Dualistic Wave: A Proposed Explanation of Zeno's Paradox

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Abstract: ***In this paper, a novel geometric object named dualistic wave is established and employed to solve the dilemma of Zeno's paradox. In one period, a dualistic wave is composed of four curved infinitesimal line segments and each of them is equivalent to a flat infinitesimal line segment. A dualistic wave can be viewed as an infinitely small wavelike space or an infinitely small flat space; they are equivalent. Intuitively, the ingredient of a straight line is obviously an infinitesimal straight line segment. However, it is shown that the ingredient can also be viewed as an infinitesimal curved line segment. An infinitesimal line segment is classified into two categories: flat infinitesimal line segment and curved infinitesimal line segment. The equivalence between a flat infinitesimal line segment and a curved infinitesimal line segment is demonstrated. This is based on the concept of equivalent infinitesimal in calculus. For example, the limit*** $\lim_{x\to0}\frac{\sin(x)}{x}=1$ ***indicates that the curved infinitesimal line segment described by the equation*** $y=\sin(x)$ $(0\le x\le L,\ L\to0)$ ***is equivalent to the flat infinitesimal line segment described by the equation*** $y=x$ $(0\le x\le L,\ L\to0)$. ***The mathematical equations describing dualistic waves constructed by different curved infinitesimal line segments are given.***

Keywords: Zeno paradox; dualistic wave; flat infinitesimal line segment; curved infinitesimal line segment

1. Introduction

Zeno's paradox [1-2] alerts mathematicians and scientists to rethink the nature of space and time. Many mathematical solutions to this paradox including the use of infinite series seemed not to touch the core point Zeno was concerned of , and it was pointed out that any solution to 'hit the point' of Zeno paradox must also make metaphysical sense [2].

In this paper, the proposed solution is based on the point that if we follow the line of thinking that insists the ingredient of a straight line is an infinitely small straight line segment, then we would end up the conclusion that Achilles can never catch the tortoise if the start position of the tortoise is ahead of that of Achilles. In mathematics, an infinitesimal [3-4] is the ingredient of any geometric object such as a straight line and a curve, and an infinitesimal is defined as an entity so small that is impossible to be measured. In this paper the classification of infinitesimal is conducted. It is stated that there are two kinds of infinitesimal line segments: flat infinitesimal line segment and curved infinitesimal line segment. Based on the concept of curved infinitesimal line segment, a geometric object called dualistic wave is built up, which shows the nature of space-time at infinitely small scale. The duality of this nature gives rise to a solution to Zeno's paradox.

2. Flat and Curved Infinitesimal Line Segments

In Cartesian coordinate, the mathematical equation $y=x$ represents a straight line (which is flat), so the equation $y=x$ $(0\le x\le L,\ L\to0)$ represents a flat infinitesimal line segment. And the mathematical equation $y=\sin(x)$ represents a wave (which is curved), so the equation $y=\sin(x)$ $(0\le x\le L,\ L\to0)$ represents a curved infinitesimal line segment. There are many curved infinitesimal line segments such as $y=\tan(x)$ $(0\le x\le L,\ L\to0)$, and $y=\ln(x+1)$ $(0\le x\le L,\ L\to0)$, and they are listed in Appendix.

3. Dualistic Wave

Generally, if $y=f(x)$ is a nonlinear function and $\lim_{x\to0}\frac{f(x)}{x}=1$, then we can use the function $y=f(x)$ to construct a dualistic wave. Figure 1 illustrates this idea.

