International Journal of Science and Research (IJSR)
ISSN: 2319-7064
ResearchGate Impact Factor (2018): 0.28 | SJIF (2019): 7.583

Robust Control Algorithm Design for Nonlinear MIMO Systems using MVPA Optimization

Musadaq A. Hadi[1], **Hazem I. Ali**[2]

[1, 2]Control and Systems Engineering Department, University of Technology-Iraq

Abstract: ***In this paper, a new robust control algorithm design is proposed for nonlinear multi-input multi-output (MIMO) systems with unknown fast time-varying parametric uncertainties. First, the system is analyzed using Lyapunov Quadratic Function (LQF) with indefinite time-derivative function. Second, the new MIMO model reference is used with desired characteristics in order to achieve the required performance. Then, a proper controller is constructed to compensate both stability and performance of the system. Afterwards, a new optimization method which is called Most Valuable Player Algorithm (MVPA) is used to optimize both the parameters of the MIMO model reference and the parameters of the proposed controller. Finally, the simulation results substantiate the efficiency of the proposed algorithm by achieving the asymptotic stable and the required performance of the system.***

Keywords: Lyapunov Quadratic Function, Most Valuable Player Algorithm, Uncertain System, Model Reference, Indefinite, Time-Varying, Parametric Uncertainties.

1. Introduction

Robust control design for parametric uncertainty has received much attention in the control literature. Many contributions studied this problem in nonlinear MIMO systems. At the beginning, slow time-varying (TV) was presented for linear SISO systems [1]. Then, some researches were published on fast time-varying (TV) for linear SISO systems [2]. Meanwhile, the parametric uncertainty was introduced with time-varying in the control literature for linear systems [3]- [5]. These researches were made as a setup for nonlinear time-varying (TV) MIMO systems. Afterwards, researches were developed to discuss the problem of time-varying parametric uncertainties for nonlinear MIMO systems [6]-[9]. A suboptimal control method was designed to eliminate the effect of parametric uncertainties on the nonlinear MIMO systems [10]. In addition, a ball screw drives with time-varying parametric uncertainties was treated using adaptive backstepping sliding mode control [11]. Furthermore, this problem was not restricted to continuous systems because researches were introduced for many discrete systems with parametric uncertainties. Robust H_2 fuzzy control was introduced for discrete-time nonlinear systems with parametric uncertainties [12]. Robust H_∞ control was designed for discrete-time systems with time-varying parametric uncertainties [13]. A robust adaptive NN output feedback control was presented to compensate a class of uncertain discrete-time nonlinear MIMO systems [14]. In addition, observer based learning control was used to handle a rapid time-varying parametric uncertainties [15]. An uncertain MIMO nonlinear system was handled by implementing robust adaptive sliding mode control [16]. Robust adaptive neural network (NN) control was addressed to deal with uncertain nonlinear MIMO system with unknown control coefficient matrices and input nonlinearities [17]. Robust decentralized tracking control was addressed to deal with uncertain MIMO nonlinear systems with time-varying delays [18]. A robust tracking control was presented for nonlinear MIMO systems with uncertainties and external disturbances [19]. Two types of sliding mode controllers were proposed for MIMO tank system [20]. A novel fuzzy-adaptive control was proposed to deal with a class of nonlinear MIMO systems [21]. Generally, the tracking controllers for MIMO systems presented in many contributions were based on second order or discrete or based on the system if the plant is Linear Time Invariant (LTI) MIMO system [22]-[24]. Moreover, the analysis was made for these contributions are based on the regular Lyapunov analysis with negative-definite time-derivative function [25]-[29]. In this paper, a robust control algorithm is presented to compensate a class of nonlinear MIMO system with time-varying parametric uncertainties. This algorithm is based on Lyapunov stability analysis with indefinite time-derivative function. A MIMO model reference is used based on the actual nonlinear system to fit the algorithm design procedure. The Most Valuable Player Algorithm (MVPA) is used to optimize the parameters of the controller and the MIMO model reference. Finally, signum function is used in the proposed control algorithm to overcome the decoupling and compensate the uncertainties of the MIMO system.

2. Most Valuable Player Algorithm

The Most Valuable Player Algorithm (MVPA) is new sport-based optimization method where the players are compete each other's collectively in teams to find the winner of the leagues' championship. In addition, they are competing with each other in order to achieve the MVP trophy. Like other metaheuristic methods. The number of population is represented as a group of skilled players which are presented design variables and the numbers of the players' skills are the dimension of the problem. Here are some sport terms related to the MVPA should be defined [30]:

- ***Team:*** a group of players who are played a sport game against another group of players.
- ***Player***: a person who is participated in a sport game.
- ***Championship***: a competition tournament to find out the best team/player in a certain sport.
- ***Franchise player***: the best player in any sports team who is played professionally.

- ***League***: a group of sports teams who are all played against each other to acquire points and figure out which team is the best.
- ***Fixture***: an event of sports that is prepared to be happened in a certain date and place.
- ***Most valuable player***: the award that is given to the best player in a sport game/series of sport games throughout a certain season.

In this algorithm, a player and a team which is a group of players both are represented as follows [30]:

$$\boldsymbol{Player_k = [S_{k,1} \quad S_{k,2} \quad \cdot \quad \cdot \quad S_{k,ProblemSize}]} \quad \textbf{(1)}$$

$$\boldsymbol{Team_i =} \begin{bmatrix} S_{1.1} & S_{1.2} & \cdot & S_{1,ProblemSize} \\ S_{2.1} & S_{2.2} & \cdot & S_{1,ProblemSize} \\ \vdots & \vdots & \cdot & \vdots \\ S_{PlayerSize,1} & S_{PlayerSize,2} & \cdot & S_{PlayerSize,ProblemSize} \end{bmatrix} \quad \textbf{(2)}$$

where $Players\ Size$ represents how many players that are played in the league, $ProblemSize$ represents the problem dimension and S represents the skills. Each team own a player who has called a franchise also the best player of the league. An example of two players with their corresponding level of skills for each one is shown in Figure 1.

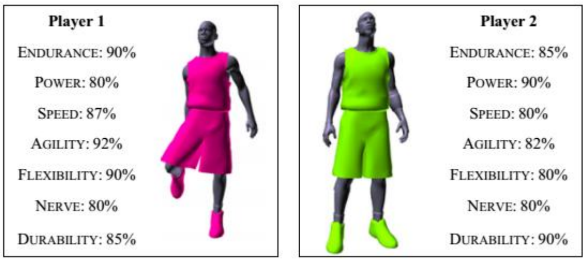


Figure 1 Two players presented with their skills [30].

