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Simulation of Power Spectrum Estimation based on AR Model

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Abstract: ***This article uses the principle of parameter model spectrum estimation, introduces the Burg algorithm in AR spectrum estimation, and gives the parameter estimation process of the Burg algorithm. Aiming at the procedural samples of the AR model, the Burg algorithm is used to estimate the power spectrum, and it is compared with the period map estimation method. The simulation results show that the power spectrum estimated by the Burg algorithm is close to the real power spectrum and is better than the result of the periodogram estimation method.***

Keywords: power spectrum estimation; AR model; AR spectrum estimation; Burg algorithm; periodogram spectrum estimation

1. Introduction

The problem of spectrum estimation is to determine the content of the spectrum of a random process based on a set of finite observations. Strictly speaking, the power spectral density (PSD) of the complex generalized stationary random process is determined by the autocorrelation function (ACF) between the observations. The PSD function describes the distribution of the power of the random process with frequency. In general, we can use a band-pass filter to filter the random process, and then measure the output power of the filter to determine its distribution. But this method must assume that the observed random process has sufficient duration so that the transient process of the filter can disappear. For many practical problems, the observed values we get are very limited, and it is impossible to get a set of effective ACF values to determine its PSD.

Therefore, the reasonable goal is to obtain a better PSD estimation value. The essential problem can be summarized as: According to the N point observation values obtained in a single realization of the generalized stationary random process, the spectral estimation method is used to estimate the PSD of the process. There are many methods of spectrum estimation, mainly including: Fourier, autoregressive (AR), moving average (MA), autoregressive moving average (ARMA), subspace spectrum estimation and state space spectrum estimation methods. Among them, Fourier method [1] is divided into periodogram, correlation diagram and BT spectrum estimation method; AR spectrum estimation method [2] can be divided into autocorrelation, covariance, Burg algorithm, etc. Subspace spectrum estimation method [3] [4] [5] Including non-linear least square method, MUSIC algorithm, ESPRIT algorithm and Capon algorithm.

In this paper, the Burg algorithm in AR spectrum estimation is used to estimate the power spectrum of a stationary random process, and the AR model of the random sequence is given, and the flowchart and derivation process of the Burg algorithm are presented. For the random process conforming to the AR model, the Burg algorithm is used for spectrum estimation, and the comparison and analysis are carried out with the real power spectrum and the estimated power spectrum using the periodogram method.

2. AR model

According to the idea of modeling, the spectral estimation method of parameter modeling is carried out in three steps: the first step is to select the model; the second step is to use the provided data samples to estimate the model parameters; and the third step is to substitute the estimated model parameters into the theoretical PSD formula, finally obtained the estimated value of the spectrum.

Commonly used parameterized models include AR model, MA model and ARMA model. The model adopted in this paper is the AR model [6], and its expression is,

$$x[n] = -\sum_{k=1}^{p} a[k]x[n-k] + u[n] \tag{1}$$

This process is a strict AR process with order p, called autoregressive process, where the sequence $x[n]$ is the sum of its own linear regression and error $u[n]$. The theoretical PSD expression of the AR process is,

$$P_{AR}(f) = \frac{\sigma^2}{|A(f)|^2} \tag{2}$$

$$A(f) = 1 + \sum_{k=1}^{p} a[k]\exp(-j2\pi fk) \tag{3}$$

Where, σ^2 is the noise variance.

This model is called the all-pole model, as shown in Figure 1, represented as a $AR(p)$ process.

