

Design of Experiments a Technique that Compares Quality in Terms of Variation - A Case Study of Process Capability Improvement of a Manufacturing Process

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Abstract: *This case study is based on the improvement of the quality of the adjustment screws by improving the process capability and process capability index of the boring operation using DOE. For the better quality, C_{pk} of the process should be a minimum of 1.33. The objective of this case study is to deal with the measurement of variation in the manufacturing process and introduce C_p and C_{pk} as parameters that compare the quality and measure the variation in the process. Also, it identifies the important variables whether they be process parameters, materials, products, and further, separates these variables, and reduces the variation of the important variables using differential diagnosis technique. This approach helps to study and analyze the effect of variables and the effect of their interaction. Also, this approach tightens the effect of interaction through process improvement using Design of Experiments (DOE).*

Keywords: Pre-control Theory, Process Capability, Design of Experiment, Differential Diagnosis, Variance, Statistical Process Control

1. Introduction

This case study was performed in an industry that manufactures pipes, valves, flanges, and fittings located in Mumbai, India. The objective of the Quality at the Source is to improve the quality and reduce the defects by analysing and improving the capability of the process. The capability of the process can be analyzed with the help of statistical tools that developed from the normal distribution curve and the control charts. Every defect may have some causes and causes may have the form of a random cause or an assignable cause. These causes can be judged by observing a pattern of data points on the control charts. Industries use either the pre-control charts or statistical process control charts to study the process capability of various processes. A pre-control chart is the easiest, more cost-effective, and time-saver tool to study the capability of the process and identify the cause of the defect. The Process Capability is the statistical tool that measures process spread with respect to the specification width provided by the customer. The Process Capability Index is the tool used to identify the centering of the sample mean with respect to the target mean provided by the customer. The process capability indices are helpful management tools, especially in the manufacturing process, which give regular quantitative measures of manufacturing capability and production quality. With the fast progression of the manufacturing innovation, suppliers require their items to be of high caliber with an exceptionally low portion of defectives generally estimated in parts per million. Nowadays most supplier certification manuals include a view of process capability analysis and describe the recommended procedure for computing a process capability index. Hence, determining the process capability and process capability index became part of Supplier Quality Requirements (SQR). This case study dealt with the measurement of variation in the process by introducing C_p and C_{pk} and study the variables causing the

variation in the process. Identify the important variables that are causing the variation in the process and hence, resulting in a defect and diagnose these causes to reduce the variation on the important variables using differential diagnosis principle. The effect of these variables and the effect of their interaction is analyzed using the Design of Experiment Technique (DOE).

2. Methodology

- Understand the concept of pre-control theory
- Understand the concept of process capability indices
- Collection of process data
- Construction of a pre-control chart
- Normality test to ensure the data is normal
- Estimation of C_p & C_{pk}
- Validate the values C_p & C_{pk} of the process with manufacturing quality standard
- Apply differential diagnosis principle to find the root cause of variation in the process
- Study & analyse the effect of various factors responsible for variation
- Adjust the process mean by improving the process
- Collection of process data after process improvement
- Construction of a pre-control chart after process improvement
- Normality test to ensure the data is normal after improving the process
- Estimation of C_p & C_{pk} after process improvement
- Validate the values C_p & C_{pk} of the process with manufacturing quality standards.

3. Pre-control Theory

Pre-control is also known as the stoplight control. Pre-control was developed to monitor the proportion of non-

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confirming units or defects produced in the manufacturing process. The pre-control chart is divided into three groups: Green, yellow, and red; where the colors loosely correspond to good, questionable, and poor quality products. The number of green, yellow, and red units observed determines whether to start the production or stop the production and when to stop and adjust the process. The objective of pre-control is to identify when the quantity of non-confirming units or defects produced is very large. Thus, the pre-control theory helps to identify and monitor the process capability of the process and provides a sampling interval to ensure that the process capability index remains large. Construction of Pre-control chart: To construct the pre-control chart, the first step is to divide the specification width by 4. The boundaries of the middle half of the specification limit are known as the Pre-control line (P-C). The area between these pre-control lines is called the green zone. The two areas between the pre-control line and the Specification line are called yellow zones. The two areas beyond the specification lines are known as red zones. When the specification width is equal to the process spread assuming a normal distribution curve, 86 % of the curve falls into the green zone and 7 % of the curve falls into the lower yellow zone, and the remaining 7 % of the curve falls in the upper yellow zone. By using the multiplication theory, the probability of falling a pair of units into the yellow-red zone is 0.49%, and there is only a 2% chance of falling the units outside the pre-control lines. But, when the process width is equal to the specification width a slight drift in the process mean will result in process variation and hence increase in the non-conforming units. Pre-control Theory gives the quickest way to determine the process capability of the process. To determine the process capability, take a sample of five consecutive units from the process. If all five units are found within the green zone, the process is in control. In fact, within this simple rule, the usual samples of fifty to one hundred units to calculate C_p and C_{pk} are not necessary. By applying the multiplication theorem of probabilities, it can be proven that a $\text{Min}C_{pk}$ of 1.33 will automatically result.

4. Process Capability (C_p) and Process Capability index (C_{pk})

4.1. Process Capability

Process Capability is the ratio of specification width provided by the customer to the actual spread of the process determined by the width of the normal distribution curve denoted as "6 σ ". It compares and relates the process width with the specification range. The Process capability is a measure of spread [1].

$$C_p = \frac{\text{Specification Width (S)}}{\text{Process width (P)}} \quad (1)$$

4.2. Process Capability based on lower specification limit (C_{pl})

C_{pl} measures how close the process mean is running to the lower specification limit. It estimates the process capability for specifications that consist of a lower limit only.

4.3. Process Capability based on upper specification limit (C_{pu})

C_{pu} measures how close the process mean is running to the upper specification limit. It estimates the process capability for specifications that consist of an upper limit only.