The phases of the MVPA are explained as follow [27]:

a) *Initialization*; a number population of the player size; players are randomly generated in the search space.

b) *Team formation*; the teams are named as 'nT_1' and 'nT_2' are first team and second team respectively. Also, the players are named such as 'nP_1' and 'nP_2' are the players of the first and second team respectively. These variables are calculated as follow [30]:

$$\boldsymbol{nP_1 = ceil\left(\frac{PlayersSize}{TeamsSize}\right)} \quad \textbf{(3)}$$

$$\boldsymbol{nP_2 = nP_1 + 1} \quad \textbf{(4)}$$

$$\boldsymbol{nT_1 = PlayersSize - nP_2 TeamsSize} \quad \textbf{(5)}$$

$$\boldsymbol{nT_2 = TeamsSize - nT_1} \quad \textbf{(6)}$$

c) *Team competition;* players are debating each other individually in order to find which one is the best player who has the best skills. This competition is calculated using the following expressions [30]:

$$\boldsymbol{TEAM_i =} \begin{bmatrix} \boldsymbol{TEAM_i + rand\begin{pmatrix} FranchisePlayer_i \\ -TEAM_i \end{pmatrix}} \\ \boldsymbol{+2rand(MVP - TEAM_i)} \end{bmatrix} \quad \textbf{(7)}$$

If $TEAM_i$ is chose to play against $TEAM_j$ and $TEAM_i$ wins the player's performance of $TEAM_i$ are expressed as follow [30]:

$$\boldsymbol{TEAM_i + rand\begin{pmatrix} TEAM_i \\ -FranchisePlayer_j \end{pmatrix}} \quad \textbf{(8)}$$

Otherwise, they are expressed as follow [30]:

$$\boldsymbol{TEAM_i + rand\begin{pmatrix} FranchisePlayer_j \\ -TEAM_i \end{pmatrix}} \quad \textbf{(9)}$$

d) *Application of greediness;* a new solution is selected after the comparison of the population is done. Each selection is made based on a better objective function value.

e) *Application of elitism*; the best (elite) players are selected and the other players are replaced with the best ones.

f) *Remove duplicates*; if the best players have been selecting twice. Then, one of them is dropped.

g) *Termination criterion;* in the MVPA, this criterion is option implemented by the user himself or the number of the iterations will be the termination criterion [30].

The reason behind using MVPA is that the method is converging faster after compared with 13 well-known optimization methods including Genetic Algorithm (GA), Particle Swarm Optimization (PSO), …, etc [30]. It can be seen in Figure 2, the steps of the MVPA in detail with respect to the system that can be used later.

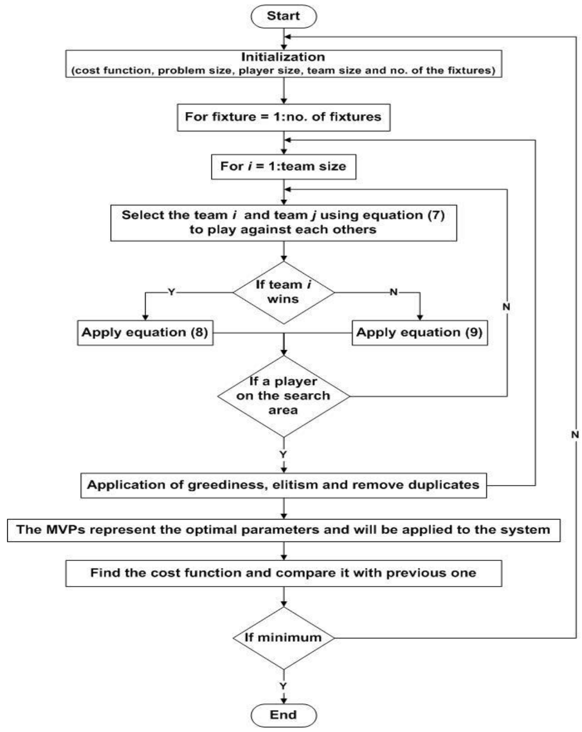


Figure 2 The flowchart of MVPA with the system.

3. Control Algorithm Design

In this section, the robust control algorithm which is based on the Lyapunov stability analysis is presented after the system is described and a proper model reference is selected.

3.1 Problem Formulation

In this subsection, the mathematical model of the nonlinear MIMO systems with unknown fast time-varying parametric uncertainties is presented as follow [31]:

$$\boldsymbol{x^{(n)} = h\left(\mathrm{x}, x^{(n-1)}, \theta_1\right) + G(\mathrm{x}, \theta_2)u} \qquad \textbf{(10)}$$

where $\mathbf{x} \triangleq \left[x^T, \dot{x}^T, \cdots, \left(x^{(n-1)}\right)^T\right]^T \in \mathcal{R}^{mn-m}, x^{(i)}(t) \in \mathcal{R}^m, i = 0, 1, \ldots, n$ denote the system states, $, u(t) \in \mathcal{R}^m$ is the control input, $x(t) \in \mathcal{R}^m$ is the system output. $h\left(\mathrm{x}, x^{(n-1)}, \theta_1\right) \in \mathcal{R}^m$ and $G(\mathrm{x}, \theta_2) \in \mathcal{R}^{m \times m}$ are locally Lipschitz in their arguments with unknown time-varying parameters $\theta_i(t) \in \mathcal{R}^{l i}\ \forall\, i = 1, 2$.

Paper ID: SR20716233855 DOI: 10.21275/SR20716233855

3.2 System Description and Preliminaries

In this subsection, Lyapunov stability analysis with indefinite time-derivative function is proved for the non-linear time-varying system [32]:

$$\dot{x}(t) = f(x,t), u(t) \qquad (11)$$

where $f: J \times R^n \times R^m \to R^n$ is continuous, locally Lipschitz on x for bounded u and such that $f(t,0,0) = 0$. The input $u: J \to R^m$ is assumed to be locally essentially bounded. In this note, we are interested in the stability analysis of system in Equation (11). Throughout this note, for any $\mathbb{C}^1$ function $V: J \times R^n \to R$, such that [32]:

$$\dot{V}(x,t)\big|_{(5)} \triangleq \frac{\partial V(x,t)}{\partial t} + \frac{\partial V(x,t)}{\partial x} f(x,t,u) \qquad (12)$$

Next, the concept of stable functions is proposed. Consider the following scalar linear time-varying system [32]:

$$\dot{y}(t) = \mu(t) y(t), \ t \in J \qquad (13)$$

where $y(t): J \to R$ is the state variable and $\mu(t) \in \mathbb{PC}(J, R)$. It is not hard to see that the state transition matrix for system as in Equation (13) is given by [32]:

$$\varphi(t,t_0) = exp\left(\int_{t_0}^{t} \mu(s) ds\right), \forall t \geq t_0 \in J \qquad (14)$$

Definition 1: The function $\mu(t) \in \mathbb{PC}(J, R)$ is said to be:

i. asymptotically stable if the scalar system as in Equation (13) is asymptotically stable;
ii. exponentially stable if the scalar system as in Equation (13) is exponentially stable, namely, there exist constants $k(t_0) > 0$ and $\alpha > 0$ such that [32]:

$$|y(t)| \leq k(t_0)|y(t_0)|\, exp(-\alpha (t - t_0)), \forall t \geq t_0 \in J \ (15)$$

iii. uniformly exponentially stable or uniformly asymptotically stable if the scalar linear time-varying system as in Equation (13) is uniformly exponentially stable, namely, the constant $k(t_0)$ in Equation (9) is independent of t_0. In Definition 1 we have noted that, for linear time-varying system, uniformly asymptotic stability and uniformly exponential stability are equivalent. By noting the transition matrix in Equation (14), Abovementioned analysis leads to the following fact

