_1 - x_0) + y_1(y_1 - y_0))) + \\
x_0(2y_0x_1y_1 - x_0(y_1^2 + z_1^2)) + y_0(2z_0y_1z_1 - y_0(x_1^2 + z_1^2)) + z_0(2x_0x_1z_1 - z_0(x_1^2 + y_1^2)) = 0
\end{gathered}$$

Cylinder

Equation results Cylinder when we add some constant in equation of line, let constant is C.

$$x^2(-(y_0-y_1)^2-(z_0-z_1)^2)+$$
$$y^2(-(x_0-x_1)^2-(z_0-z_1)^2)+$$
$$z^2(-(x_0-x_1)^2-(y_0-y_1)^2)+$$
$$2*x*y(x_0-x_1)(y_0-y_1)+2*x*z(x_0-x_1)(z_0-z_1)+2*y*z(y_0-y_1)(z_0-z_1)+$$
$$2x(x_0(y_1(y_1-y_0)+z_1(z_1-z_0))+x_1(y_0(y_0-y_1)+z_0(z_0-z_1)))+$$
$$2y(y_1(x_0(x_0-x_1)+z_0(z_0-z_1))+y_0(x_1(x_1-x_0)+z_1(z_1-z_0)))+$$
$$2z(z_1(x_0(x_0-x_1)+y_0(y_0-y_1))+z_0(x_1(x_1-x_0)+y_1(y_1-y_0)))+$$
$$x_0(2y_0x_1y_1-x_0(y_1^2+z_1^2))+y_0(2z_0y_1z_1-y_0(x_1^2+z_1^2))+$$
$$z_0(2x_0x_1z_1-z_0(x_1^2+y_1^2))+C=0$$

After some observation we found, putting (x, y, z)in Equation at any surrounding coordinates of line result some value, which is equals to multiplication of square of distance between focal points and square of minimum distance from equation of line that is.

$$C=((x_0-x_1)^2+(y_0-y_1)^2+(z_0-z_1)^2)r^2$$

Replacing C in equation result equation of Cylinder.

$$x^2(-(y_0-y_1)^2-(z_0-z_1)^2)+$$
$$y^2(-(x_0-x_1)^2-(z_0-z_1)^2)+$$
$$z^2(-(x_0-x_1)^2-(y_0-y_1)^2)+$$
$$2*x*y(x_0-x_1)(y_0-y_1)+2*x*z(x_0-x_1)(z_0-z_1)+2*y*z(y_0-y_1)(z_0-z_1)+$$
$$2x(x_0(y_1(y_1-y_0)+z_1(z_1-z_0))+x_1(y_0(y_0-y_1)+z_0(z_0-z_1)))+$$
$$2y(y_1(x_0(x_0-x_1)+z_0(z_0-z_1))+y_0(x_1(x_1-x_0)+z_1(z_1-z_0)))+$$
$$2z(z_1(x_0(x_0-x_1)+y_0(y_0-y_1))+z_0(x_1(x_1-x_0)+y_1(y_1-y_0)))+$$
$$x_0(2y_0x_1y_1-x_0(y_1^2+z_1^2))+y_0(2z_0y_1z_1-y_0(x_1^2+z_1^2))+$$
$$z_0(2x_0x_1z_1-z_0(x_1^2+y_1^2))+((x_0-x_1)^2+(y_0-y_1)^2+(z_0-z_1)^2)r^2=0$$

Where r is radius of Cylinder.

Elliptic Cone

In equation we can add constant to see its behavior

As we increase C the equation starts to take the shape of a Elliptic Cone, and as we increase it further, this equation takes the shape of a Hyperboloid of one sheet, So now we know that by taking K up to a point, the equation takes the shape of a cone, So now we will find out what that will be the equation has to pass through midpoint of both focal points to form a cone.

Mid-points are.

$$x=\frac{(x_0+x_1)}{2}$$
$$y=\frac{(y_0+y_1)}{2}$$
$$z=\frac{(z_0+z_1)}{2}$$

General form of equation is.

$$Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz+K=0$$

$$K-=Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz$$

Since the cone will pass through the midpoint of the two focus points, we can replace x, y, z with $\frac{(x_0+x_1)}{2}, \frac{(y_0+y_1)}{2}, \frac{(z_0+z_1)}{2}$.