4.4. Process Capability Index

S = Specification Width
 P = Process Width
 D = Design Center
 \bar{X} = Process Average
 k = Correction factor

$$C_p = \frac{S}{P}$$

$$K = \frac{\bar{X}-D}{S/2} \quad \text{OR} \quad \frac{D-\bar{X}}{S/2} \quad (2)$$

$$C_{pk} = (1 - K)C_p \quad (3)$$

Bhote&Bhote [1] states that the process capability C_p only deals with the ratio of the specification width and the process width, process capability does not take non-centering of the process mean into account. Whereas, the process capability index C_{pk} is the parameter which considers the non-centering of the process mean with respect to specification limit. This non-centering of the process mean involves variation and thus, reduces the margin of safety. In order to interpret this non-centering a correction factor introduced as "K". When the process average \bar{X} coincides with the design center D at the target value, then K reduced to zero and the magnitude of C_p and C_{pk} becomes equal. When the process average \bar{X} skews towards one end of the specification limit away from the design center D then, the magnitude of K increases, and the value of C_{pk} reduces as compared to the value of C_p . Process spread and non-centering of process mean are the important parameters that affect the quality of the process and also causes variation. In the process improvement, non-centering of the process mean is easy to drift but, reducing the process spread is a difficult task. C_{pk} is an excellent tool to measure the variability and process capability because it considers both process spread and non-centering of process width. Achieving $C_{pk} = 2$ is nothing but achieving the zero defect and zero variation in the process.

5. Problem Analysis

5.1 Objective

To analyse the process capability of the boring operation and determine the defect rate. Verify whether the process capability index of the process is 1.33 or more than 1.33 if not, improve the process by using DOE Technique.

5.2 Product Specifications

The diameter of the adjustment screw is the critical quality parameter to qualify the customer's requirement. A 3.6 mm tool is used for the boring operation as per the SOP.

Detailed specifications are shown in table 1 along with the product drawing shown in fig 1.

5.3 Data collection in sub-groups of 5 units

For refined accuracy of estimated C_p and C_{pk} of the process, at least 50 samples are necessary. The data is collected in of sample size of 100 units in the sub-group of 5 units.

5.4 Construction of a Pre-control Chart

Data of 100 adjustment screws are recorded as shown in the table 2 and plotted on the pre-control chart in the form of 20 observations of subgroup size 5 as shown in the fig 2. Data point of each sample is the average of 5 readings. All the 20 data points are observed in the yellow zone within Specification limits.

5.5 Normal probability plot and Histogram to validate the data is Normal

To check whether the recorded data in this case is normal, a graphical method of normal probability plot and the histogram is used. This data is analyzed in MINITAB-19. The data appears to be normal in the histogram diagram shown in fig 4. The normal probability plot yields Anderson Darling test static value of 0.710 and p value of 0.062 which is greater than the significance level $\alpha=0.05$ as shown in the fig 3 hence, we can conclude that the data is normal and the curve is normally distributed.

Table 1: Product Specifications

Product Name: Adjustment Screw	Specifications: 4.05 ± 0.05
Material: Steel	Instrument: Boring Dial Gauge
Operation: Boring	All dimensions are in "mm"