Lemma 1*:* The scalar function $\mu(t) \in \mathbb{PC}(J, R)$ is [32]:

i. asymptotically stable if and only if

$$lim_{t\to\infty} \int_{t_0}^{t} \mu(s)\, ds = -\infty \qquad (16)$$

ii. exponentially stable if and only if there exist $\alpha > 0$ and $\beta(t_0)$, which is non-negative, piece-wise continuous, and non-decreasing, such that

$$\int_{t_0}^{t} \mu(s)\, ds \leq -\alpha(t - t_0) + \beta(t_0), \forall t \geq t_0 \in J \ (17)$$

iii. uniformly exponentially stable if and only if Equation (14) is satisfied, where β is independent of t_0. Of course, if $\mu(t) \in \mathbb{PC}(J, R)$ is a periodic function with period T, then the three different stability concepts in Definition 1 are equivalent, and moreover, they are equivalent to the existence of a $c > 0$ such that

$$\int_{t+T}^{t} \mu(s)\, ds \leq -c \qquad (18)$$

Theorem 1: Assume that there exists a C^1 function $V: J \times \mathcal{R}^n \to J$, two $\mathcal{NK}_\infty$ functions $\alpha_i, i = 1,2$, and a scalar function $\mu(t) \in \mathbb{PC}(J, \mathcal{R})$, $\forall t \in J$ and $x \in \mathcal{R}^n$ such that [32]:

$$\alpha_1(t,x) \leq V(x,t) \leq \alpha_2(t,x) \qquad (19)$$

$$\dot{V}(x,t)\big|_{(5) \text{ where } u \equiv 0} \leq \mu(t) V(x,t) \qquad (20)$$

Then the nonlinear time-varying system of Equation (11) with $u \equiv 0$ is:

- Globally asymptotically stable if $\mu(t)$ is asymptotically stable.
- Globally uniformly and asymptotically stable if $\mu(t)$ is uniformly exponentially stable and $\alpha_i(s,t), s, i = 1,2$ are independent of t.
- Globally exponentially stable if $\mu(t)$ is exponentially stable and there exist $m > 0$ and $k_i(\cdot) \in \mathcal{N}, i = 1,2$ such that $\alpha_i(s,t) = k_i(t)\, s^m, i = 1,2$.
- Globally uniformly and exponentially stable if $\mu(t)$ is uniformly exponentially stable and there exist $m > 0, k_i > 0, i = 1,2$ such that $\alpha_i(s,t) = k_i s^m$, $i = 1,2$.

Proof: note that Equation (20) includes

$$\frac{d}{dt} \ln V(x,t) = \frac{\dot{V}(x,t)}{V(x,t)} \leq \mu(t), \ \ \forall t \geq t_0 \in J$$

From which it follows that

$$\begin{aligned} \alpha_1(t_0, |x(t)|) &\leq \alpha_1(t, |x(t)|) \\ &\leq V(x(t), t) \\ &\leq V(x(t_0), t_0)\varphi(t, t_0) \qquad (21) \\ &\leq \alpha_2(t_0, |x(t_0)|)\varphi(t, t_0) \end{aligned}$$

Proof of Item 1: Since $\lim_{t\to\infty} \varphi(t,t_0) = 0$, there exists a $T = T(t_0)$ such that $\varphi(t,t_0) \leq 1, \forall t \geq t_0 + T(t_0)$. Let

$$\gamma(t_0) \triangleq \max_{s \in [t_0,\ t_0 + T(t_0)]} \{\varphi(s, t_0)\} \geq 1$$

Then from Equation (21) we have

$$\alpha_1(t_0, |x(t)|) \leq \alpha_2(t_0, |x(t_0)|)\gamma(t_0), \forall t \geq t_0 \quad (22)$$

Consequently, for a function $\alpha \in \mathcal{NK}_\infty$, so $\alpha^{-1}(t,s)$ is used to denote the inverse function of $\alpha(t,s)$ with respect to the second variable, namely $\alpha^{-1}(t, \alpha(t,s)) \equiv s$. Then

$$\delta(t_0) = \alpha_2^{-1}\left(t_0, \left(\frac{1}{\gamma(t_0)}\right)\alpha_1(t_0, \varepsilon)\right)$$

Equivalently, $\alpha_2(t_0, \delta(t_0)\gamma(t_0)) = \alpha_1(t_0, \varepsilon)$. Then from Equation (22) that, for any $x(t_0) \leq \delta(t_0)$,

$$\begin{aligned} \alpha_1(t_0, |x(t)|) &\leq \alpha_2(t_0, \delta(t_0))\gamma(t_0), \\ &= \alpha_1(t_0, \varepsilon), \ \forall t \geq t_0 \in J \end{aligned}$$

which is just $x(t) \leq \varepsilon, \forall t \geq t_0$. On the other hand, from Equations (16) and (21) that $\lim_{t\to\infty} x(t) = 0$. This proves that the system is globally asymptotically stable.

Proof of Item 2: It follows from Item 3 of Lemma 1 and Equation (21) such that, for any $t \geq t_0 \in J$,

$$\begin{aligned} x(t) &\leq \alpha_1^{-1}(\alpha 2(|x(t_0)|)\phi(t,t_0)) \\ &\leq \alpha_1^{-1}(\alpha 2(|x(t_0)|)) exp(\beta) exp(-\alpha(t - t_0) \in \mathcal{KL} \end{aligned}$$

which shows that the system is globally uniformly asymptotically stable.

Proof of Item 3: By noting that $\alpha_i^{-1}(t,s) = s^{1/m} k_i^{-\left(\frac{1}{m}\right)}(t)$ and Equation (17)

$$x(t) \leq \left(\frac{k_2(t_0)}{k_1(t_0)}\right)^{1/m} e^{\beta(t_0)/m} |x(t_0)| e^{-(\alpha/m)(t-t_0)} \quad (23)$$

which indicates the system is globally exponentially stable.

Proof of Item 4: This follows from Equation (23) since is $k_i(t_0), i = 1,2$ and $\beta(t_0)$are independent of t_0. The proof is finished. To go further, the following technical lemma is introduced.

Lemma 2: *(Generalised Gronwall–Bellman Inequality):* Assume that $\pi(t) \in \mathbb{PC}(J, R), \mu(t) \in \mathbb{C}(J, R)$ and $y(t): J \to J$ is such that [32]:

$$\dot{y}(t) \leq \mu(t) y(t) + \pi(t), \ \ t \in J \qquad (24)$$

Then, for any $t \geq s \in J$, the inequality holds true:

$$y(t) \leq \varphi(t,s) y(s) + \int_{s}^{t} \varphi(t,\lambda)\pi(\lambda)\, d\lambda \qquad (25)$$

Lemma 2 can be regarded as the Gronwall–Bellman inequality in the differential form. Note that differently from the generalized integral Gronwall–Bellman inequality in, the function $\mu(t)$ is not required to be positive for all t.