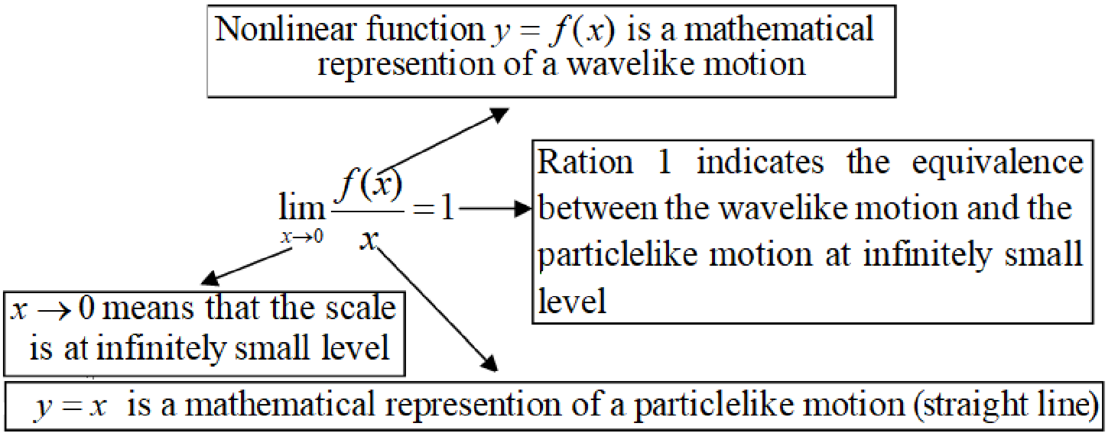

Figure 1: Mathematical description of wave-particle duality based on equivalent infinitesimals

A specific example is that we can use the nonlinear function $y = \sin(x)$ to construct a dualistic wave because $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$. Figure 2 shows the equivalence between the infinitely small curved line segment $y = \sin(x)$ $0 \le x \le L, L \to 0$ and the infinitely small straight line segment $y = x$ $0 \le x \le L, L \to 0$.

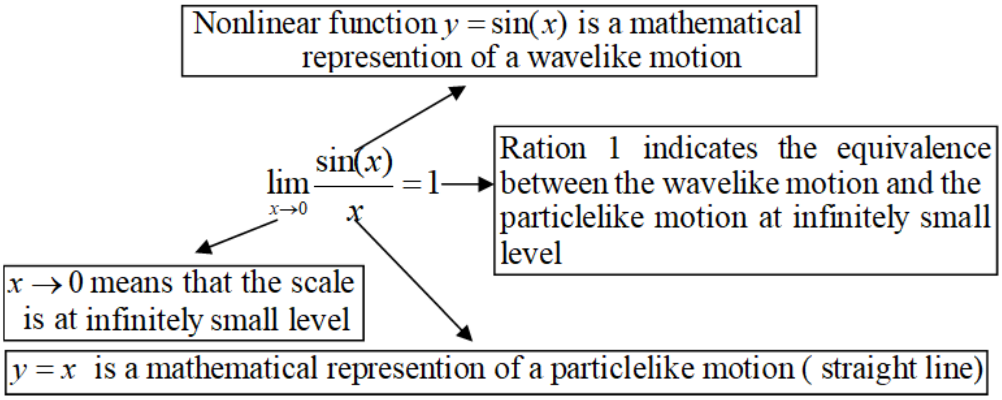

Figure 2: Mathematical description of wave-particle duality based on $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

In Figure 3, the dualistic wave built upon $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$ is plotted and eqs (1) is the system of equations describing it.

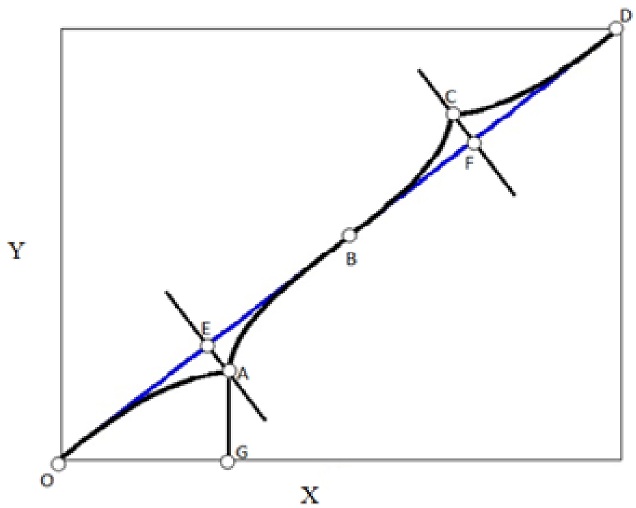

Figure 3: Dualistic wave constructed by $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

$$\begin{cases} y=\sin(x) & 0\le x<x_0 \\ \sin(-y+\frac{\sqrt{2}}{4}\lambda)=-x+\frac{\sqrt{2}}{4}\lambda & x_0\le x<\frac{\sqrt{2}}{4}\lambda \\ x-\frac{\sqrt{2}}{4}\lambda=\sin(y-\frac{\sqrt{2}}{4}\lambda) & \frac{\sqrt{2}}{4}\lambda\le x<\frac{\sqrt{2}}{4}\lambda+\sin(x_0) \\ -y+2x_0+\frac{\sqrt{2}}{4}\lambda=\sin(-x+2x_0+\frac{\sqrt{2}}{4}\lambda) & \frac{\sqrt{2}}{4}\lambda+\sin(x_0)\le x\le\lambda \end{cases} \quad (1)$$

where x_0 is the root of the equation: $\sin(x)+x=\frac{\sqrt{2}}{4}\lambda$, λ is the wavelength. The following is the derivation of the above eqs (1).