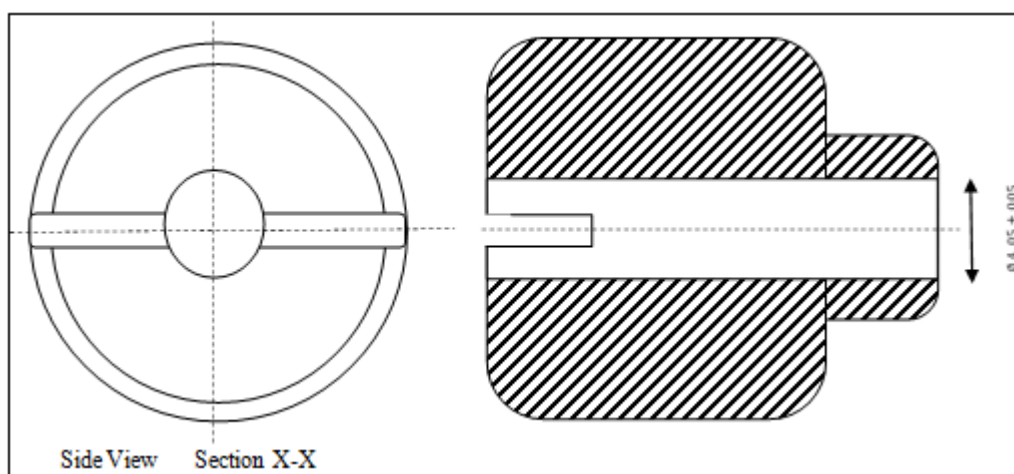


Figure 1: Product Drawing

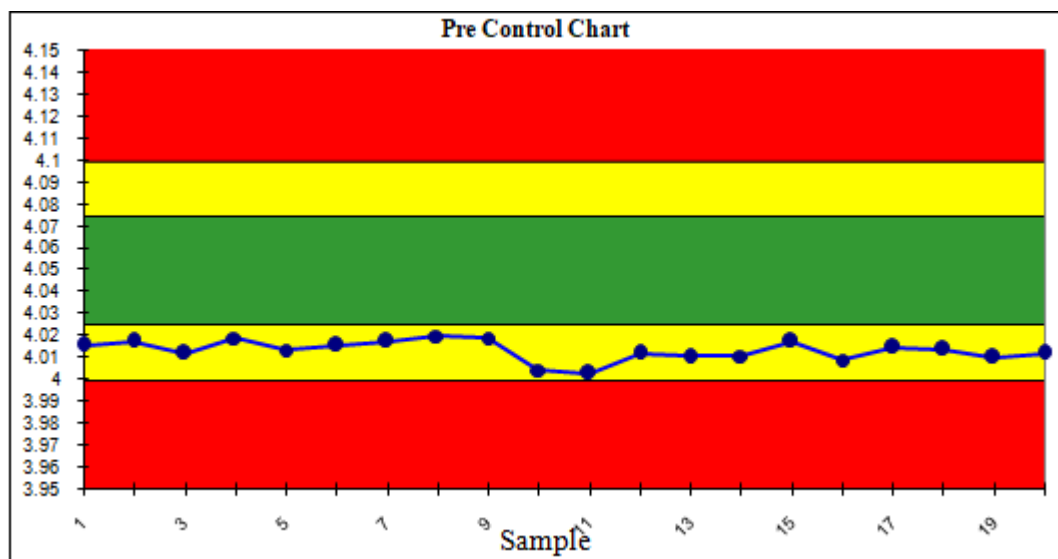


Figure 2: Construction of Pre-control Chart before process improvement

Table 2: Data of 100 samples of Sub-group 5

Sample	1	2	3	4	5	\bar{X}	R
1	4.021	4.03	4.027	4.002	3.998	4.0156	0.032
2	4.022	4.026	4.006	4.029	4.005	4.0176	0.024
3	4.023	4.008	3.999	4.025	4.004	4.0118	0.026
4	4.031	4.028	4.02	4.007	4.009	4.019	0.024
5	4.01	4.012	4.013	4.029	4.003	4.0134	0.026
6	4.011	4.014	4.01	4.023	4.02	4.0156	0.013

7	3.995	4.028	4.026	4.018	4.021	4.0176	0.033
8	4.022	4.027	4.019	4.016	4.015	4.0198	0.012
9	4.024	4.035	4.02	4.003	4.013	4.019	0.032
10	3.975	4.011	4.016	4.006	4.014	4.0044	0.041
11	3.965	4.005	4.004	4.007	4.034	4.003	0.069
12	4.039	4.015	4.012	4.009	3.986	4.0122	0.053
13	4.018	3.992	4.012	4.013	4.017	4.0104	0.026
14	4.021	4.026	4.006	4.002	3.999	4.0108	0.027
15	4.022	4.028	4.001	4.018	4.019	4.0176	0.027
16	4.016	4.011	3.998	4.004	4.015	4.0088	0.018
17	4.027	4.03	4.013	3.994	4.01	4.0148	0.036
18	4.029	4.005	4.003	4.02	4.012	4.0138	0.026
19	4.003	4.007	4.009	4.008	4.024	4.012	0.021
20	3.982	4.014	4.012	4.025	4.026	4.0118	0.044

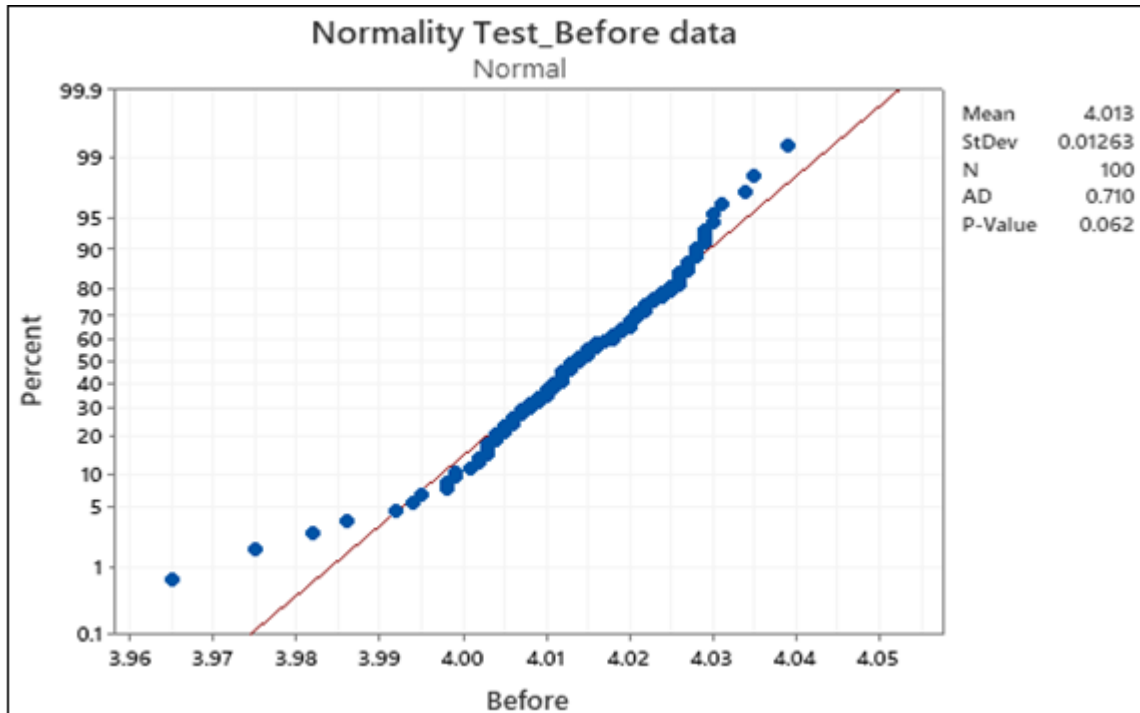


Figure 3: Normality Test for the recorded data

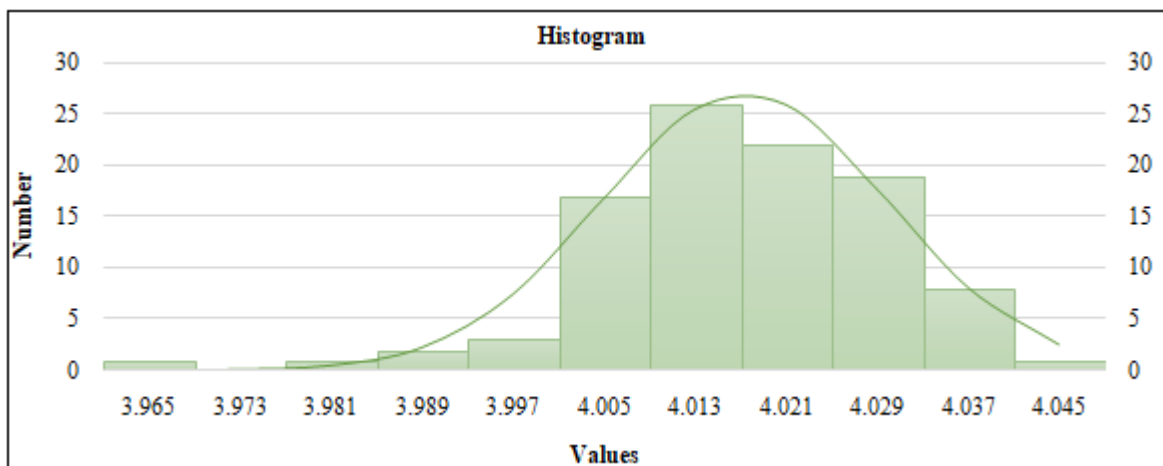


Figure 4: Histogram showing normally distributed curve for recorded data

5.6 Estimation of Sample Average and Standard Deviation

Sample Mean:

$$\bar{X} = \frac{80.269}{20}$$

From the collected data, we can calculate the sample mean and standard deviation as follows,

$$\bar{X} = 4.01336$$

Standard Deviation:

