

Particle Swarm Optimization Design of Digital Filters

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Abstract: Aiming at the need of designing digital filters with finite length impulse response, a method of designing digital filters with particle swarm optimization is proposed. The particle swarm optimization algorithm is used to optimize the filter coefficients. Based on the mean square error minimization criterion and the maximum error minimization criterion, the optimization objective function is given, and the realization steps of the particle swarm optimization algorithm are designed. The program is used for simulation design, and the low-pass filter obtained by the optimized design meets the design requirements, indicating that the digital filter design based on particle swarm optimization is an effective design method.

Keywords: Particle swarm optimization; Digital filter; Coefficient optimization; Objective function.

1. Introduction

Digital signal filters have always played an irreplaceable role in modern signal acquisition and processing systems. As the units with the most basic functions, they have been widely researched and applied in the field of communication systems and automatic control. According to the unit impulse response, digital filters can be divided into finite-length impulse response (Finite Impulse Response, FIR) digital filters and infinite-length impulse response (Infinite Impulse Response, IIR) digital filters. Digital filter design needs to be able to meet multiple technical indicators at the same time, and the design workload is large and complicated. In the actual application process, it needs to be close to the specific technical index requirements.

The digital signal filter is actually an algorithm that can change the relative proportion of the frequency components contained in the input signal, or filter out certain frequency components through a certain mathematical relationship. In the early days, the computational cost of designing FIR digital filters with global optimization algorithms was too large [1], and it was not easy to find the optimal solution using local neighborhood search technology [2]. Since then, more evolutionary algorithms have been used in the design of FIR filters, including simulated annealing algorithm [3], genetic algorithm [4], evolutionary programming method [5], immune algorithm [6], ant colony algorithm [7], Neural network algorithm [8] and so on.

This paper designs FIR digital filter based on particle swarm optimization. Combine the particle swarm optimization algorithm with the design requirements of the digital filter parameters, use the mean square error minimization criterion and the maximum error minimization criterion to construct a fitness function, and use the particle swarm optimization algorithm to design the digital filter parameters to obtain optimization digital filter. The method in the paper is used to simulate the low-pass filter, which proves that the proposed method can meet the design requirements.

2. Analysis method

2.1 Optimal model of FIR digital filter

The amplitude-frequency characteristic of N -order linear phase filter is expressed as

$$H(\omega) = \sum_{n=0}^{\frac{N-1}{2}} a_n \cos(\omega n) \quad (1)$$

Where H is the frequency response of the filter. ω is the real frequency. N is the length of the time series. n is the discrete time. And a_n is the filter coefficient, expressed as

$$\begin{cases} a_0 = h\left(\frac{N-1}{2}\right) \\ a_n = 2h\left(\frac{N-1}{2} - n\right) \quad n = 1, 2, \dots, \frac{N-1}{2} \end{cases} \quad (2)$$

Here, $h(n)$ is the impulse response of the filter. The main goal of designing a FIR digital filter is to design sequence a_0, a_1, \dots, a_n , that is, the optimization target is the filter coefficient sequence a_0, a_1, \dots, a_n .

The premise of FIR digital filter design is to determine the optimal criterion. In this paper, the criterion of minimum mean square error or maximum error is selected as the design criterion.

(1) Minimum square error criterion

The optimization objective function is the root mean square error of the amplitude response $|H(\omega)|$ of the filter to be designed and the expected amplitude response $|H_d(\omega)|$

$$E(\omega) = \sum_{i=1}^M (|H_d(\omega)| - |H(\omega)|)^2 \quad (3)$$

The optimization criterion is to make the root mean square error $E(\omega)$ as small as possible.

(2) Maximum error minimization criterion

The optimization objective function is the maximum error between the amplitude response $|H(\omega)|$ of the filter to be designed and the expected amplitude response $|H_d(\omega)|$

$$E(\omega) = W(\omega)[H_d(\omega) - H(\omega)] \tag{4}$$

The optimization criterion is to make the maximum error as small as possible.

Therefore, the optimal objective function of FIR digital filter optimization design is

$$\min_{a(n)} [E(\omega)] \tag{5}$$

Among them, $E(\omega)$ can choose the mean square error or the maximum error according to the design requirements.

2.2 Realization of particle swarm algorithm for digital filter design

In the FIR digital filter design process, the position A of the particle swarm represents the optimization variable a_0, a_1, \dots, a_n , and the fitness evaluation function E of the particle swarm is the optimization goal $\min_{a(n)} [E(\omega)]$. Then the particle coding structure is represented by the particle's current position A , particle velocity V , and fitness value E .