Theorem 2: Assume that there exists a C^1 function $V: J \times \boldsymbol{R^n} \to J$, *two* $\mathcal{N}K_\infty$ functions $\alpha_i,\ i = 1,2$, an asymptotically stable function $\mu(t) \in \mathbb{C}(J, \boldsymbol{R})$, and a scalar function $\pi(t) \in \mathbb{PC}(J, J)$ such that, for all $(x,t) \in J \times \boldsymbol{R^n}$, Equation (17) and the following inequality are satisfied [32]:

$$\dot{V}(x,t)_{(2) where\ u \equiv 0} \leq \mu(t)V(x,t) + \pi(t) \qquad (26)$$

Denote $\kappa(t,t_0) : J \times J \to \boldsymbol{R}$ as:

$$\kappa(t,t_0) = \int_{t_0}^{t} \varphi(t,s)\pi(s)\,ds \qquad (27)$$

Then system in Equation (11) is globally asymptotically stable if $\kappa(t,t_0)$ is bounded for any $t \geq t_0 \in J$ and:

$$\lim_{t\to\infty} \kappa(t,t_0) = \lim_{t\to\infty} \int_{t_0}^{t} \varphi(t,s)\pi(s)ds = 0, \qquad \forall t_0 \in J \qquad (28)$$

Proof: Applying Lemma 2 on inequality in Equation (26) gives, for all $t \geq t_0 \in J$,

$$\begin{aligned} \alpha_1(t_0, |x(t)|) &\leq \alpha_1(t\,|x(t)|) \\ &\leq V(x(t),t) \\ &\leq V(x(t_0),t_0)\varphi(t,t_0) + \kappa(t,t_0) \end{aligned}$$

Hence, by noting that $lim_{t\to\infty}\,\varphi(t,t_0) = 0$ and Equation (26), we have $lim_{t\to\infty}\,\alpha_1(t_0, x(t)) = 0$, which in turn implies $lim_{t\to\infty}\,x(t) = 0$. Hence the origin of the system is globally attractive. On the other hand, as $\kappa(t,t_0)$ and $\varphi(t,t_0)$ are bounded, $\alpha_1(t_0, x(t))$ is bounded, which in turn implies that $x(t)$ is bounded, namely, the system is Lagrange stable. Then, by Proposition 2.5 in [33], the system is Lyapunov stable. Consequently, the non-linear time-varying system in Equation (11) is globally asymptotically stable. The most important advantage of Theorem 2 is that the right-hand-side of Equation (26) is not required to be negative for all time. Moreover, it is even allowed to have a drift term that is non-negative for all time.

3.3 MIMO Model Reference Selection

The selection of the model reference for Linear Time Invariant (LTI) MIMO systems depends on the relative degree of the plant. However, the system is nonlinear and fast parametric time-varying. Therefore, the model reference is designed based on the system to fit the algorithm design procedure. The parameters of the MIMO model reference are optimized using MVPA in order to obtain the optimum model reference for MIMO system. The following is the selected MIMO model reference [22]-[24]:

$$\dot{x}_d = Ax_d + Br$$
$$y = cx_d$$

Or
$$\begin{bmatrix} \dot{x}_{1d} \\ \dot{x}_{2d} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} x_{1d} \\ x_{2d} \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$
$$\begin{bmatrix} y_{1d} \\ y_{2d} \end{bmatrix} = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{bmatrix} x_{1d} \\ x_{2d} \end{bmatrix} \qquad (29)$$

where a_1, a_2, b_1, b_2, c_1 and c_2 represent the coefficients of the model reference which are positive constants. Also, x_{1d} and x_{2d} represent the states of the model reference and r_1, r_2, y_{1d} and y_{2d} are the model reference inputs and outputs respectively.

3.4 Lyapunov Stability Analysis

In this subsection, Lyapunov Quadratic Function (LQF) is selected in order to analyze the MIMO system as follow:

$$e = x_d - x \qquad (30)$$
$$\dot{e} = \dot{x}_d - \dot{x} \qquad (31)$$

Let the system be $\quad \dot{x} = f(x,u,t) \qquad (32)$

Then substituting Equations (29), (30) and (32) in equation (31), gives:

$$\dot{e} = Ax_d + Br - f(x,u,t)$$
$$\dot{e} = Ax_d + Br - f(x,u,t) + Ax - Ax$$
$$\dot{e} = A(x_d - x) + Az + Br - f(x,u,t)$$
$$\dot{e} = Ae + Ax + Br - f(x,u,t) \qquad (23)$$

By using LQF as follow:

$$V(e) = e^T P\, e \qquad (34)$$

The time-derivative of LQF $\dot{V}(e)$ yields:

$$\dot{V}(e) = \dot{e}^T P\, e + e^T P\, \dot{e} \qquad (35)$$

Substituting Equation (33) in equation (35) gives:

$$\dot{V}(e) = [Ae + Ax + Br - f(x,u,t)]^T\, P\, e + e^T P\, [Ae + Ax + Br - f(x,u,t)]$$
$$\dot{V}(e) = e^T(A^T P + PA)e + 2e^T P[Ax - f(x,u,t) + Br]$$
$$\dot{V}(e) = -e^T Q e + 2M \qquad (36)$$

with $\quad M = e^T P[Ax + Br - f(x,u,t)] \qquad (37)$

where Q and P are positive matrix.

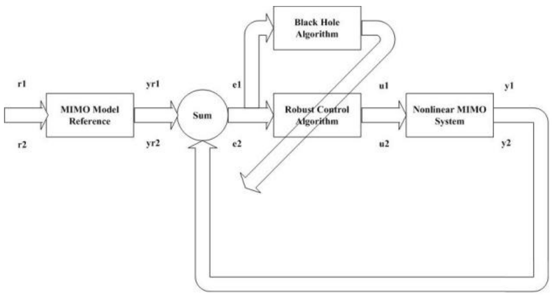


Figure 3: General nonlinear MIMO system with MVPA

Figure 3 shows the interconnections between the general form of the nonlinear MIMO system and the MVPA. Aforementioned analysis is applied on the following MIMO system in order to overcome the parametric uncertainty. Considering MIMO nonlinear system with fast time-varying parametric uncertainties [31]:

$$\dot{x}_1 = u_1\varphi_1(x_1,\theta_1) + x_2 + x_1x_2^2\varphi_1(x_1,\theta_1)$$
$$\dot{x}_2 = u_2\varphi_2(x_2,\theta_2) - x_1 + x_1^2x_2\varphi_1(x_1,\theta_1) - \varphi_2(x_2,\theta_2) \qquad (38)$$

where $\varphi_1(x_1,\theta_1) = (1 + x_1^2)^{\theta_1(t)}$, $\quad \varphi_2(x_2,\theta_2) = e^{\theta_2(t)x_2}$ and $\theta_1(t),\ \theta_2(t)$ are time-varying parameters as $\theta = [\theta_1(t)\ \theta_2(t)]^T = [sin(0.5t) - cos(t)\ sin(0.5t) - cos(t)]^T$.

Or $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \varphi_1(x_1,\theta_1) & 0 \\ 0 & \varphi_2(x_2,\theta_2) \end{bmatrix} \times \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \qquad (39)$$

where $\delta_1 = x_1x_2^2\varphi_1(x_1,\theta_1)$, $\delta_2 = x_1^2x_2\varphi_1(x_1,\theta_1) - \varphi_2(x_2,\theta_2)$.

