Comment on a New Quadratic Formulation to Ensure Maximum Profit of a Textile Industry and a Modified Harmonic Average Technique to Solve Multi Objective Quadratic Programming Problem (MOQPP) by Margia Yesmin and Md. Abdul Alim

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Abstract: ***The present analysis observes few weaknesses with the methodologies and most critically with the interpretation of the results in the study to solve multi objective quadratic programming problems by Yesmin and Alim. All the results and their interpretations have been reviewed and improvements in the erroneous conclusions have also been suggested.***

Keywords: Multi Objective Quadratic Programming Problem, Averaging and Modified

Averaging Technique, Chandra Sen's Multi Objective Optimization Approach

1. Introduction

The present study reviews the research paper of Dr. Yesmin & Dr. Alim [12] on various techniques to solve multi objective quadratic programming (MOQP) problems. The MOQP techniques have been explained with an example along with an application of these techniques in the textile industry. The results of various MOQP techniques have been compared with the Chandra Sen's multi objective programming (MOP) technique [22] and concluded that Chandra Sen's technique is inferior to other MOQP techniques. The similar conclusions have also been drawn in several studies [1]-----[5], [10], [11], [13]-----[21], [23]----[26]. It was noticed that most of the studies on MOP are lacking the basic conceptual clarity and using inappropriate examples. The interpretation of the results was not found to be logical. All these issues have been well attended in the recent studies [6], [7], [8], [9].

2. Solution and its Interpretation

Yesmin & Alim discussed various averaging techniques to solve MOQP problems. The study has been evaluated with respect to the solving the example and interpretation of the results. The example 1 of the study has been reproduced here below.

Example 1

$$\text{Max } Z_1 = 4x_1 + 2x_2 - x_1^2 - x_2^2 + 5$$
$$\text{Max } Z_2 = 2x_1 + x_2 - x_1^2$$
$$\text{Min } Z_3 = 6 - 6x_1 + 2x_1^2 - 2x_1x_2 + x_2^2$$
$$\text{Min } Z_4 = 2x_1 + 3x_2 - 2x_1^2$$

Subject to

$$x_1 + 4x_2 \leq 9$$
$$x_1 + x_2 \leq 3$$
$$3x_1 + 2x_2 \leq 8$$
$$x_1, x_2 \geq 0$$

It is to point out here that the optimal values of third and fourth objective would be zero or negative. How the combined objective function can be formulated by any averaging techniques? All the four objectives have been optimized individually and results are presented in Table 1.

Table 1: Solution of Individual Optimization

Item	Individual Optimization			
	Max. Z_1	Max. Z_2	Min. Z_3	Min. Z_4
X_1, X_2	2, 1	0.875, 2.0313	0, 2.25	0.3125, 2.1719
Z_1	10	7.6708	4.4375	5.779
Z_2	1	3.0156	2.25	2.6992
Z_3	0	6.9788	15 (16.125)	12.3972
Z_4	-1	6.3127	6.75	6.9453

All the four solutions are different and indicate the presence of conflicts amongst objectives. The optimization of first objective Z_1 achieved its value of 10 with the values of remaining objectives Z_2, Z_3 and Z_4, 1,0 and -1 respectively. Similarly other three objective have been optimized. The optimal values of Z_2, Z_3 and Z_4 were 3.0156, ,0, and -1 respectively. It is not clear that the minimum values of Z_3 and Z_4 were 15 (corrected value 16.125) and 6.9453 respectively. The minimum values of these objectives have already been achieved in the optimization of first objective Z_1. The example was also solved using several MOQP techniques and the results have been presented in Table 2.