For $N-1$ -order filters, the particle structure is expressed as

$$\begin{cases} A = \{a_0, a_1, \dots, a_{(N-1)/2}\} \\ V = \{v_0, v_1, \dots, v_{(N-1)/2}\} \\ E(A) = \sum_{i=1}^M (|H_d(\omega) - |H(\omega)|)^2 \end{cases} \tag{6}$$

or

$$\begin{cases} A = \{a_0, a_1, \dots, a_{(N-1)/2}\} \\ V = \{v_0, v_1, \dots, v_{(N-1)/2}\} \\ E(A) = W(\omega)[H_d(\omega) - H(\omega)] \end{cases} \tag{7}$$

The program design flow chart is shown in Figure 1. The steps are:

- 1) Build a digital filter optimization model, given an ideal filter model, determine the digital filter parameters, such as design stage, sampling frequency, etc., and build a minimum optimization objective function.
- 2) Build a particle swarm optimization algorithm model to complete the initialization of parameters, such as population size, spatial dimension, position value range, speed value range, inertia weight, learning factor, number of iterations, etc.
- 3) Particle swarm initialization, including random initialization of particle position and velocity.
- 4) Calculate the fitness of particles based on the optimization objective function.
- 5) Update the individual and global extreme values of the particles.
- 6) Update the particle position and speed.
- 7) Repeat steps (4)~(6) until the iteration termination condition is satisfied.
- 8) Output the global extremum to obtain the digital filter coefficient a_0, a_1, \dots, a_n .

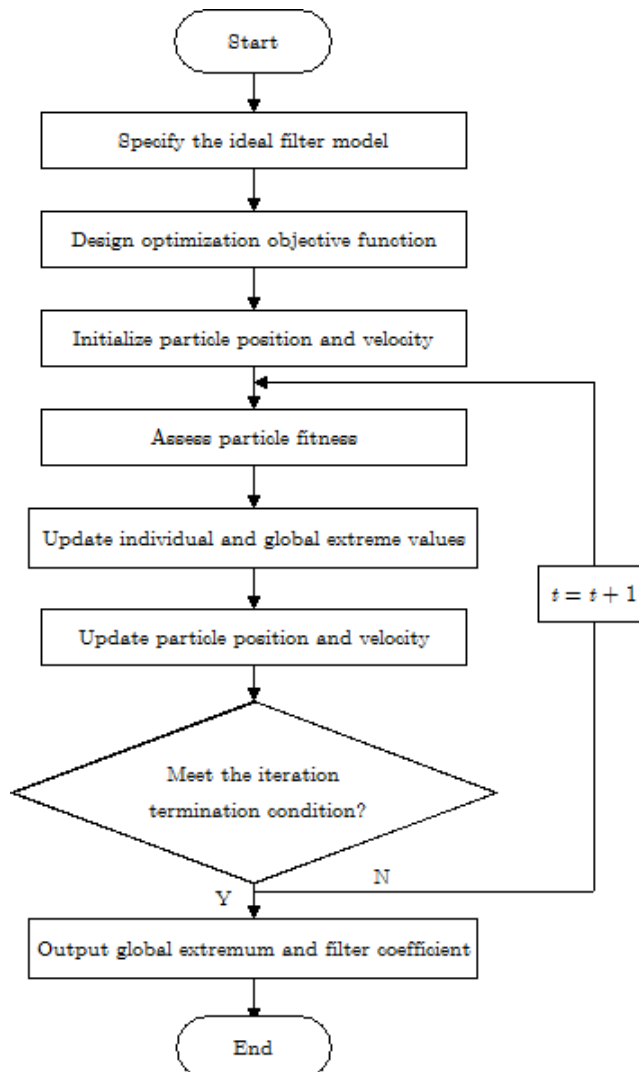


Figure 1: Digital filter design flow chart

3. Calculation and analysis

Design a low-pass filter with order 30, the technical specifications are as follows

$$H_d(e^{j\omega}) = \begin{cases} 1 & \omega \leq 0.5\pi \\ 0 & 0.5\pi \leq \omega \leq \pi \end{cases} \tag{8}$$

In the design process, ω is evenly sampled, and point $M=100$. Find the ideal filter characteristic $H_d(\omega)$ according to ω_i , and substitute the filter coefficient a_n into formula (1) to get the design characteristic $H(\omega)$. Take the error between design characteristics and ideal characteristics as the optimization goal $E(\omega)$. In the particle swarm optimization algorithm, the number of populations $N=50$ and particle dimensions $D = \frac{N+1}{2}$. Inertial weight $\omega=0.5$, self-learning factor $c_1=3$, group learning factor $c_2=3$. The particle position value range is $-1 \leq a_i \leq 1$, and the particle velocity value range is $-0.2 \leq v_i \leq 0.2$. The maximum number of iterations $iter_{max} = 5000$.

Using the mean square error minimization as the optimization criterion, the convergence process of the optimization goal is shown in Figure 3, and the comparison between the ideal filter

and the designed filter is shown in Figure 4. Using maximum error minimization as the optimization criterion, the convergence process of the optimization goal is shown in Figure 5, and the comparison between the ideal filter and the designed filter is shown in Figure 6. The filter coefficient $A = \{a_0, a_1, \dots, a_{(N-1)/2}\}$ and the corresponding error $E(A)$ designed with two design criteria are shown in Table 1.