3. Burg algorithm

3.1 Burg algorithm theory

There are many methods for AR spectrum estimation, including autocorrelation function method, covariance method, etc. In this paper, Burg algorithm [7] [8] is used. The Burg algorithm first estimates the reflection coefficient, and then uses Levinson's recursion to obtain the AR parameter estimates [9]. Taking the minimum estimated value of the prediction error power as the goal, the recursive method is used to obtain the estimated value of the reflection coefficient. If the estimated value of the reflection coefficient

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$\{k_1, k_2, \cdots k_p\}$ is known, the AR parameter estimation method is as follows,

$$\hat{r}_{xx}[0] = \frac{1}{N}\sum_{n=0}^{N-1}|x[n]|^2 \tag{4}$$

$$\hat{a}[1] = \hat{k}_1 \tag{5}$$

$$\hat{\rho}_1 = \left(1 - |\hat{a}_1[1]|^2\right)\hat{r}_{xx}[0] \tag{6}$$

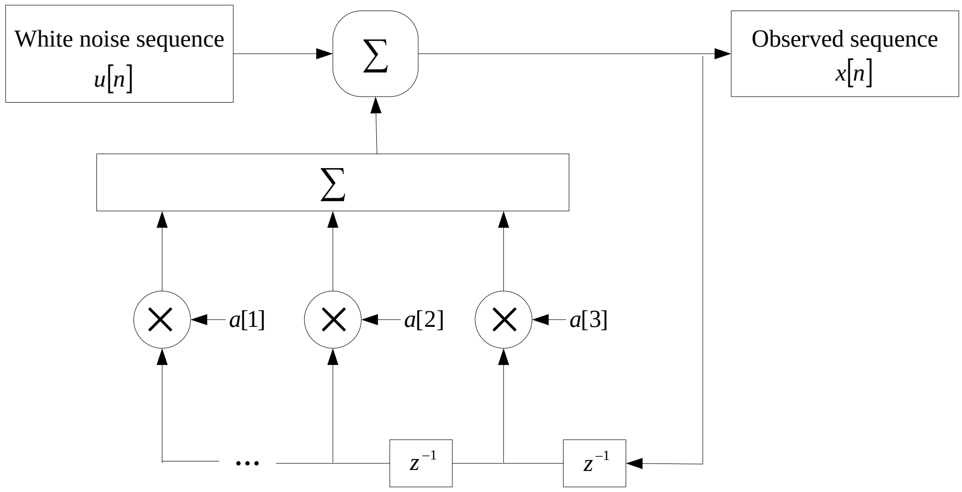

Figure 1: Autoregressive model of stochastic process.

As for $k = 2, 3, \cdots, p$,

$$\hat{a}_k[i] = \begin{cases} \hat{a}_{k-1}[i] + \hat{k}_k \hat{a}^*_{k-1}[k-i] & i = 1, 2, \cdots k-1 \\ \hat{k}_k & i = k \end{cases} \tag{7}$$

$$\hat{\rho}_k = \left(1 - |\hat{a}_k[k]|^2\right)\hat{\rho}_{k-1} \tag{8}$$

The estimated value of AR filter parameters is $\{\hat{a}_p[1], \hat{a}_p[2], \cdots, \hat{a}_p[p]\}$, and the estimated value of white noise variance is $\hat{\rho}_p$. When deriving the estimated value of the k-th reflection coefficient, Burg assumes that the $(k-1)$-order prediction error power is extremely small, and the coefficient of the $(k-1)$-order prediction error filter can be estimated, expressed as $\{\hat{a}_{k-1}[1], \hat{a}_{k-1}[2], \cdots, \hat{a}_{k-1}[k-1]\}$. Using Levinson's recursion, according to equation (7), we know that the coefficients of the k-order prediction error filter are only related to k_k, so the k-order prediction error power estimation is also related to k_k only.

In order to obtain an estimate of k_k, Burg proposed to minimize the average value of the forward and backward prediction error power estimates, namely,

$$\hat{\rho}_k = \frac{1}{2}\left(\hat{\rho}^f_k + \hat{\rho}^b_k\right) \tag{9}$$

Where,

$$\hat{\rho}^f_k = \frac{1}{N-k}\sum_{n=k}^{N-1}\left|x[n] + \sum_{i=1}^{p} a_k[i]x[n-i]\right|^2 \tag{10}$$

$$\hat{\rho}^b_k = \frac{1}{N-k}\sum_{n=0}^{N-1-k}\left|x[n] + \sum_{i=1}^{p} a^*_k[i]x[n+i]\right|^2 \tag{11}$$

$$a_k[i] = \begin{cases} \hat{a}_{k-1}[i] + k_k \hat{a}^*_{k-1}[k-i] & i = 1, 2, \cdots k-1 \\ k_k & i = k \end{cases} \tag{12}$$