By substituting Equations (29) and (39) in Equation (37) as follow:

$$M = [e_1 \quad e_2] \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \times \left\{ \begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} \varphi_1(x_1,\theta_1) & 0 \\ 0 & \varphi_2(x_2,\theta_2) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} - \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \right\}$$

$$= [e_1 \quad e_2]\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\left\{\begin{bmatrix} 0 & 0 \\ -a_1+1 & -a_2-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1r_1 - \varphi_1(x_1,\theta_1)u_1 - \delta_1 \\ b_2r_2 - \varphi_2(x_1,\theta_1)u_2 - \delta_2 \end{bmatrix}\right\}$$

$$= [e_1 \quad e_2]\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\left\{\begin{bmatrix} 0 \\ (-a_1+1)x_1 + (-a_2-1)x_2 \end{bmatrix} + \begin{bmatrix} b_1r_1 - \varphi_1(x_1,\theta_1)u_1 - \delta_1 \\ b_2r_2 - \varphi_2(x_1,\theta_1)u_2 - \delta_2 \end{bmatrix}\right\}$$

$$= \left[\begin{pmatrix} e_1p_{11} \\ +e_2p_{12} \end{pmatrix} \quad \begin{pmatrix} e_1p_{12} \\ +e_2p_{22} \end{pmatrix}\right] \times \begin{bmatrix} b_1r_1 - \varphi_1(x_1,\theta_1)u_1 - \delta_1 \\ (-a_1+1)x_1 + (-a_2-1)x_2 + b_2r_2 - \varphi_2(x_1,\theta_1)u_2 - \delta_2 \end{bmatrix}$$

$$= \begin{pmatrix} e_1p_{11} \\ +e_2p_{12} \end{pmatrix}[b_1r_1 - \varphi_1(x_1,\theta_1)u_1 - \delta_1] + \begin{pmatrix} e_1p_{12} \\ +e_2p_{22} \end{pmatrix} \times \begin{bmatrix} (-a_1+1)x_1 + (-a_2-1)x_2 \\ +b_2r_2 - \varphi_2(x_1,\theta_1)u_2 - \delta_2 \end{bmatrix} \quad \textbf{(40)}$$

Let $$u_1 = \frac{1}{\varphi_1(x_1,\theta_1)} \times \begin{bmatrix} b_1r_1 - \delta_1 - c_1x_1^2 \\ +c_2x_1^2 sign\begin{pmatrix} e_1p_{11} \\ +e_2p_{12} \end{pmatrix} \end{bmatrix} \quad \textbf{(41)}$$

$$u_2 = \frac{1}{\varphi_2(x_2,\theta_2)}\begin{bmatrix} b_2r_2 - \delta_2 + \begin{pmatrix} -a_1 \\ +1 \end{pmatrix}x_1 + \begin{pmatrix} -a_2 \\ -1 \end{pmatrix}x_2 \\ -c_3x_2^2 + c_4x_2^2 sign\begin{pmatrix} e_1p_{12} \\ +e_2p_{22} \end{pmatrix} \end{bmatrix} \quad \textbf{(42)}$$

Then $$M = \begin{pmatrix} e_1p_{11} \\ +e_2p_{12} \end{pmatrix} \times \left[\begin{pmatrix} c_1 \\ -c_2 sign\begin{pmatrix} e_1p_{12} \\ +e_2p_{22} \end{pmatrix} \end{pmatrix}x_1^2\right] + \begin{pmatrix} e_1p_{12} \\ +e_2p_{22} \end{pmatrix}\left[\begin{pmatrix} c_3 \\ -c_4 sign\begin{pmatrix} e_1p_{12} \\ +e_2p_{22} \end{pmatrix} \end{pmatrix}x_2^2 + \begin{pmatrix} k_dx_1^2 \\ +k_bx_2^2 \end{pmatrix}^2\right] \quad \textbf{(43)}$$

By substituting Equation (43) in Equation (36) gives:

$$\dot{V}(e) = -eQe^T + 2\begin{bmatrix} \begin{pmatrix} e_1p_{11} \\ +e_2p_{12} \end{pmatrix}\left[\begin{pmatrix} c_1 \\ -c_2 sign\begin{pmatrix} e_1p_{12} \\ +e_2p_{22} \end{pmatrix} \end{pmatrix}x_1^2\right] + \\ \begin{pmatrix} e_1p_{12} \\ +e_2p_{22} \end{pmatrix}\left[\begin{pmatrix} c_3 \\ -c_4 sign\begin{pmatrix} e_1p_{12} \\ +e_2p_{22} \end{pmatrix} \end{pmatrix}x_2^2 + \begin{pmatrix} k_dx_1^2 \\ +k_bx_2^2 \end{pmatrix}^2\right] \end{bmatrix} \quad \textbf{(44)}$$

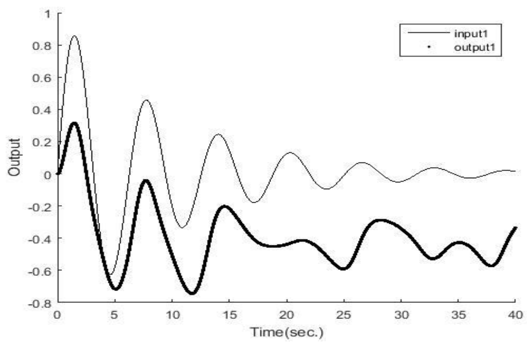
It is obvious that the time-derivative function is indefinite function. Therefore, according to the Theorems 1 and 2 if $\dot{V}(e) < 0$ then the MIMO system is asymptotically stable. Moreover, , the integral square error performance index (ISE) is used. It is expressed as [34]:

$$J = \int_0^t e^2(t)\, dt \quad \textbf{(45)}$$

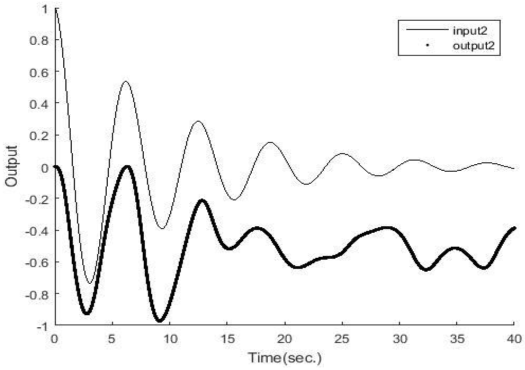
where $e(t)$represents the difference between the output of the desired model and the output of the actual system.

4. Simulation Results

This section presents the simulation results of the MIMO system with and without the proposed controller. Figures 4 to 6 show the closed loop time response properties of the system $(y_1(t), y_2(t))$, the system state trajectories $(x_1(t), x_2(t))$ and the phase-plane trajectory of the system respectively; before applying the controller.



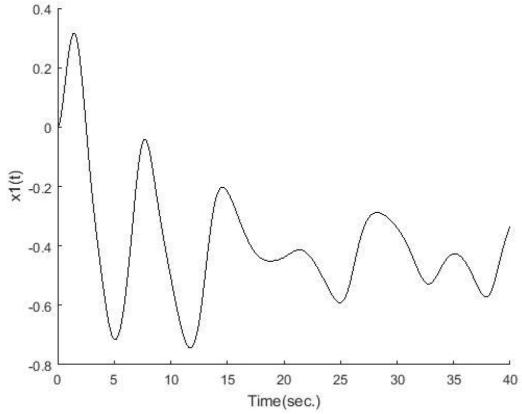
(a)



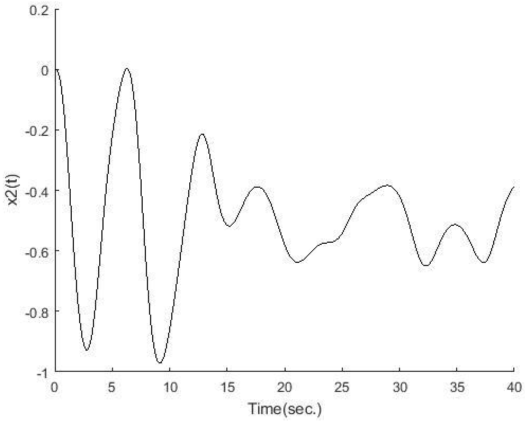
(b)

Figure 4 (a) and (b) are the closed loop time response of $y_1(t),\ y_2(t)$.