As to the curved line segment OA in figure 3, the following is the mathematical equation describing it

$$y=\sin(x) \quad (2)$$

To find the mathematical equation describing the curved line segment AB in figure 3, we notice that AB and OA are symmetric about the straight line AE, and the point $E=(\frac{\sqrt{2}}{8}\lambda, \frac{\sqrt{2}}{8}\lambda)$, λ is the wavelength of the dualistic wave. The coordinates of point $A(x_0, y_0)$ can be obtained by considering that it is the cross point of the straight line AE and the curved line OA.

The mathematical equation for the straight line AE is $(y-\frac{\sqrt{2}}{8}\lambda)=-1\cdot(x-\frac{\sqrt{2}}{8}\lambda)$ which is equal to the following:

$$y=-x+\frac{\sqrt{2}}{4}\lambda \quad (3)$$

Plugging the coordinates of point A into eq (2) and eq (3) results in:

$$\sin(x_0)+x_0=\frac{\sqrt{2}}{4}\lambda \quad (4)$$

So x_0 is the root of the equation $\sin(x)+x=\frac{\sqrt{2}}{4}\lambda$.

Point $A=(x_0, \sin(x_0))$.

Consider that there are two points of $P(x,y)$ and $P'(x',y')$ such that they are symmetric about the straight line AE, and $P(x,y)$ is on the curved line segment AB and $P'(x',y')$ is on the curved line segment OA. This leads to that the midpoint of the straight line segment PP' is on the straight line AE.

$$\frac{y+y'}{2}=-\frac{x+x'}{2}+\frac{\sqrt{2}}{4}\lambda \quad (5)$$

That PP' is parallel to OD gives rise to the following:

$$\frac{y-y'}{x-x'}=1 \quad (6)$$

Based on eq (5) and eq (6), we get the coordinates of point $P'(x',y')$ as:

$$\begin{cases} x'=-y+\frac{\sqrt{2}}{4}\lambda \\ y'=-x+\frac{\sqrt{2}}{4}\lambda \end{cases} \quad (7)$$

Plugging the coordinates of point $P'(x',y')$ into eq(2) which describes the curved line segment OA on which P' is, we have :

$$-x+\frac{\sqrt{2}}{4}\lambda=\sin(-y+\frac{\sqrt{2}}{4}\lambda) \quad (8)$$

The above eq(8) is the mathematical equation for the curved line segment AB.

To obtain the mathematical equation for the curved line segment BC, it is not difficult to see that BC can be formed by manipulating OA: first reflecting OA with respect to the straight line AE, and then shifting it from point O and point B to form BC.

Based on symmetry, we know that $x=\sin(y)$ describes the curved line segment which is the mirror image of the curved line segment OA with respect to the straight line OA. Then we can shift the curved line segment described

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by $x = \sin(y)$ from O to B along the straight line OD to form the curved line segment BC. Because the wavelength is λ, $B = (\frac{\sqrt{2}}{4}\lambda, \frac{\sqrt{2}}{4}\lambda)$. The following eq(9) describes the curved line segment BC:

$$x - \frac{\sqrt{2}}{4}\lambda = \sin(y - \frac{\sqrt{2}}{4}\lambda) \qquad (9)$$

Point $C = (\ \sin(x_0) + \frac{\sqrt{2}}{4}\lambda\ ,\ x_0 + \frac{\sqrt{2}}{4}\lambda\ )$.