Define the estimated forward and backward prediction errors as,

$$\hat{e}^f_k[n] = x[n] + \sum_{i=1}^{k} a_k[i]x[n-i] \tag{13}$$

$$\hat{e}^b_k[n] = x[n-k] + \sum_{i=1}^{k} a^*_k[i]x[n-k+i] \tag{14}$$

The estimated value of forward prediction error power is,

$$\hat{\rho}^f_k = \frac{1}{N-k}\sum_{n=k}^{N-1}\left|\hat{e}^f_k[n]\right|^2 \tag{15}$$

The estimated value of backward prediction error power is,

$$\hat{\rho}^b_k = \frac{1}{N-k}\sum_{n=k}^{N-1}\left|\hat{e}^b_k[n]\right|^2 \tag{16}$$

Substituting equation (12) into equation (13) and equation (14), we get,

$$\hat{e}^f_k[n] = \hat{e}^f_{k-1}[n] + k_k \hat{e}^b_{k-1}[n-1] \tag{17}$$

$$\hat{e}^b_k[n] = \hat{e}^b_{k-1}[n] + k^*_k \hat{e}^f_{k-1}[n-1] \tag{18}$$

Where,

$$\hat{e}^f_0[n] = \hat{e}^b_0[n] = x[n] \tag{19}$$

When these relationships are substituted into equations (15) and (16), and then into equation (9), the average estimated prediction error power becomes,

$$\hat{\rho}_k = \frac{1}{2(N-k)}\sum_{n=k}^{N-1}\left[\left|\hat{e}^f_{k-1}[n] + k_k \hat{e}^b_{k-1}[n-1]\right|^2 + \left|\hat{e}^b_{k-1}[n-1] + k^*_k \hat{e}^f_{k-1}[n]\right|^2\right] \tag{20}$$

Using the complex gradient method, find the differentiation of the real and imaginary parts of $\hat{\rho}_k$ to k_k, and make the result equal to zero, we get,

$$\frac{\partial \hat{\rho}_k}{\partial k_k} = \frac{1}{N-k}\sum_{n=k}^{N-1}\left\{\left(\hat{e}^f_{k-1}[n] + k_k \hat{e}^b_{k-1}[n-1]\right)\hat{e}^b_{k-1}[n-1]^* + \left(\hat{e}^b_{k-1}[n-1]^* + k_k \hat{e}^f_{k-1}[n]^*\right)\hat{e}^f_{k-1}[n]\right\} = 0 \tag{21}$$

Solve k_k, there is,

$$\hat{k}_k = \frac{-2\sum_{n=k}^{N-1}\hat{e}^f_{k-1}[n]\hat{e}^b_{k-1}[n-1]^*}{\sum_{n=k}^{N-1}\left(\left|\hat{e}^f_{k-1}[n]\right|^2 + \left|\hat{e}^b_{k-1}[n-1]\right|^2\right)} \tag{22}$$

In general, the Burg algorithm for AR parameter estimation is expressed as,

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$$\begin{cases}\hat{r}_{xx}[0]=\frac{1}{N}\sum_{n=0}^{N-1}|x[n]|^2\\ \hat{\rho}_0=\hat{r}_{xx}[0]\\ \hat{e}_0^f[n]=x[n], \quad n=1,2,\cdots N-1\\ \hat{e}_0^b[n]=x[n], \quad n=0,1,\cdots N-2\end{cases} \tag{23}$$

As for $k=1,2,\cdots,p$,

$$\begin{cases}\hat{k}_k=\dfrac{-2\sum_{n=k}^{N-1}\hat{e}_{k-1}^f[n]\hat{e}_{k-1}^b[n-1]^*}{\sum_{n=k}^{N-1}\left(\left|\hat{e}_{k-1}^f[n]\right|^2+\left|\hat{e}_{k-1}^b[n-1]\right|^2\right)}\\ \hat{\rho}_k=\left(1-\left|\hat{k}_k\right|^2\right)\hat{\rho}_{k-1}\end{cases} \tag{24}$$

$$\hat{a}_k[i]=\begin{cases}\hat{a}_{k-1}[i]+\hat{k}_k\hat{a}_{k-1}^*[k-i] & i=1,2,\cdots,k-1\\ \hat{k}_k & i=k\end{cases} \tag{25}$$