Figure 4 that the system tracking between the output and input signals are unstable and has bad performance corresponding to the applied input $r = e^{-0.1t}sin(t)$. This result is due to nonlinearity effects and parametric time varying uncertainties.



(a)



(b)

Figure 5 (a) and (b) are system state trajectories.

Figure 5 explains the system state trajectories due to the instability and the fast time-varying of the MIMO system.

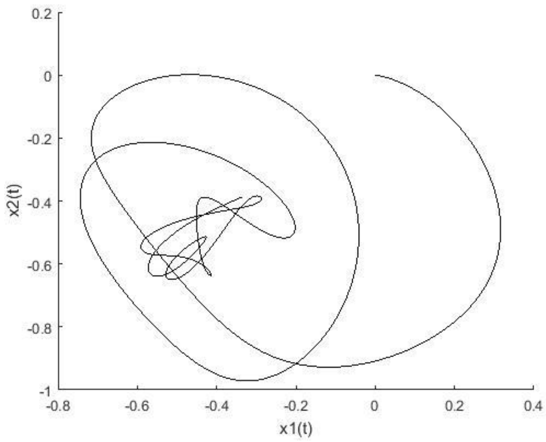
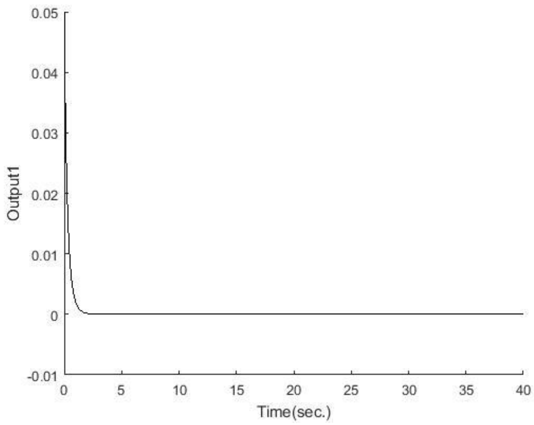
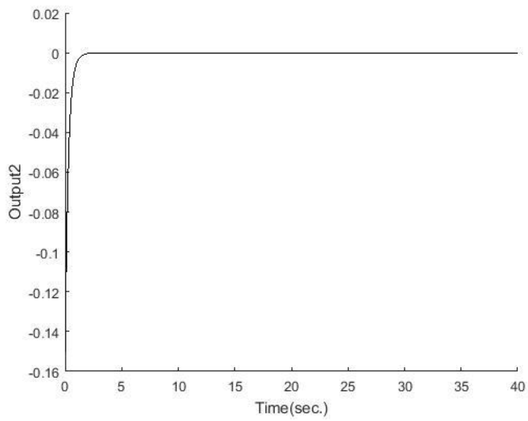


Figure 6 The phase-plane trajectory

The phase-plane in Figure 6 confirms that the system is unstable because the trajectory starts at the origin zero and changing in unstable way.



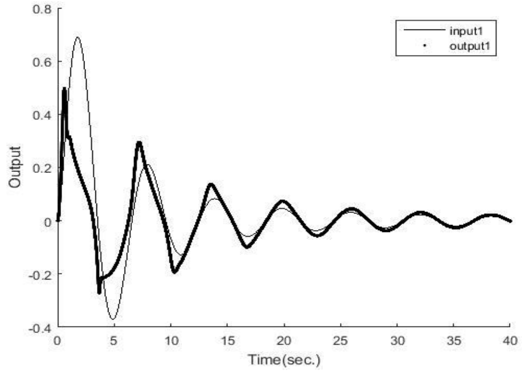
(a)



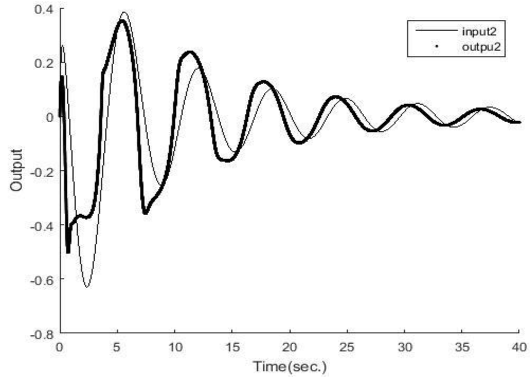
(b)

Figure 7 (a) and (b) are the stability properties of $x_1(t), x_2(t)$

Figure 7 explains the stabilization properties of the MIMO system state trajectories and proved that the proposed controller can stabilize the system efficiently.



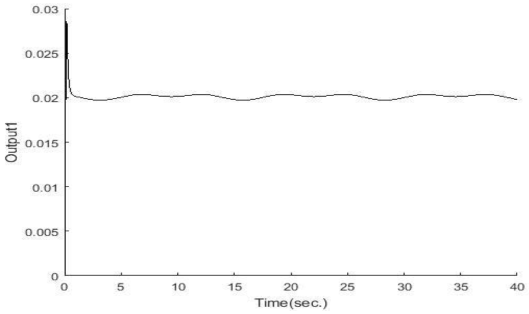
(a)



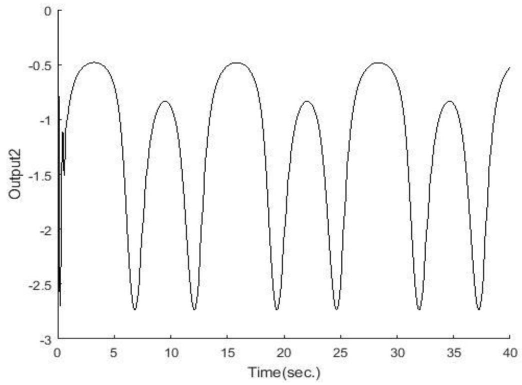
(b)

Figure 8 (a) and (b) are the time response properties of $y_1(t),\ y_2(t)$.

It can be seen in Figure 8 the effectiveness of the controller throughout the ability to compensate the MIMO system under the effects of the parametric time-varying uncertainties.



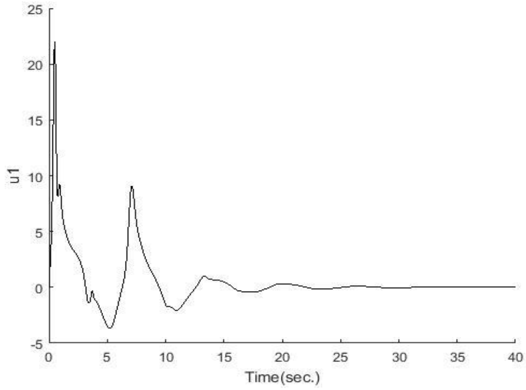
(a)



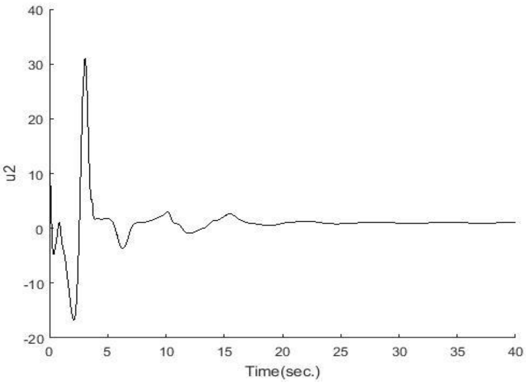
(b)

Figure 9: (a) and (b) are the coupling properties of $y_1(t), y_2(t)$

Figure 9 shows the efficiency/robustness of the controller by the ability to decoupling the MIMO system under the effects of the uncertainties.