We have,

X_i = Observed value of sample item

N = Number of Observation

\bar{X} = Sample Mean

$\hat{\sigma}$ = Standard Deviation for Sample

To calculate the process standard deviation we have,

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

Where,

\bar{R} = Average of range

$d_2 = 2.236$ from statistical control chart

$$\hat{\sigma} = \frac{0.0305}{2.236} = 0.01311$$

So now we have data;

Data	Value
Sample Mean (\bar{X})	4.01336
Standard Deviation ($\hat{\sigma}$)	0.01311
Upper Specification Limit (USL)	4.10
Lower Specification Limit (LSL)	4.00

5.7 Estimation of Process Capability:

$$C_p = \frac{(USL - LSL)}{6 \times \hat{\sigma}}$$

$$C_p = \frac{(4.10 - 4.00)}{6 \times 0.01311}$$

$$C_p = 1.27$$

5.8 Estimation of Capability ratio

Percentage of specification band used by the manufacturing process is denoted by C_r .

$$C_r = \frac{6 \times \hat{\sigma}}{(HSL - USL)} \times 100$$

$$C_r = \frac{6 \times 0.01311}{(4.10 - 4.00)} \times 100$$

$$C_r = 79 \%$$

This means that the manufacturing process uses 79 % of specification band.

5.9 Estimation of Process Capability Index

$$C_{pk} = \text{Min}(C_{pu}, C_{pl})$$

$$C_{pk} = \text{Min}\left(\frac{(USL - \bar{X})}{3\sigma}, \frac{(\bar{X} - LSL)}{3\sigma}\right)$$

$$\text{Min}\left(\frac{(4.10 - 4.05126)}{3 \times 0.01311}, \frac{(4.05126 - 4.00)}{3 \times 0.01311}\right)$$

$$C_{pk} = \text{Min}(2.02, 0.34)$$

$$C_{pk} = 0.34$$

Process Capability Analysis

The recorded data further analyzed in MINITAB-19 Statistical software. Process capability analysis is shown in the fig 5, provides the process capability C_p of 1.30 and process capability index C_{pk} of 0.35. The process capability analysis shows, the process is within the specification limits. There would be rejections of 148100 PPM adjustment screws due to falling outside the lower specification limit.

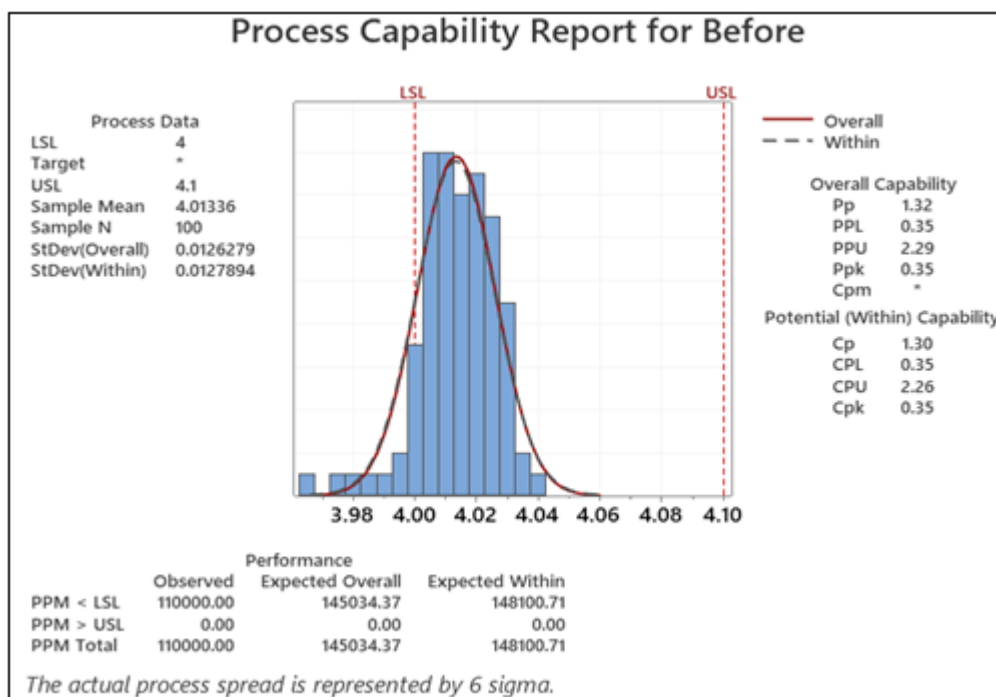


Figure 5: Process Capability analysis in MINITAB before process improvement

6. Root Cause Analysis using Differential Diagnosis Principle

The Differential Diagnosis is an approach which helps in solving the problem by differentiating the two processes which have similar characteristics provided that one of the processes is totally fine with its working and other process is having some problem. If there is a problem with the process, a typical approach is by finding the root cause after studying the same process & its characteristics or by studying the past data of the process. Differential diagnosis is an approach which compares the characteristics of the problematic process with the characteristics of good process. This approach helps to solve the problem in the shortest period of time. This systematic approach of Differential Diagnosis is structured in the fig 6;

6.1 Problem Specification

Diameter of the bore is the critical quality parameter. The bore diameter recorded and found in the lower specification zone.