Now we derive the equation of the curved line segment CD in figure 3. Because $C = (\ \sin(x_0) + \frac{\sqrt{2}}{4}\lambda\ ,\ x_0 + \frac{\sqrt{2}}{4}\lambda\ )$ and the slope of the straight line CF is -1, the equation of the straight line CF is $(y - x_0 - \frac{\sqrt{2}}{4}\lambda) = -1 \cdot (x - x_0 - \frac{\sqrt{2}}{4}\lambda)$ that is:

$$y + x = 2x_0 + \frac{\sqrt{2}}{2}\lambda \qquad (10)$$

We notice that CD and BC are symmetric with respect to the straight line CF. Consider that there are two points: one is $Q(x,y)$ on CD; the other is $Q'(x',y')$ on BC, and they are symmetric with respect to the straight line CF. That the midpoint on the straight line segment QQ' is on the straight line CF described by eq (9) leads to the following:

$$\frac{y+y'}{2} + \frac{x+x'}{2} = 2x_0 + \frac{\sqrt{2}}{2}\lambda \qquad (11)$$

That the slope of the straight line QQ' is 1 results in:

$$\frac{y-y'}{x-x'} = 1 \qquad (12)$$

Combining eq(11) and eq(12), we have:

$$\begin{cases} x' = -y + 2x_0 + \frac{\sqrt{2}}{2}\lambda \\ y' = -x + 2x_0 + \frac{\sqrt{2}}{2}\lambda \end{cases} \qquad (13)$$

Plugging the coordinates of the point Q' into eq (8) due to that Q' is on the curved line segment BC described by eq (9),we have:

$$-y + 2x_0 + \frac{\sqrt{2}}{4}\lambda = \sin(-x + 2x_0 + \frac{\sqrt{2}}{4}\lambda) \qquad (14)$$

The above eq(13) describes the curved line segment CD.

Now we see that the combination of eq(2), eq(7), eq(9), and eq(14) construct a system of equations (as shown in eqs(1)) to describe the dualistic wave in figure 3.

If we rotate counterclockwise the coordinate system in figure 3 through an angle of 45 degrees, we get the following figure 4:

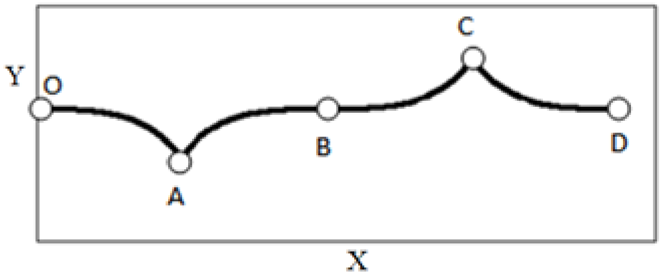

Figure 3: Duality wave

Figure 4 The dualistic wave viewed by the observer in the line $y = x$ in the coordinate system in figure 3. The system of equations describing the wave in figure 4 is the following:

$$\begin{cases} \frac{\sqrt{2}}{2}(x+y) = \sin(\ \frac{\sqrt{2}}{2}(x-y)\ ) & 0 \le x < \lambda/4 \\ \frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x + y) = \sin(\ \frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x - y)\ ) & \lambda/4 \le x < \lambda/2 \\ \frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} + y) = \sin(\ \frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} + y)\ ) & \lambda/2 \le x < 3\lambda/4 \\ \frac{\sqrt{2}}{2}(\lambda - x - y) = \sin(\ \frac{\sqrt{2}}{2}(\lambda - x + y)\ ) & 3\lambda/4 \le x \le \lambda \end{cases}$$

(15)

The following shows the derivation of eqs (15).

First we conduct the derivation of the equation of the curved line segment OA in figure 3. For a point, the relationship between its coordinates (x', y') in figure 3 and its coordinates (x, y) in figure 4 is described by eq(16) because we rotate counterclockwise the coordinate system in figure 3 through 45 degree to form the coordinate system in figure 4

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos(45^\circ) & -\sin(45^\circ) \\ \sin(45^\circ) & \cos(45^\circ) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} \quad (16)$$

Consider that there is a point $P(x, y)$ on OA in figure 4 and the coordinates of the point are (x', y') in figure 3. So the following holds according to eq(15):

$$\begin{cases} x' = \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y \\ y' = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \end{cases} \quad (17)$$

Because $P'(x', y')$ is on the curved line $y = \sin(x)$, we have:

$$\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y = \sin(\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y) \quad (18)$$

The above eq(18) describes OA in figure 4.