(a)



(b)

Figure 10: (a) and (b) are the resulting control action $u_1(t), u_2(t)$

Figure 10 explains the control actions $(u_1(t), u_2(t))$. The control signal $u_1(t)$ jumped at value 22 and then decreased rapidly after 3 *sec*. The other control signal $u_2(t)$ jumped between -17 and 31 then decreased during the first 5 *sec*. These control signals explained the first 5 *sec*. in the unmatched tracking part of the output responses in Figure 8. After that, the tracking is achieved due to the effectiveness of the control actions. Also, they are preferable for many applicable MIMO systems. Moreover, the chattering parts that supposed to be appear in the controller actions due to $sign(\cdot)$ function has been treated with the help of the optimization method.

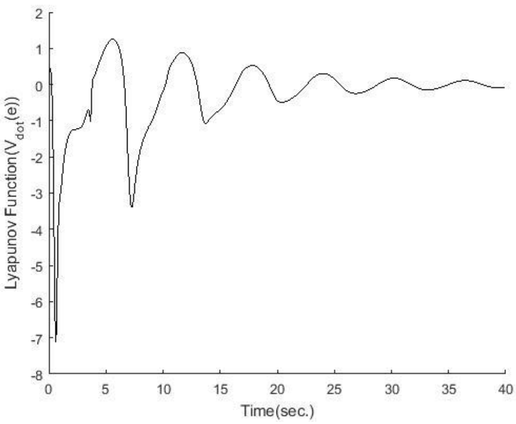


Figure 11: The time derivative of the LQF.

It can be shown in Figure 11 the controlled system becomes asymptotically stable (Theorems 1 and 2) despite of the indefinite property of the time-derivative LQF.

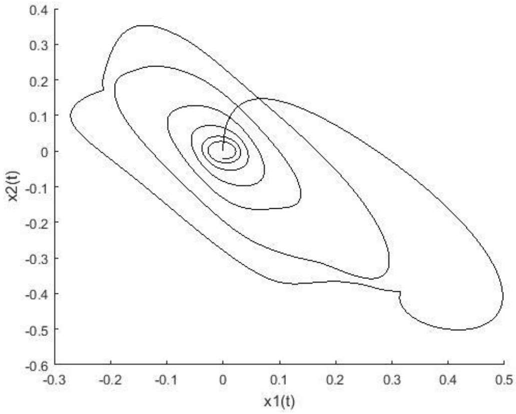


Figure 12: The phase-plane trajectory of the system.

Figure 12 clarifies that the phase-plane trajectory is led to zero due to the proposed controller which indicates that the system is stabilized. The following table contains the optimization settings and the resulting optimal parameters. Eventually, it was observed that the settings of MVPA were adequate for this application. In particular, when the number of iteration is equal to 9; MVPA achieves the best minimization of the cost function. Also it is found that the increasing the number of iteration did not improve the convergence properties. The following table contains the optimization settings and the resulting optimal parameters.

Table 1: MVPA Optimization Settings, Optimal Parameters and Model Reference Parameters

Parameters	Optimal Values	Parameters	Optimal Values
Lower bound	9.9×10^{-9}	k_3	0.0142042
Upper bound	0.2089899	k_4	0.0386814
Problem dims.	11	k_a	0.1083136
No. of pop.	20	k_b	0.18760542
No. iterations	30	k_c	0.17651952
P_{11}	0.1845522	k_d	0.16064578
P_{12}	0.0254792	a_1	2.47055964
P_{22}	0.0606605	a_2	9.79999218
k_1	0.0130289	b_1	6.20283109
k_2	0.0299971	b_2	0.81833922

5. Conclusion

In this paper, a new robust control algorithm is proposed for a class of nonlinear MIMO systems with time-varying parametric uncertainty. The stability of the indefinite time-derivative Lyapunov function is achieved successively. After using MVPA, the optimal parameters have been obtained for both MIMO model reference and the constructed controller. The proposed algorithm has effectively decoupled of the nonlinear MIMO system. The proposed robust control design can be extended to solve many problems in the nonlinear systems.

References

[1] Marino, R., and P. Tomei. "Adaptive Control of Linear Time-Varying Systems." *Automatica* 39, no. 4 (2003):

651–59. https://doi.org/10.1016/s0005-1098(02)00287-x.

[2] Song, Mingchao, Yan Lin, and Ran Huang. "Robust Adaptive Dynamic Surface Control for Linear Time-Varying Systems." *International Journal of Adaptive Control and Signal Processing* 28, no. 10 (2013): 932–48. https://doi.org/10.1002/acs.2418.

[3] Han, Qing-Long. "Robust Stability for a Class of Linear Systems with Time-Varying Delay and Nonlinear Perturbations." *Computers & Mathematics with Applications*, vol. 47, no. 8–9, Apr. 2004, pp. 1201–1209, 10.1016/s0898-1221(04)90114-9.

[4] Fan Wang, and V. Balakrishnan. "Robust Kalman Filters for Linear Time-Varying Systems with Stochastic Parametric Uncertainties." *IEEE Transactions on Signal Processing*, vol. 50, no. 4, Apr. 2002, pp. 803–813, 10.1109/78.992124.

[5] Nguyen, Dinh Hoa, and David Banjerdpongchai. "A Convex Optimization Approach to Robust Iterative Learning Control for Linear Systems with Time-Varying Parametric Uncertainties." *Automatica*, vol. 47, no. 9, Sept. 2011, pp. 2039–2043, 10.1016/j.automatica.2011. 05.022.

[6] Boulkroune, A., et al. "Adaptive Fuzzy Tracking Control for a Class of MIMO Nonaffine Uncertain Systems." *Neurocomputing*, vol. 93, Sept. 2012, pp. 48–55, 10.1016/j.neucom.2012.04.006.

[7] Wang, J., and Z. Qu. "Robust Adaptive Control of a Class of Nonlinearly Parameterised Time-Varying Uncertain Systems." *IET Control Theory & Applications*, vol. 3, no. 6, 1 June 2009, pp. 617–630, 10.1049/iet-cta.2008.0163.

[8] Du, Jialu, et al. "Adaptive Control for a Class of Time-Varying Uncertain Nonlinear Systems." *2006 6th World Congress on Intelligent Control and Automation*, 2006, 10.1109/wcica.2006.1712509.

[9] Wang, Chunxiao, et al. "TABLF-Based Adaptive Control for Uncertain Nonlinear Systems with Time-Varying Asymmetric Full-State Constraints." *International Journal of Control*, 9 July 2019, pp. 1–9, 10.1080/00207179.2019.1639825.

[10] Xu, Jian-Xin, and Ying Tan. "A Suboptimal Learning Control Scheme for Non-Linear Systems with Time-Varying Parametric Uncertainties." *Optimal Control Applications and Methods*, vol. 22, no. 3, 2001, pp. 111–126, 10.1002/oca.687.

[11] Dong, Liang, and Wen Cheng Tang. "Adaptive Backstepping Sliding Mode Control of Flexible Ball Screw Drives with Time-Varying Parametric Uncertainties and Disturbances." *ISA Transactions*, vol. 53, no. 1, Jan. 2014, pp. 110–116, 10.1016/j.isatra.2013 .08.009.