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$$4\left(\frac{x_0+x_1}{2}\right)^2((x_0-x_1)^2-d_1^2)+$$
$$4\left(\frac{y_0+y_1}{2}\right)^2((y_0-y_1)^2-d_1^2)+$$
$$4\left(\frac{z_0+z_1}{2}\right)^2((z_0-z_1)^2-d_1^2)+$$
$$8*\left(\frac{x_0+x_1}{2}\right)*\left(\frac{y_0+y_1}{2}\right)(x_0-x_1)(y_0-y_1)+$$
$$8*\left(\frac{x_0+x_1}{2}\right)*\left(\frac{z_0+z_1}{2}\right)(x_0-x_1)(z_0-z_1)+$$
$$8*\left(\frac{y_0+y_1}{2}\right)*\left(\frac{z_0+z_1}{2}\right)(y_0-y_1)(z_0-z_1)+$$
$$4\left(\frac{x_0+x_1}{2}\right)((x_0-x_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+x_0d_1^2+x_1d_1^2)+$$
$$4\left(\frac{y_0+y_1}{2}\right)((y_0-y_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+y_0d_1^2+y_1d_1^2)+$$
$$4\left(\frac{z_0+z_1}{2}\right)((z_0-z_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+z_0d_1^2+z_1d_1^2)=-K$$

By expanding and simplifying the equation, K is.

$$K=-(x_1^2+z_1^2+y_1^2)(x_0^2-x_1^2+y_0^2+z_0^2-z_1^2-y_1^2+d_1^2)+$$
$$(x_0^2+y_0^2+z_0^2)(x_0^2-x_1^2+y_0^2+z_0^2-z_1^2-y_1^2-d_1^2)-$$
$$d_1^2(x_0x_1+y_0y_1+z_0z_1+x_0x_1+y_0y_1+z_0z_1)$$

$$4x^2((x_0-x_1)^2-d_1^2)+4y^2((y_0-y_1)^2-d_1^2)+4z^2((z_0-z_1)^2-d_1^2)+$$
$$8*x*y(x_0-x_1)(y_0-y_1)+8*x*z(x_0-x_1)(z_0-z_1)+8*y*z(y_0-y_1)(z_0-z_1)+$$
$$4x((x_0-x_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+x_0d_1^2+x_1d_1^2)+$$
$$4y((y_0-y_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+y_0d_1^2+y_1d_1^2)+$$
$$4z((z_0-z_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+z_0d_1^2+z_1d_1^2)+K=0$$

If we compare with the general equation and change from the K found in the equation, then we get an equation, it will always be a cone equation.

$$4x^2((x_0-x_1)^2-d_1^2)+4y^2((y_0-y_1)^2-d_1^2)+4z^2((z_0-z_1)^2-d_1^2)+$$
$$8*x*y(x_0-x_1)(y_0-y_1)+8*x*z(x_0-x_1)(z_0-z_1)+8*y*z(y_0-y_1)(z_0-z_1)+$$
$$4x((x_0-x_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+x_0d_1^2+x_1d_1^2)+$$
$$4y((y_0-y_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+y_0d_1^2+y_1d_1^2)+$$
$$4z((z_0-z_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+z_0d_1^2+z_1d_1^2)-$$
$$(x_1^2+z_1^2+y_1^2)(x_0^2-x_1^2+y_0^2+z_0^2-z_1^2-y_1^2+d_1^2)+$$
$$(x_0^2+y_0^2+z_0^2)(x_0^2-x_1^2+y_0^2+z_0^2-z_1^2-y_1^2-d_1^2)-$$
$$d_1^2(x_0x_1+y_0y_1+z_0z_1+x_0x_1+y_0y_1+z_0z_1)=0$$

Hyperboloid of one sheet

In equation if constant is greater than the equation do not generate Hyperboloid of one sheet but it do when we add some constant in equation.

Suppose $Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz+J=0$ is equation and we add extra value to J for example let L

$Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz+J+L=0$ like this.