Table 2: Solution of different Multiobjective optimization techniques

Item	Multiobjective Optimization						
	Chandra Sen's Approach	Arithmetic Averaging Technique	Geometric Averaging Technique	Harmonic Averaging Technique	Modified Arithmetic Averaging Technique	Modified Geometric Averaging Technique	Modified Harmonic Averaging Technique
Z*	1.5	1.7	1.99	2.35	2.51	2.73	2.85
X_i	2.18, 0.73	2.24, 0.49	2.04, 0.44	2.11, 0.52	2.43, 0.35	2.43, 0.35	2.16, 0.26
Z_1	9.9	9.6823	9.6848	9.7575	9.3926	9.3926	9,4268
Z_2	0.34	-0.0476	0.3584	0.2879	-0.6949	-0.6949	-0.0856
Z_3	0.3	0.8802	0.6752	0.5906	1.7738	1.7738	1.3832
Z_4	-2.95	-4.0852	-2.9232	-3.1242	-5.8998	-5.8998	-4.2312

Z* = Multiobjective Function,

The values of the combined objective function Z* are presented in the first row. The combined objective function is formulated using various scalarizing techniques for solving the MOQP problems. The values of combined objective functions are non comparable. The combined objective function helps in generating a compromising solution. However, the values of the basic objectives Z_1, Z_2, Z_3 and Z_4 should be compared. The results indicated that none of the seven solutions achieved all the objectives simultaneously. The geometric averaging technique achieved Z_2 with its highest value 0.3584. The modified arithmetic and geometric averaging techniques have achieved Z_1 and Z_4 only. Chandra Sen's approach achieved Z_1 (9.9) and Z_3 (0.3) with Z_2 (0.34) very near to its optimal level 0.3584 of geometric averaging technique. The achievement of all the objectives using Chandra Sen's approach seems better over all other techniques.

Yesmin and Alim have also made an application of the averaging techniques for improving the performance of Textile Industry. The problem has been formulated as below:

Example 2

Max $Z_1 = -0.5x_1^2 - 0.13x_2^2 + 3.5x_1 + 6.5x_2$

Max $Z_2 = -0.09x_1^2 - 0.24x_2^2 + .12x_1 + 13.204x_2$

Max $Z_3 = -0.12x_1^2 - 0.19x_2^2 + 8.5x_1 + 10.73x_2$

Subject to

$0.29x_1 + 0.2x_2 \leq 50000$..........(1)
$0.29x_1 + 0.2x_2 \leq 7000$ ....(2)
$0.22x_1 + 0.23x_2 \leq 10000$ ....(3)
$x_1, x_2 \geq 0$

The left side equations of the above mentioned constraints 1 and 2 are exactly the same and hence the first constraint is redundant. Similarly the third constraint is also redundant due to higher value in right hand side making the non achievable values of x_1 and X_2. All the three objectives have been optimized individually and the results are presented in Table 3.

Table 3: Individual Optimization

Item	Individual Optimization		
	Max. Z_1	Max. Z_2	Max. Z_3
X_1, X_2	4, 25	34, 28	35, 28.2
Z_1	87.38	-378.92	-410.081
Z_2	203.14	285.54	285.44
Z_3	241.42	301.76	302.01

All the three solutions were different revealing the presence of conflicts in the objectives. The maximization of first objective achieves it with the appropriate values of second and third objectives. However, the maximization of second and third objectives have reduced the value of first objective to the unacceptable levels of -378.92 and -410.081 respectively. The problem has been solved using Chandra Sen's approach and modified harmonic averaging technique. The solution is given in Table 4.

Table 4: Multiobjective Optimization

Item	Multiobjective Optimization	
	Chandra Sen's Approach	Modified Harmonic Averaging Technique
Z*	2.36	3.9
X_1, X_2	7, 26	13, 27
Z_1	87.3	41.73
Z_2	285.54	245.898
Z_3	302.01	241.42

Z*= Multiobjective Function

The values of multiobjectve function Z* are mentioned in the first row of the table, but these are non comparable. The achievements of the all the three objectives reveal the superiority of Chandra Sen's approach over modified harmonic averaging technique.

3. Conclusion

The present paper analyzed the methodologies and the results of several MOQP techniques and Chandra Sen's approach to solve MOO problems. Few weaknesses in the interpretation of the results have been identified. The comparative analyses of various MOO techniques have also been rectified.

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