[12] Wu, Huai-Ning. "Robust H2 Fuzzy Output Feedback Control for Discrete-Time Nonlinear Systems with Parametric Uncertainties." *International Journal of Approximate Reasoning*, vol. 46, no. 1, Sept. 2007, pp. 151–165, 10.1016/j.ijar.2006.11.002. Accessed

[13] Gao, Y., et al. "Robust H∞ Control for Constrained Discrete-Time Piecewise Affine Systems with Time-Varying Parametric Uncertainties." *IET Control Theory & Applications*, vol. 3, no. 8, 1 Aug. 2009, pp. 1132–1144, 10.1049/iet-cta.2008.0182.

[14] Cui, Yang, et al. "Robust Adaptive NN Control for a Class of Uncertain Discrete-Time Nonlinear MIMO Systems." *Neural Computing and Applications*, vol. 22, no. 3–4, 7 Dec. 2011, pp. 747–754, 10.1007/s00521-011-0766-4.

[15] Xu, Jian-Xin, and Jing Xu. "Observer Based Learning Control for a Class of Nonlinear Systems With Time-Varying Parametric Uncertainties." *IEEE Transactions on Automatic Control*, vol. 49, no. 2, Feb. 2004, pp. 275–281, 10.1109/tac.2003.822874.

[16] Zhang, Y.A., et al. "Robust Adaptive Sliding Mode Control Using Fuzzy Modelling for a Class of Uncertain MIMO Nonlinear Systems." *IEE Proceedings - Control Theory and Applications*, vol. 151, no. 4, 1 July 2004, pp. 522–524, 10.1049/ip-cta:20040439.

[17] Mou Chen, et al. "Robust Adaptive Neural Network Control for a Class of Uncertain MIMO Nonlinear Systems With Input Nonlinearities." *IEEE Transactions on Neural Networks*, vol. 21, no. 5, May 2010, pp. 796–812, 10.1109/tnn.2010.2042611.

[18] Yu, Yao, et al. "Robust-Decentralized Tracking Control for a Class of Uncertain MIMO Nonlinear Systems with Time-Varying Delays." *International Journal of Robust and Nonlinear Control*, vol. 24, no. 18, 27 Aug. 2013, pp. 3474–3490, 10.1002/rnc.3068.

[19] Chang, Yeong-Chan. "Robust Tracking Control for Nonlinear MIMO Systems via Fuzzy Approaches." *Automatica*, vol. 36, no. 10, Oct. 2000, pp. 1535–1545, 10.1016/s0005-1098(00)00083-2.

[20] Aksu, Inayet Ozge, and Ramazan Coban. "Sliding Mode PI Control with Backstepping Approach for MIMO Nonlinear Cross-coupled Tank Systems." *International Journal of Robust and Nonlinear Control*, vol. 29, no. 6, 16 Jan. 2019, pp. 1854–1871, 10.1002/rnc.4469.

[21] Zhang, Huaguang, and Zeungnam Bien. "Adaptive Fuzzy Control of MIMO Nonlinear Systems." *Fuzzy Sets and Systems*, vol. 115, no. 2, Oct. 2000, pp. 191–204, 10.1016/s0165-0114(98)00394-7.

[22] Sam Ge, S., and C. Wang. "Adaptive Neural Control of Uncertain MIMO Nonlinear Systems." *IEEE Transactions on Neural Networks*, vol. 15, no. 3, May 2004, pp. 674–692, 10.1109/tnn.2004.826130.

[23] Belikov, Juri, and Eduard Petlenkov. "Model Reference Control of Nonlinear MIMO Systems by Dynamic Output Feedback Linearization of ANARX Models." *2009 IEEE International Conference on Control and Automation*, Dec. 2009, 10.1109/icca.2009.5410177.

[24] Kurek, Jerzy E. "Model Reference Control for a Class of MIMO System with Dead-Time." *Advances in Intelligent Systems and Computing*, 19 Aug. 2017, pp. 436–443, 10.1007/978-3-319-65960-2_54.

[25] Shotorbani, Amin, and Ebrahim Babaei. "Robust Nonlinear Controller Based on Control Lyapunov Function and Terminal Sliding Mode for Buck Converter." *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, vol. 29, no. 6, 12 Apr. 2016, pp. 1055–1069, 10.1002/jnm.2164.

[26] Zhen, Hong-tao, et al. "Neural NetworkL1Adaptive Control of MIMO Systems with Nonlinear Uncertainty." *The Scientific World Journal*, vol. 2014, 2014, pp. 1–8, 10.1155/2014/942094.

[27] Bidikli, Baris, et al. "An Asymptotically Stable Robust Controller Formulation for a Class of MIMO Nonlinear Systems with Uncertain Dynamics." *International Journal of Systems Science*, vol. 47, no. 12, 6 May 2015, pp. 2913–2924, 10.1080/00207721.2015.1039627.

[28] O'Connor, William J., and David J. McKeown. "Time-Optimal Control of Flexible Robots Made Robust Through Wave-Based Feedback." *Journal of Dynamic Systems, Measurement, and Control*, vol. 133, no. 1, 29 Nov. 2010, 10.1115/1.4002714.

[29] Mahmood, Maaz, and Prashant Mhaskar. "Constrained Control Lyapunov Function Based Model Predictive Control Design." *International Journal of Robust and Nonlinear Control*, vol. 24, no. 2, 31 Aug. 2012, pp. 374–388, 10.1002/rnc.2896.

[30] Bouchekara, H. (2017). Most Valuable Player Algorithm: a novel optimization algorithm inspired from sport. *Operational Research*.

[31] Wu, Zhongle, et al. "Adaptive Tracking for MIMO Nonlinear Systems with Unknown Fast Time-Varying Parameters." *International Journal of Robust and Nonlinear Control*, vol. 28, no. 13, 27 Apr. 2018, pp. 4058–4074, 10.1002/rnc.4121.

[32] Zhou, Bin. "Stability Analysis of Non-Linear Time-Varying Systems by Lyapunov Functions with Indefinite Derivatives." *IET Control Theory & Applications*, vol. 11, no. 9, 2 June 2017, pp. 1434–1442, 10.1049/iet-cta.2016.1538. Accessed 20 Feb. 2020.

[33] Karafyllis, I., and J. Tsinias. "A Converse Lyapunov Theorem for Nonuniform in Time Global Asymptotic Stability and Its Application to Feedback Stabilization." *SIAM Journal on Control and Optimization*, vol. 42, no. 3, Jan. 2003, pp. 936–965, 10.1137/s0363012901392967. Accessed 20 Feb. 2020.

[34] Rahimian, M. and Tavazoei, M. (2013). Improving integral square error performance with implementable fractional-order PI controllers. *Optimal Control Applications and Methods*, 35(3), pp.303-323.

Author Profile

Musadaq A. Hadi is graduated from the department of Control and Systems Engineering, University of Technology, Baghdad, Iraq in 2014. His research interests include optimal control, robust control, automatic control design, and nonlinear control and optimization methods. (Corresponding author).
ORCID iD: 0000-0002-3884-495X.

Hazem I. Ali is graduated from the department of Control and Systems Engineering, University of Technology, Baghdad, Iraq in 1997. He obtained the MSc degree in Mechatronic Engineering from University of Technology, Baghdad, Iraq in 2000 and the PhD in Control and Automation from the department of Electrical and Electronic Engineering, University Putra Malaysia, Malaysia in 2010. Currently, he is a Professor in Control and Systems Engineering Department, University of Technology, Baghdad, Iraq. His research interests include robust control, intelligent control and optimization techniques. He is a member of IEEE and IAENG.