As we increase L the equation starts to take the shape of an Elliptic Cone, and as we increase it further, this equation takes the shape of a Hyperboloid of one sheet.

When J+L is more than K equation will form Hyperboloid of one sheet.

$$K = -(x_1^2 + z_1^2 + y_1^2)(x_0^2 - x_1^2 + y_0^2 + z_0^2 - z_1^2 - y_1^2 + d_1^2) + (x_0^2 + y_0^2 + z_0^2)(x_0^2 - x_1^2 + y_0^2 + z_0^2 - z_1^2 - y_1^2 - d_1^2) - d_1^2(x_0x_1 + y_0y_1 + z_0z_1 + x_0x_1 + y_0y_1 + z_0z_1)$$

2. Conclusion

The intention of this paper was, to give an equation which can plot Conics and Quadric Surfaces of any shape and orientation using any two focal points and a distance.

First equation we made in this paper plots different type of Quadric Surfaces only using two point and distance. This equation Sphere, Ellipsoid, Hyperboloid of two sheet, Line in different cases.

Also plots Hyperboloid of one sheet need to add extra constant. Only takes $A(x_0, y_0, z_0)$, $A(x_1, y_1, z_1)$ and D Distance.

$$\begin{gathered}4x^2((x_0 - x_1)^2 - D^2) + 4y^2((y_0 - y_1)^2 - D^2) + 4z^2((z_0 - z_1)^2 - D^2) + \\ 8xy(x_0 - x_1)(y_0 - y_1) + 8xz(x_0 - x_1)(z_0 - z_1) + 8yz(y_0 - y_1)(z_0 - z_1) + \\ 4x((x_0 - x_1)(x_1^2 + y_1^2 + z_1^2 - x_0^2 - y_0^2 - z_0^2) + x_0D^2 + x_1D^2) + \\ 4y((y_0 - y_1)(x_1^2 + y_1^2 + z_1^2 - x_0^2 - y_0^2 - z_0^2) + y_0D^2 + y_1D^2) + \\ 4z((z_0 - z_1)(x_1^2 + y_1^2 + z_1^2 - x_0^2 - y_0^2 - z_0^2) + z_0D^2 + z_1D^2) + \\ (x_0^2 + y_0^2 + z_0^2)(x_0^2 + y_0^2 + z_0^2 - D^2 - x_1^2 - y_1^2 - z_1^2) - \\ D^2(x_0^2 + y_0^2 + z_0^2 - D^2 + x_1^2 + y_1^2 + z_1^2) - \\ (x_1^2 + y_1^2 + z_1^2)(x_0^2 + y_0^2 + z_0^2 + D^2 - x_1^2 - y_1^2 - z_1^2) = 0\end{gathered}$$

Second equation we made in this paper plots different type of conics only using two point and distance. This equation Circle, Ellipse, Hyperbola, Line in different cases. Only takes $A(x_0, y_0, z_0)$, $A(x_1, y_1, z_1)$ and D Distance.

$$\begin{gathered}4x^2((x_0 - x_1)^2 - D^2) + 4y^2((y_0 - y_1)^2 - D^2) + \\ 8 * x * y(x_0 - x_1)(y_0 - y_1) + 4x((x_0 - x_1)(x_1^2 + y_1^2 - x_0^2 - y_0^2) + x_0D^2 + x_1D^2) + \\ 4y((y_0 - y_1)(x_1^2 + y_1^2 - x_0^2 - y_0^2) + y_0D^2 + y_1D^2) + \\ (x_0^2 + y_0^2)(x_0^2 + y_0^2 - D^2 - x_1^2 - y_1^2) - D^2(x_0^2 + y_0^2 - D^2 + x_1^2 + y_1^2) \\ -(x_1^2 + y_1^2)(x_0^2 + y_0^2 + D^2 - x_1^2 - y_1^2) = 0\end{gathered}$$

Third equation we made in this paper plots line in three dimensional system.