Secondly we derive the equation of AB in figure 3. We notice that OA and AB are symmetric with respect to the straight line $x = \frac{\lambda}{4}$. Consider there is a point $P(x, y)$ on AB and a point $P'(x', y')$ is on OA, and the two points are symmetric with respect to the straight line $x = \frac{\lambda}{4}$. That the midpoint of PP' is on the straight line $x = \frac{\lambda}{4}$ results in:

$$\begin{cases} \frac{x + x'}{2} = \frac{\lambda}{4} \\ y' = y \end{cases} \quad (19)$$

So the following stands

$$\begin{cases} x' = \frac{\lambda}{2} - x \\ y' = y \end{cases} \quad (20)$$

Plugging the coordinates of x' and y' in eq(20) into eq(18) due to that $P'(x', y')$ is on OA, we have the following:

$$\frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x + y) = \sin(\frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x - y)) \quad (21)$$

The above eq(21) describes the curved line segment AB in figure 4.

Thirdly, we derive the equation of BC in figure 4. It is not difficult to see that BC can be formed by first reflecting OA with respect to the straight line $y = 0$ and then shifting it from point O to point B along the straight line $y = 0$. Since the point $B = (\frac{\lambda}{2}, 0)$, the shifted distance is $\frac{\lambda}{2}$ and the equation of BC is the following:

$$\frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} - y) = \sin(\frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} + y)) \quad (22)$$

Fourthly, we derive the equation of CD in figure 4. We notice that BC and CD are symmetric about the straight line $x = \frac{3}{4}\lambda$, and $C = (\frac{3}{4}\lambda, 0)$. Consider that there are two points of $P(x, y)$ and $P'(x', y')$. $P(x, y)$ is on CD and $P'(x', y')$ is on BC, and P and P' are symmetric about the straight line $x = \frac{3}{4}\lambda$. That the midpoint of PP' is on the straight line $x = \frac{3}{4}\lambda$ results in:

$$\begin{cases} \frac{x + x'}{2} = \frac{3}{4}\lambda \\ y' = y \end{cases} \quad (23)$$

So

$$\begin{cases} x' = \frac{3}{2}\lambda - x \\ y' = y \end{cases} \quad (24)$$

Plugging the coordinates of x' and y' in eq(24) into eq(22) due to that $P'(x', y')$ is on BC, we have the following:

$$\frac{\sqrt{2}}{2}(\lambda - x - y) = \sin(\frac{\sqrt{2}}{2}(\lambda - x + y)) \quad (25)$$

The above eq(25) describes the curved line segment CD in figure 4.

Now we see that the combination of eq(18), eq(21), eq(22), and eq(25) construct a system of equations (as shown in eqs(15)) to describe the dualistic wave in figure 4. If the wavelength λ is infinitely small, the wave described by eqs(1) or eqs(15) can be thought as both a wave and a straight line.

In Appendix, it shows other equivalent infinitesimals in addition to $\lim_{x\to 0}\frac{\sin(x)}{x} = 1$. For example we can also construct a dualistic wave based on $\lim_{x\to 0}\frac{\tan(x)}{x} = 1$, and the corresponding system of equations describing this wave is the following:

$$\begin{cases} \frac{\sqrt{2}}{2}(x+y) = \tan(\frac{\sqrt{2}}{2}(x-y)) & 0 \le x < \lambda/4 \\ \frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x + y) = \tan(\frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x - y)) & \lambda/4 \le x < \lambda/2 \\ \frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} + y) = \tan(\frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} + y)) & \lambda/2 \le x < 3\lambda/4 \\ \frac{\sqrt{2}}{2}(\lambda - x - y) = \tan(\frac{\sqrt{2}}{2}(\lambda - x + y)) & 3\lambda/4 \le x \le \lambda \end{cases}$$

(26)

The system of equations describing a dualistic wave constructed based on $\lim_{x\to 0}\frac{e^x - 1}{x} = 1$ is the following:

$$\begin{cases} \frac{\sqrt{2}}{2}(x+y) = e^{(\frac{\sqrt{2}}{2}(x-y))} - 1 & 0 \le x < \lambda/4 \\ \frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x + y) = e^{(\frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x - y))} - 1 & \lambda/4 \le x < \lambda/2 \\ \frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} + y) = e^{(\frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} - y))} - 1 & \lambda/2 \le x < 3\lambda/4 \\ \frac{\sqrt{2}}{2}(\lambda - x - y) = e^{(\frac{\sqrt{2}}{2}(\lambda - x + y))} - 1 & 3\lambda/4 \le x \le \lambda \end{cases}$$