If $k=1$, then $\hat{a}_1[1]=\hat{k}_1$.

$$\begin{aligned}\hat{e}_k^f[n]&=\hat{e}_{k-1}^f[n]+\hat{k}_k\hat{e}_{k-1}^b[n-1], \quad n=k+1,k+2,\cdots,N-1\\ \hat{e}_k^b[n]&=\hat{e}_{k-1}^b[n-1]+\hat{k}_k^*\hat{e}_{k-1}^f[n], \quad n=k,k+1,\cdots,N-2\end{aligned} \tag{26}$$

The esimated values are $\{\hat{a}_p[1],\hat{a}_p[2],\cdots,\hat{a}_p[p],\hat{\rho}_p\}$.

3.2 Burg Algorithm implementation

The flow chart of spectrum estimation based on Levinson's recursive method is shown in Figure 2. The implementation process is summarized as follows.

Step1: According to the initial conditions (23) and formula (24), find the reflection coefficient k_1.
Step2: According to the autocorrelation function $\hat{r}_{xx}[0]=\frac{1}{N}\sum_{n=0}^{N-1}|x[n]|^2$ of the sequence $x[n]$, find the AR model at the order $k=1$, and sum of the forward and backward prediction error power.
Step3: According to k_1, calculate the forward prediction error and backward prediction error by formula (26), and then calculate k_2 by formula (24).
Step4: Calculate the AR model parameters from equation (25) at the order $k=2$, and the sum of the forward and backward prediction error power.
Step5: Repeat the above process until $k=p$.

4. Experimental Results and Analysis

4.1 Simulation environment

Using MATLAB software for simulation, the observation signal sequence is $x[n]=-\sum_{k=1}^{p}a[k]x[n-k]+u[n]$, where $u[n]$ is the white noise sequence with mean 0 and variance 1, p is the order of the AR model ($p=4$ is taken in this article), $a[k]$ is the coefficient of the AR model, in the simulation experiment $a[k]$ is set as $a=[-1.352 \quad 1.338 \quad -0.662 \quad 0.240]$. The selection of $a[k]$ is to ensure that the pole of $H(z)=1\Big/\left(1+\sum_{k=1}^{p}a[k]\right)$ is located in the unit circle. The data length in the experiment is 1024, and the number of Monte Carlo simulations is 300.

4.2 Simulation results

The AR spectrum estimation result of the given signal using Burg algorithm is shown in Figure 3. Compared with the real power spectrum, the estimated power spectrum has been very smooth, and it is in good agreement with the real power spectrum.

The power spectrum estimated by the periodogram method is shown in Figure 4. It can be seen that the periodogram spectrum estimation can roughly reflect the shape and trend of the real power spectrum, but due to the addition of random white noise, the spectrum estimation obtained each time is different, and the real power spectrum cannot be obtained.

The comparison between the two shows that the Burg algorithm using Levinson's recursion can more effectively estimate the power spectrum of the AR model.