It can be seen from Figure 2 and Figure 4 that the particle swarm optimization-based algorithm can quickly converge in the direction of minimizing the objective function until the iteration conditions are satisfied to obtain the global best. It can be seen from Figure 3 and Figure 5 that the characteristics of the designed filter and the expected filter are basically the same and meet the design requirements. Comparing Figure 3 and Figure 5, it is found that the filter designed based on the maximum error minimization criterion has a smaller deviation from the ideal filter compared to the filter designed based on the mean square error minimization. It can be seen that based on particle swarm optimization algorithm can obtain better filter coefficients, the characteristics of the designed filter and the expected filter are basically the same, to meet the design requirements.

4. Conclusion

In this paper, the particle swarm optimization algorithm is used to optimize the design of the FIR digital filter. The mathematical optimization model, basic design criteria, and design steps of the particle swarm optimization digital filter are studied. Given the design parameters of the low-pass filter, the proposed method is used to complete the simulation design, and the amplitude-frequency response of the designed filter and the ideal filter are compared. The experimental results show that the optimization by the particle swarm optimization algorithm, the designed filter has basically the same characteristics as the expected filter, and meets the design index requirements.

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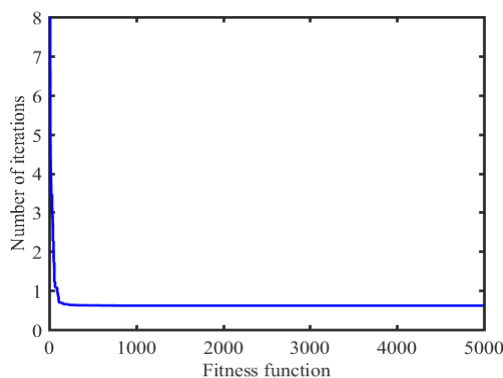


Figure 2: Convergence process of optimization target based on mean square error minimization design

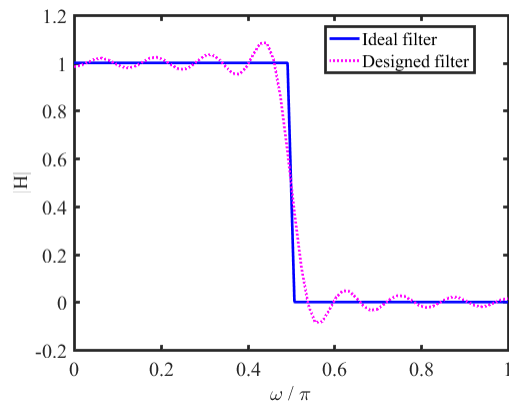


Figure 3: Design result of low-pass digital filter based on mean square error minimization design

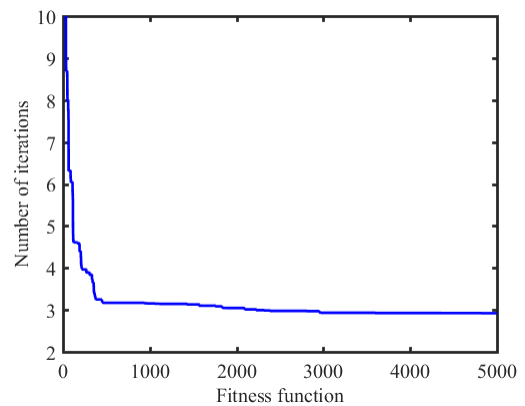


Figure 4: Convergence process of optimization target based on maximum error minimization design

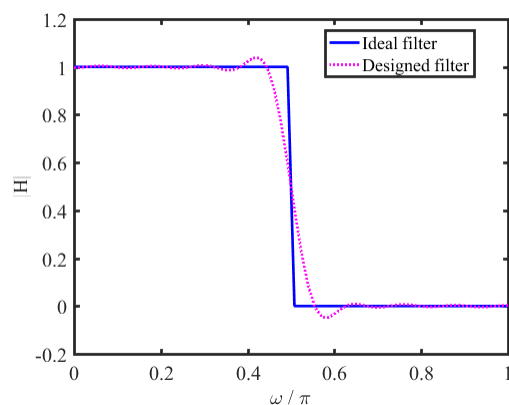


Figure 5: Design result of low-pass digital filter based on maximum error minimization design

Table 1: Coefficient and error of the designed filter

	Mean square error minimization design	Maximum error minimization design
a_0	0.49993	0.49907
a_1	0.6369	0.63546
a_2	3.3311e-05	0.0013369
a_3	-0.2119	-0.20905
a_4	-2.5153e-05	-0.00065224
a_5	0.12777	0.12252
a_6	0.00013972	-0.0006381
a_7	-0.090792	-0.082021
a_8	-3.9117e-05	0.0016413
a_9	0.071427	0.056837

a_{10}	5.2484e-05	-0.001223
a_{11}	-0.057715	-0.03864
a_{12}	-2.5185e-05	0.00067081
a_{13}	0.049643	0.023898
a_{14}	2.4611e-05	-0.00047438
a_{15}	-0.04256	-0.012472
$E(A)$	0.61294	2.9206

interests include antenna and radome design, electromagnetic scattering analysis and intelligent optimization algorithm.

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Author Profile



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