(27)

The system of equations describing a dualistic wave constructed based on $\lim_{x\to 0}\frac{\frac{1}{\ln(a)}\cdot(a^x - 1)}{x} = 1$ is the following:

$$\begin{cases} \frac{\sqrt{2}}{2}(x+y) = \frac{1}{\ln(a)}(a^{(\frac{\sqrt{2}}{2}(x-y))} - 1) & 0 \le x < \lambda/4 \\ \frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x + y) = \frac{1}{\ln(a)}(a^{(\frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x - y))} - 1) & \lambda/4 \le x < \lambda/2 \\ \frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} + y) = \frac{1}{\ln(a)}(a^{(\frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} - y))} - 1) & \lambda/2 \le x < 3\lambda/4 \\ \frac{\sqrt{2}}{2}(\lambda - x - y) = \frac{1}{\ln(a)}(a^{(\frac{\sqrt{2}}{2}(\lambda - x + y))} - 1) & 3\lambda/4 \le x \le \lambda \end{cases}$$

(28)

The system of equations describing a dualistic wave constructed based on $\lim_{x\to 0}\frac{\frac{1}{ab}((1+ax)^b - 1)}{x} = 1$ is the following:

$$\begin{cases} \frac{\sqrt{2}}{2}(x+y) = \frac{1}{ab}((1 + a(\frac{\sqrt{2}}{2}(x-y))^b - 1) & 0 \le x < \lambda/4 \\ \frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x + y) = \frac{1}{ab}((1 + a(\frac{\sqrt{2}}{2}(\frac{\lambda}{2} - x - y))^b - 1) & \lambda/4 \le x < \lambda/2 \\ \frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} + y) = \frac{1}{ab}((1 + a(\frac{\sqrt{2}}{2}(x - \frac{\lambda}{2} + y))^b - 1) & \lambda/2 \le x < 3\lambda/4 \\ \frac{\sqrt{2}}{2}(\lambda - x - y) = \frac{1}{ab}((1 + a(\frac{\sqrt{2}}{2}(\lambda - x + y))^b - 1) & 3\lambda/4 \le x \le \lambda \end{cases}$$

(29)

The system of equations describing a dualistic wave constructed based on $\lim_{x\to 0}\frac{n\cdot(\sqrt[n]{1+x} - 1)}{x} = 1$ is the following:

$$\begin{cases} \frac{\sqrt{2}}{2}(x+y)=n\cdot(\sqrt[n]{1+\frac{\sqrt{2}}{2}(x-y)}-1) & 0\le x<\lambda/4 \\ \frac{\sqrt{2}}{2}(\frac{\lambda}{2}-x+y)=n\cdot(\sqrt[n]{1+\frac{\sqrt{2}}{2}(\frac{\lambda}{2}-x-y)}-1) & \lambda/4\le x<\lambda/2 \\ \frac{\sqrt{2}}{2}(x-\frac{\lambda}{2}+y)=n\cdot(\sqrt[n]{1+\frac{\sqrt{2}}{2}(x-\frac{\lambda}{2}+y)}-1) & \lambda/2\le x<3\lambda/4 \\ \frac{\sqrt{2}}{2}(\lambda-x-y)=n\cdot(\sqrt[n]{1+\frac{\sqrt{2}}{2}(\lambda-x+y)}-1) & 3\lambda/4\le x\le\lambda \end{cases} \quad (30)$$

It is found that if we construct two dualistic waves based on $\lim_{x\to0}\frac{\sin(x)}{x}=1$ and $\lim_{x\to0}\frac{\arcsin(x)}{x}=1$ respectively, we will find that the two waves are same. It is due to the symmetry that $y=\sin(x)$ and $y=\arcsin(x)$ are two mutually inverse functions. The same happens when we construct dualistic waves based on $\lim_{x\to0}\frac{\tan(x)}{x}=1$ and $\lim_{x\to0}\frac{\arctan(x)}{x}=1$, $\lim_{x\to0}\frac{e^x-1}{x}=1$ and $\lim_{x\to0}\frac{\ln(1+x)}{x}=1$, etc.