Only takes $A(x_0, y_0, z_0)$ and $A(x_1, y_1, z_1)$.

$$\begin{gathered}x^2(-(y_0 - y_1)^2 - (z_0 - z_1)^2) + \\ y^2(-(x_0 - x_1)^2 - (z_0 - z_1)^2) + \\ z^2(-(x_0 - x_1)^2 - (y_0 - y_1)^2) + \\ 2 * x * y(x_0 - x_1)(y_0 - y_1) + 2 * x * z(x_0 - x_1)(z_0 - z_1) + 2 * y * z(y_0 - y_1)(z_0 - z_1) + \\ 2x(x_0(y_1(y_1 - y_0) + z_1(z_1 - z_0)) + x_1(y_0(y_0 - y_1) + z_0(z_0 - z_1))) + \\ 2y(y_1(x_0(x_0 - x_1) + z_0(z_0 - z_1)) + y_0(x_1(x_1 - x_0) + z_1(z_1 - z_0))) + \\ 2z(z_1(x_0(x_0 - x_1) + y_0(y_0 - y_1)) + z_0(x_1(x_1 - x_0) + y_1(y_1 - y_0))) + \\ x_0(2y_0x_1y_1 - x_0(y_1^2 + z_1^2)) + y_0(2z_0y_1z_1 - y_0(x_1^2 + z_1^2)) + z_0(2x_0x_1z_1 - z_0(x_1^2 + y_1^2)) = 0\end{gathered}$$

Forth equation we made in this paper plots Cylinder in three dimensional system.

Only takes $A(x_0, y_0, z_0)$, $A(x_1, y_1, z_1)$ and Radius r.

$$
\begin{gathered}
x^2(-(y_0-y_1)^2-(z_0-z_1)^2)+\\
y^2(-(x_0-x_1)^2-(z_0-z_1)^2)+\\
z^2(-(x_0-x_1)^2-(y_0-y_1)^2)+\\
2*x*y(x_0-x_1)(y_0-y_1)+2*x*z(x_0-x_1)(z_0-z_1)+2*y*z(y_0-y_1)(z_0-z_1)+\\
2x(x_0(y_1(y_1-y_0)+z_1(z_1-z_0))+x_1(y_0(y_0-y_1)+z_0(z_0-z_1)))+\\
2y(y_1(x_0(x_0-x_1)+z_0(z_0-z_1))+y_0(x_1(x_1-x_0)+z_1(z_1-z_0)))+\\
2z(z_1(x_0(x_0-x_1)+y_0(y_0-y_1))+z_0(x_1(x_1-x_0)+y_1(y_1-y_0)))+\\
x_0(2y_0x_1y_1-x_0(y_1^2+z_1^2))+y_0(2z_0y_1z_1-y_0(x_1^2+z_1^2))+\\
z_0(2x_0x_1z_1-z_0(x_1^2+y_1^2))+((x_0-x_1)^2+(y_0-y_1)^2+(z_0-z_1)^2)r^2=0
\end{gathered}
$$

Fifth equation we made in this paper plots Elliptic Cone in three dimensional system.

Only takes $A(x_0,y_0,z_0)$, $A(x_1,y_1,z_1)$ and d_1.

$$
\begin{gathered}
4x^2((x_0-x_1)^2-d_1^2)+4y^2((y_0-y_1)^2-d_1^2)+4z^2((z_0-z_1)^2-d_1^2)+\\
8*x*y(x_0-x_1)(y_0-y_1)+8*x*z(x_0-x_1)(z_0-z_1)+8*y*z(y_0-y_1)(z_0-z_1)+\\
4x((x_0-x_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+x_0d_1^2+x_1d_1^2)+\\
4y((y_0-y_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+y_0d_1^2+y_1d_1^2)+\\
4z((z_0-z_1)(x_1^2+y_1^2+z_1^2-x_0^2-y_0^2-z_0^2)+z_0d_1^2+z_1d_1^2)-\\
(x_1^2+z_1^2+y_1^2)(x_0^2-x_1^2+y_0^2+z_0^2-z_1^2-y_1^2+d_1^2)+\\
(x_0^2+y_0^2+z_0^2)(x_0^2-x_1^2+y_0^2+z_0^2-z_1^2-y_1^2-d_1^2)-\\
d_1^2(x_0x_1+y_0y_1+z_0z_1+x_0x_1+y_0y_1+z_0z_1)=0
\end{gathered}
$$