6.2 Data collection

The data is collected in a sample size of 100 of subgroup of 5 units as shown in table 2.

6.3 Define the problem

The process mean falls in the lower specification limit that resulted in to the process capability index of 0.34.

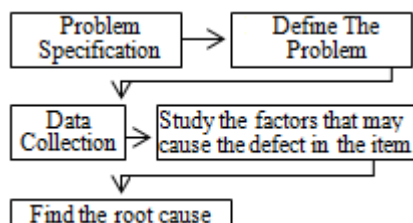


Figure 6: Differential Diagnosis Approach

6.4 Study the factors that may cause the defect in the item

In the boring operation Tool diameter, Point angle, Material, Feed inches per revolution and Speed (SFM) are the factors which may influence the bore diameter of the adjustment screw. All these factors excluding point angle found to be similar on both the CNC machines. So, the only differentiating variable was the point angle having magnitude 90° on CNC machine 1 and 125° on CNC machine 3 as shown in table3.

Table 3: Factor's Comparison between M/C 1 & M/C 3

Parameters	Machine 1	Machine 3
Tool Diameter	3.96	3.96
Point Angle	90	126
Material	Steel	Steel
Feed inches per revolution	0.004	0.004
Speed (SFM)	60	60

6.5 Find the root cause by comparing Bad (Defective sample) with Good (Quality Assured Item):

Differential diagnosis suggests an approach to reach the root cause of the problem in a shortest period of time. The root cause analysis is shown in the table no 4. Here, the point angle and the tool diameter are the only parameters may change manually as the other parameters like speed and feed are as per the SOP and programmed same on both the machines 1 & 3. Based on the above assumption made, designed an experiment to study the effect of point angle and tool diameter. DOE of 2 factors 2 levels and 3 replicates is constructed.

Table 4: Root Cause Analysis

0	What is the item with the defect?	Adjustment Screw	
	What is the defect?	The diameter of a hole is a critical parameter of an adjustment screw and is out of the lower specification limit.	
	Data Pertaining to "WHAT"	PRESENT	ABSENT
1.1	What symptoms in the defective item are?	Diameter is out of lower specification limit	None
	Data Pertaining to "WHERE"	PRESENT	ABSENT
2.1	On which part of the item is the defect	The lower bottom part of an adjustment screw	Upper body part, threaded portion of the adjustment screw
2.2	On which variety of the item is the defect	Variety of items made on M/C 1	Variety of items made on M/C 3
2.3	After which operation is the defect	Facing Operation	None
2.4	On which station or machine is the defect	CNC Machine -1	CNC Machine- 3
2.5	Parameters which are same on M/C 1 & M/C 2	Material, tool diameter, feed inches/revolution and speed in SFM	Point Angle

7. Measuring the effect of variables using Design of Experiment

The main purpose of Design of Experiment is to improve the quality of the product or process. A reduced variation in the process results into reduced scraps, less rejections of

product, and fewer warranty returns, consequently reducing costs and improving customer satisfaction. Design of Experiment is a statistical technique used to study the effects of multiple variables simultaneously. Improving quality will require that we reduce variations around the target by achieving consistency of performance. To achieve

consistency will require that both the distance of the population mean from target and the standard deviation of the population be reduced. Design of Experiment is a technique that can be applied to affect these performance characteristics. A four run, two-level experiment was conducted based on 2 factors.

Alias Structure

Aliases
A
B
AB

Factor Name
A Point Angle
B Tool Diameter

Table 5: Response Table

Level	Point Angle	Tool Diameter
1	90	3.96
2	126	4.00

Standard Order	Factors		Response
	A	B	(y)
1	1	1	y ₁
2	1	2	y ₂
3	2	1	y ₃
4	2	2	y ₄

The levels selected for the factors are shown below. Based on the levels selected for the factors, a coded design of experiment is shown in the table no 5. With the same factor settings of levels, three experimental runs have been conducted known as replicates. R₁, R₂&R₃ are the replicates in the design of experiment shown in the table no 5. Multiple replicates increase the precision of the results. In the performed design of experiment, the predetermined levels of the factors were set and started the production. After that one quality measurement was recorded. After taking one quality measurement, reset the equipment, and continued production. After that second quality measurement was recorded. In random order, the operators run each combination of factor settings three times, taking one measurement at each run. y₁, y₂, y₃, y₄ are the average of 3 responses recorded as shown in the table no 6.

Table 6: Response Record

Standard Order	Point Angle	Tool Diameter	R1	R2	R3	Sum
1	90	3.96	4.014	4.019	4.015	12.048
2	90	4	4.354	4.332	4.353	13.039
3	126	3.96	4.055	4.062	4.058	12.175
4	126	4	4.404	4.41	4.405	13.219

Table 7: Design of Experiment

Random Order Trial No	Standard Order Trial No	Response Observed Values (Y)	A		B		AB	
			Point Angle		Tool Diameter			
			1 (90)	2 (126)	1 (3.96)	2 (4.0)	1	2
4	1	12.048	12.048		12.048			12.048
3	2	13.039	13.039			13.039		13.039
1	3	12.175		12.175	12.175			12.175
2	4	13.219		13.219		13.219		13.219
Total		50.481	25.087	25.394	24.223	26.258		25.214
Number of values		12	6	6	6	6		6
Average		4.207	4.181	4.233	4.037	4.376		4.202
Effect			0.052		0.34			0.009

The experiment was conducted with a random order trial sequence of 4, 3, 1 & 2. This experiment was performed with two factors A & B with two levels, low and high. Factor A represents Point Angle, whereas factor B represents Tool Diameter. Factor AB represents interaction of Point angle and Tool Diameter. Point angle have low level of 90° and a high level of 126°. Tool diameter have a low level of 3.96 mm and high level of 4 mm. The combination of factors A & B designed in an experiment yields the result shown in the response observed values.

- Point angle has less effect as compared to tool diameter.
- Point Angle and Tool Diameter has very low interaction effect on the diameter of the hole.
- Graphical display estimates that the effect of tool diameter is more as compared to other factors.
- Ultimate goal of this experiment is to study the effect of point angle and tool diameter and their interaction, choose the one factor that has a moderate effect on the diameter of the hole.