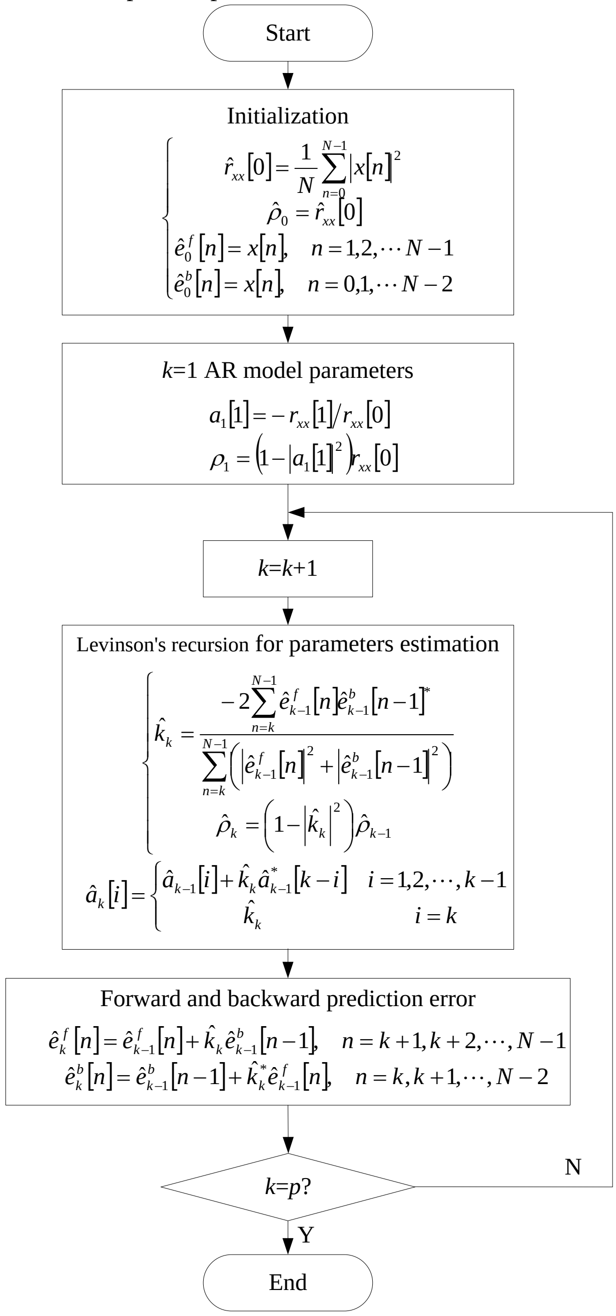

Figure 2: Flowchart for Levinson's recursion.

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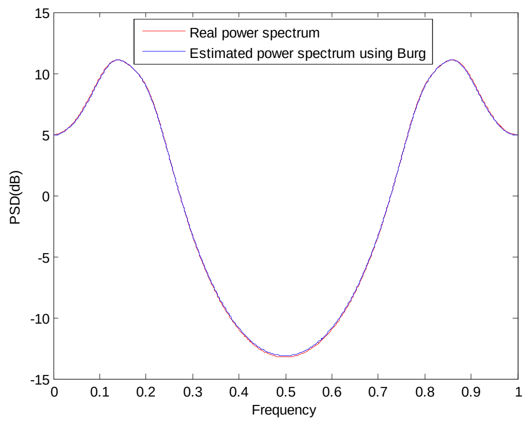

Figure 3: Spectrum estimation result using Burg algorithm.

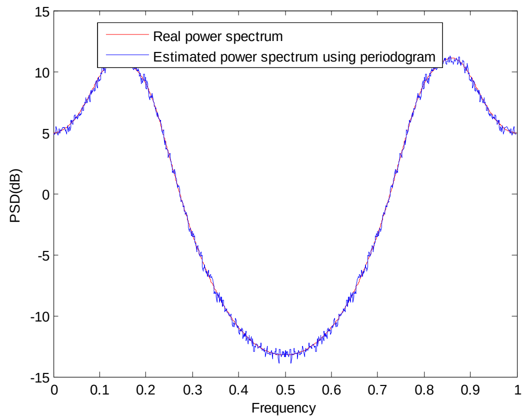

Figure 4: Spectrum estimation result using periodogram method

5. Conclusion

In this paper, the power spectrum estimation based on the AR model is studied, and the Burg algorithm is used to estimate the spectrum of the random stationary process signal. It can be seen from the results of the simulation experiment that the power spectrum of the observed signal sequence can basically be estimated whether it is the average period map estimation or the Burg algorithm spectrum estimation. Compared with the real power spectrum, it is obvious that the estimation effect of Burg algorithm is better.

6. Acknowledgements

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