4. Solution to Zeno's paradox based on dualistic wave

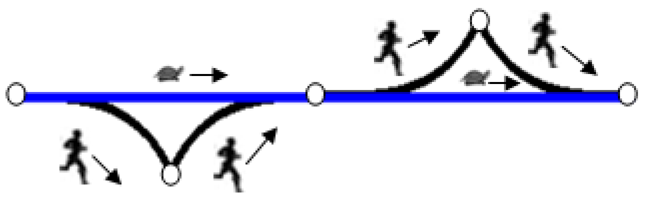

Figure 5: Achilles travels along the dualistic wave path and the tortoise travels along a straight line path

In figure 5 it is seen that we assume Achilles runs along a dualistic wave path and the tortoise along a straight line path, then it is possible that Archilles overtakes the tortoise without passing the points the tortoise took previously. The underlying mechanism of this explanation is the equivalence of flat and curved space-times at infinitely small scales.

5. Conclusion

The classification of infinitesimal line segments including flat and curved infinitesimal line segments is proposed. A novel type of geometric object named dualistic wave is given, and it serves as a mathematical tool to reveal the equivalence between wavelike and particlelike motions at infinitely small scale. The duality of the space-time structure at infinitely small scale is demonstrated by dualistic wave and it provides a solution to Zeno's paradox.

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Appendix

Examples and meanings of equivalent infinitesimals

(1) $\lim_{x\to0}\frac{\sin(x)}{x}=1$ reveals that curved line segment $y=\sin(x),\ 0\le x\le L$ is equivalent to straight line segment $y=x,\ 0\le x\le L$ as $L\to0$.

(2) $\lim_{x\to0}\frac{\arcsin(x)}{x}=1$ reveals that curved line segment $y=\arcsin(x),\ 0\le x\le L$ is equivalent to straight line segment $y=x,\ 0\le x\le L$ as $L\to0$.

(3) $\lim_{x\to0}\frac{\tan(x)}{x}=1$ reveals that curved line segment $y=\tan(x),\ 0\le x\le L$ is equivalent to straight line segment $y=x,\ 0\le x\le L$ as $L\to0$.

(4) $\lim_{x\to0}\frac{\arctan(x)}{x}=1$ reveals that curved line segment $y=\arctan(x),\ 0\le x\le L$ is equivalent to straight line segment $y=x,\ 0\le x\le L$ as $L\to0$.

(5) $\lim_{x\to0}\frac{e^x-1}{x}=1$ reveals that curved line segment $y=e^x-1,\ 0\le x\le L$ is equivalent to straight line segment $y=x,\ 0\le x\le L$ as $L\to0$.

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(6) $\lim_{x\to 0}\frac{\ln(1+x)}{x}=1$ reveals that curved line segment $y=\ln(1+x),\ 0\le \mathrm{x}\le \mathrm{L}$ is equivalent to straight line segment $y=x,\ 0\le \mathrm{x}\le \mathrm{L}$ as $L\to 0$.

(7) $\lim_{x\to 0}\frac{\frac{1}{\ln(a)}\cdot(a^x-1)}{x}=1$ reveals that curved line segment $y=\frac{1}{\ln(a)}\cdot(a^x-1),\ 0\le \mathrm{x}\le \mathrm{L}$ is equivalent to straight line segment $y=x,\ 0\le \mathrm{x}\le \mathrm{L}$ as $L\to 0$.

(8) $\lim_{x\to 0}\frac{\ln(a)\cdot\log_a(1+x)}{x}=1$ reveals that curved line segment $y=\ln(a)\cdot\log_a(1+x),\ 0\le \mathrm{x}\le \mathrm{L}$ is equivalent to straight line segment $y=x,\ 0\le \mathrm{x}\le \mathrm{L}$ as $L\to 0$.

(9) $\lim_{x\to 0}\frac{\frac{1}{ab}((1+ax)^b-1)}{x}=1$ reveals that curved line segment $y=\frac{1}{ab}((1+ax)^b-1),\ 0\le \mathrm{x}\le \mathrm{L}$ is equivalent to straight line segment $y=x,\ 0\le \mathrm{x}\le \mathrm{L}$ as $L\to 0$.

(10) $\lim_{x\to 0}\frac{n\cdot(\sqrt[n]{1+x}-1)}{x}=1$ reveals that curved line segment $y=n.(\sqrt[n]{1+x}-1),\ 0\le \mathrm{x}\le \mathrm{L}$ is equivalent to straight line segment $y=x,\ 0\le \mathrm{x}\le \mathrm{L}$ as $L\to 0$.