- To shift the process mean of the diameter of the adjustment screw by controlling the factor i.e. by controlling the point angle.
- Objective is to shift the process mean towards the target of 4.05 mm in order to increase the process capability index.
- Shifting of the process mean towards standard mean that will improve the process capability index and hence, the process capability.
- Using the graphical display, calculated the maximum and minimum average response values. The target diameter of 4.05 mm falls in minimize average response region. Considering only point angle has moderate effect, kept the point angle at high and tool diameter at low. Keeping the point angle high and tool diameter low, an average response will be around 4.059 mm.

$$y_{max} = \bar{y} + (B_1 - \bar{y}) + (A_1 - \bar{y})$$

$$y_{max} = 4.207 + (4.377 - 4.207) + (4.231 - 4.207)$$

$$y_{max} = 4.207 + 0.024 + 0.17$$

$$y_{max} = 4.401$$

$$y_{min} = \bar{y} + (B_2 - \bar{y}) + (A_2 - \bar{y})$$

$$y_{min} = 4.207 + (4.037 - 4.207) + (4.181 - 4.207)$$

$$y_{min} = 4.207 - 0.024 - 0.17$$

$$y_{min} = 4.013$$

- Regression Equation in Uncoded Units
 Result = $-9.9 - 0.1706$ (Point Angle)
 $+ 3.45$ (Tool Diameter)
 $+ 0.0437$ (Point Angle)
 $*$ (Tool Diameter)

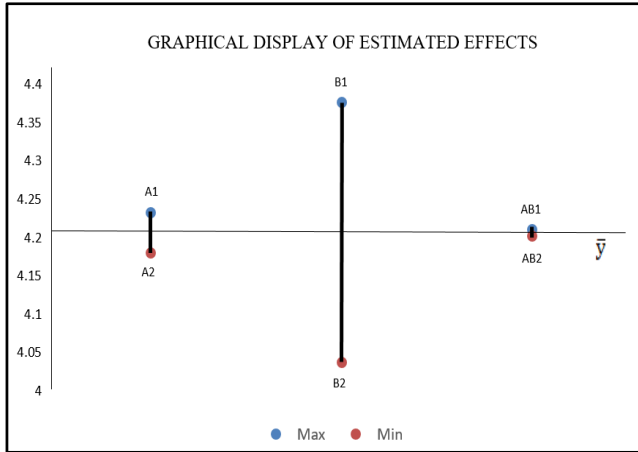


Figure 7: Estimated effect of factors using DOE

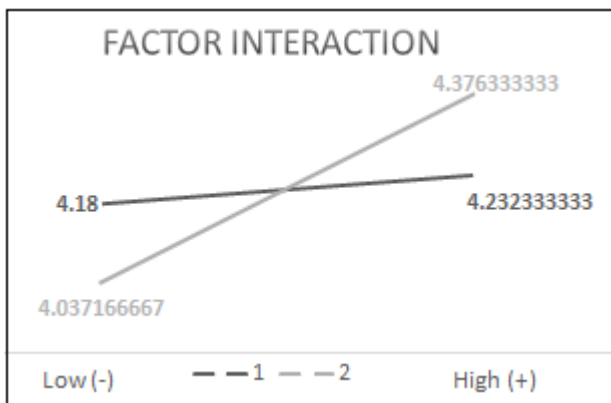


Figure 8: Effect of factors and their interaction

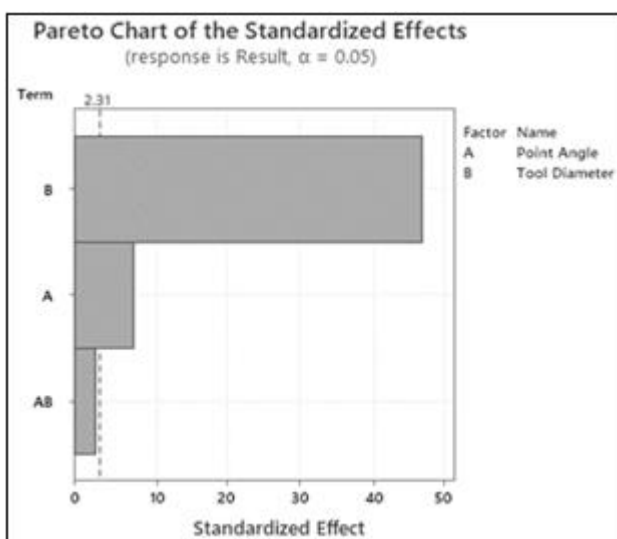


Figure 9: Pareto Chart for standardized effect

Table 8: Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value
Model	3	0.365805	0.121935	789.22	0
Linear	2	0.365217	0.182608	1181.93	0
Point angle	1	0.00952	0.00952	61.62	0
Tool Diameter	1	0.355696	0.355696	2302.24	0
2-way Interactions	1	0.000588	0.0000588	3.81	0.087
Point angle*Tool diameter	1	0.000588	0.0000588	3.81	0.087
Error	8	0.001236	0.000154		
Total	11	0.367041			

Table 10: Analysis of Variation

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		4.20933	0.00359	1173.11	0	
Point Angle	0.05633	0.02817	0.00359	7.85	0	1
Tool Diameter	0.34433	0.17217	0.00359	47.98	0	1
Point Angle*Tool Diameter	0.014	0.001	0.00359	1.95	0	1

8. Evaluation of Process Capability after improving the process

The tool angle is the reason why the diameter of the bore was in the lower specification limit. The data has been collected by adjusting the tool angle from 90° to 126°. The mean diameter found to be 4.051 mm which is in the green zone within the control limits of the pre-control chart and hence process considered to be in control.

8.1 Data collection in sub-groups of 5 units

Data collected in the form of 100 samples in the sub-group of 5 units.

8.2 Construction of a Pre-control Chart

Data of 100 adjustment screws are recorded as shown in the table 10 and plotted on the pre-control chart in the form of 20 observations of subgroup size 5 as shown in the fig 10. Data point of each sample is the average of 5 readings. All the 20 data points are observed in the Green Zone within control limits.

8.3 Normal probability plot and Histogram to validate the data is Normal:

To check whether the recorded data in this case is normal, a graphical method of normal probability plot and the histogram is used. This data is analyzed in MINITAB-19 as shown in figure 11. The data appears to be normal in the histogram diagram as shown in the fig 12. The normal probability plot yields Anderson Darling test Static value 0.571 and p value of 0.135 which is greater than the significance level $\alpha = 0.05$ hence, we can conclude that the data is normal and the curve is normally distributed.

Table 10: Data recorded of 100 units in sub-group of 5

Sample No	1	2	3	4	5	\bar{X}	R
1	4.039	4.06	4.041	4.047	4.053	4.048	0.021
2	4.045	4.04	4.052	4.057	4.055	4.0498	0.017
3	4.05	4.046	4.049	4.043	4.051	4.0478	0.008
4	4.042	4.044	4.048	4.058	4.056	4.0496	0.016
5	4.046	4.041	4.05	4.055	4.058	4.05	0.017
6	4.048	4.045	4.054	4.04	4.039	4.0452	0.015
7	4.042	4.047	4.053	4.052	4.071	4.053	0.029
8	4.043	4.049	4.05	4.078	4.03	4.05	0.048
9	4.051	4.072	4.079	4.073	4.07	4.069	0.028
10	4.056	4.044	4.078	4.059	4.06	4.0594	0.034
11	4.057	4.043	4.047	4.069	4.057	4.0546	0.026
12	4.06	4.048	4.046	4.029	4.063	4.0492	0.034
13	4.068	4.056	4.065	4.049	4.055	4.0586	0.019
14	4.058	4.054	4.052	4.059	4.05	4.0546	0.009
15	4.039	4.066	4.042	4.05	4.029	4.0452	0.037
16	4.061	4.053	4.051	4.045	4.044	4.0508	0.017
17	4.06	4.054	4.055	4.057	4.025	4.0502	0.035
18	4.05	4.053	4.022	4.056	4.058	4.0478	0.036
19	4.034	4.052	4.062	4.051	4.059	4.0516	0.028
0	4.032	4.045	4.033	4.049	4.046	4.041	0.017

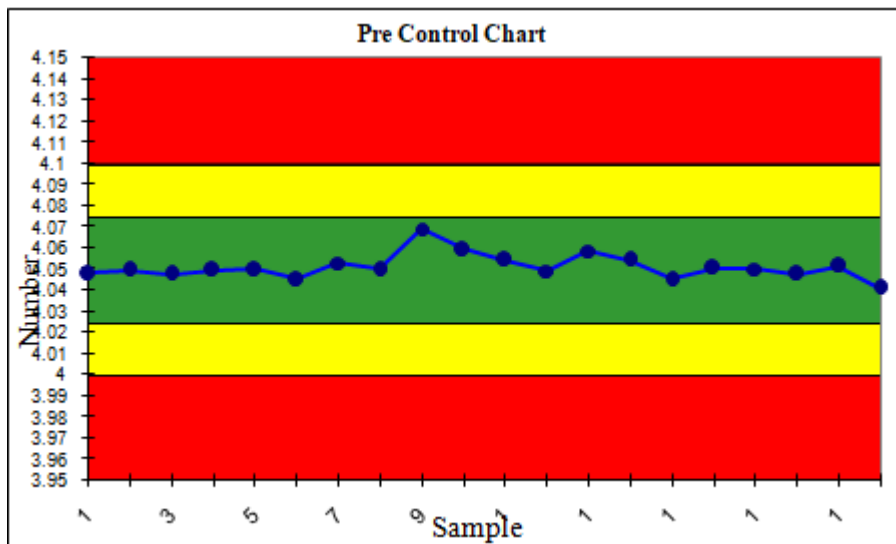


Figure 10: Construction of Pre-control chart after improving the process

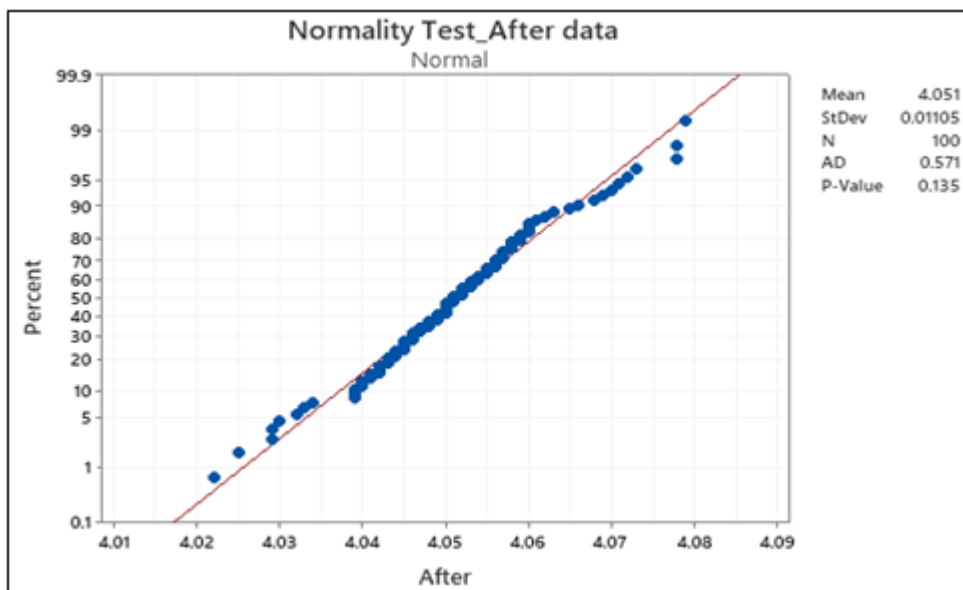


Figure 11: Normality Test Result for the recorded data after process improvement

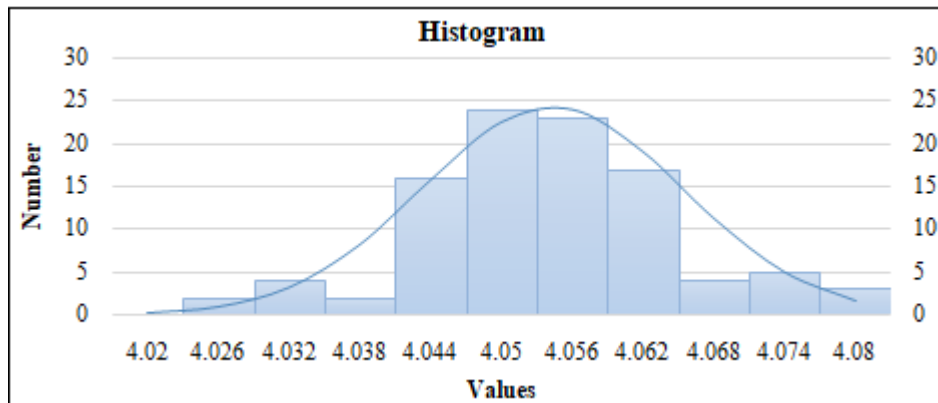


Figure 12: Histogram for the recorded data after process improvement

8.4 Estimation of Sample Average and Standard Deviation

From the collected data, calculated the sample mean and standard deviation as follows,

Sample Mean:

$$\bar{X} = \frac{81.0252}{20}$$

$$\bar{X} = 4.05127$$

Standard Deviation:

We have,

X_i = Observed value of sample item

N = Number of Observation

\bar{X} = Sample Mean

$\hat{\sigma}$ = Standard Deviation for Sample

To calculate the process standard deviation we have,

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

Where,

\bar{R} = Average of range

d_2 = 2.236 from statistical control chart

$$\hat{\sigma} = \frac{0.02455}{2.236} = 0.0105$$

So now we have data;

Data	Value
Sample Mean (\bar{X})	4.05127
Standard Deviation ($\hat{\sigma}$)	0.0105
Upper Specification Limit (USL)	4.10
Lower Specification Limit (LSL)	4.00

8.5 Estimation of Process Capability:

Let, the average of average of observed values of the samples is \bar{X} Then, Process Capability C_p will be,

$$C_p = \frac{(USL - LSL)}{6 \times \hat{\sigma}}$$

$$C_p = \frac{(4.10 - 4.00)}{6 \times 0.0105}$$

$$C_p = 1.58$$

8.6 Estimation of Capability ratio:

Percentage of specification band used by the manufacturing process is denoted by C_r .

$$C_r = \frac{6 \times \hat{\sigma}}{(HSL - USL)} \times 100$$

$$C_r = \frac{6 \times 0.0105}{(4.10 - 4.00)} \times 100$$

$$C_r = 63 \%$$

This means that the manufacturing process uses 63 % of specification band.

8.7 Estimation of Process Capability Index

$$C_{pk} = \text{Min}(C_{pu}, C_{pl})$$

$$C_{pk} = \text{Min}\left(\frac{(USL - \bar{X})}{3\sigma}, \frac{(\bar{X} - LSL)}{3\sigma}\right)$$

$$= \text{Min}\left(\frac{(4.10 - 4.05126)}{3 \times 0.0105}, \frac{(4.05126 - 4.00)}{3 \times 0.0105}\right)$$

$$C_{pk} = \text{Min}(1.54, 1.62)$$

$$C_{pk} = 1.54$$

8.8 Process Capability and Process Capability Index

The recorded data further analyzed in MINITAB-19 statistical software as shown in the figure no 13. The process capability analysis shows, the process is in control and stable to meet the specification limits. There would be rejections of 2 adjustment screws due to falling outside the lower specification limit and 6 adjustment screws will get scraped due to diameter is over the upper specification limit.

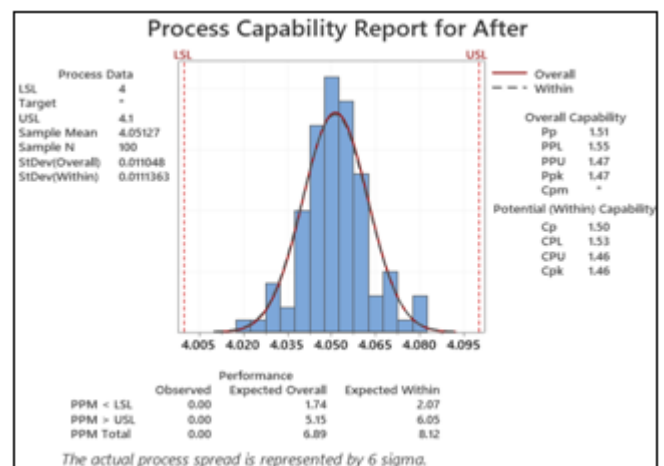


Figure 13: Process Capability Analysis in MINITAB after improvement of process

9. Results and Discussion

The process is in control if the process capability index of the process is 1.33 or greater than 1.33. In this case, the process capability index was 0.34 which was far less than the desired process capability index. After improvement in the process, process yields the process capability index of 1.54 and process capability of 1.58. That automatically helped to reduce the defect rate as shown in the table 11.

Table 11: Results

Parameter	Before Process Improvement	After Process Improvement
Process Capability	1.27	1.58
Process Capability Index	0.34	1.54
Defect Rate	148100	9

10. Conclusion

This case study drives a conclusion of an improvement in the quality of the product as well as the process that made think about the unwavering contribution of the Differential Diagnosis and the Design of Experiments in finding the factors responsible for the variation in the process and the effect of those factors and their interaction on the process capability. The process is assumed to be in control if the process capability index of the process is 1.33 or more than 1.33. The process capability and the process capability index quantifies the variation in the process. In this case Differential Diagnosis principle used to identify the variables and to find the root cause of variation in the process whereas Design of Experiment technique helped to quantify the effect of those variables on the process stability. After eliminating the variation of the process, the process capability improved from 1.27 to 1.58 and the process capability index improved from 0.34 to 1.54. The overall elimination of variation and improvement in the process capability, helped to reduce the defect rate from 148100 PPM to 9 PPM.

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