Optimal Design of Finite Impulse Response Digital Filters through Ant Colony Algorithm

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Abstract: ***Aiming at the need of high-precision digital filter design, a method to improve the design of finite impulse response digital filters with an ant colony algorithm is proposed. The design result of the window function method is further optimized using the ant colony algorithm, and the optimization objective function and implementation steps are designed. The ant colony algorithm and Chebyshev approximation are used to optimize the filter coefficients, and the optimization goals and implementation steps are given. Using software for simulation design, the experimental results show that the filters obtained by the two design methods can meet the design requirements, and the error with the ideal filter is small.***

Keywords: Ant colony algorithm; Digital filter; Window function method; Chebyshev approximation.

1. Introduction

Digital filters are an important branch of signal processing. They are algorithms and devices composed of multipliers, adders, and delays. They can be implemented by computer software and digital hardware. Their functions are to process discrete digital signals to achieve the effect of changing the amplitude-frequency characteristics. According to the unit impulse response, digital filters can be divided into finite impulse response (FIR) digital filters and infinite impulse response (IR) digital filters. They are used in signal processing, pattern recognition, Image processing, biomedicine and other fields are widely used.

The development of computer optimization algorithms has opened the way to solve the complex calculation problems of filter design. Through computer-aided calculations, multiple technical indicators of the filter can be achieved. However, no optimization design method can achieve the ideal filter amplitude-frequency characteristics, but can only approximate the ideal filter amplitude-frequency characteristics. In order to achieve better optimization results, a variety of optimization algorithms have been used in the filter design process. The global optimization algorithm used in the early stage of FIR digital filter optimization is highly complicated [1], and although the local neighborhood search technology has a small calculation amount, it is difficult to find the optimal solution [2]. Since then, simulated annealing algorithm [3], genetic algorithm [4], evolutionary programming method [5], immune algorithm [6], particle swarm optimization algorithm [7], neural network algorithm [8] have been applied to the FIR digital filter designing.

This paper designs FIR digital filter based on ant colony algorithm. Firstly, combined with the window function method, the ant colony algorithm to derive the window function optimizes the objective function. Then combined with Chebyshev approximation method, the objective function of Chebyshev approximation using ant colony algorithm is derived. After that, the design steps of designing FIR digital filter with ant colony algorithm are given. Finally, the method in the paper is used to simulate the low-pass filter, which proves that the proposed method can effectively reduce the design error.

2. Design Method

2.1 Ant colony optimization window function

The transfer function of the FIR digital filter is

$$H(z)=\sum_{n=0}^{N-1} h(n) z^{-n} \tag{1}$$

where n is the discrete moment, N is the length of the time series, $h(n)$ is the impulse response, and z is the complex frequency. Take $z=e^{j\omega}$, let ω be the real frequency, then the frequency response of the filter is

$$H\left(e^{j\omega}\right)=\sum_{n=0}^{N-1} h(n) e^{-j\omega n} \tag{2}$$

The ideal frequency response is $H_d\left(e^{j\omega}\right)$, and there is an error function E for the designed filter amplitude $\left|H\left(e^{j\omega}\right)\right|$ and ideal filter amplitude $\left|H_d\left(e^{j\omega}\right)\right|$, expressed as

$$E=\left|H\left(e^{j\omega}\right)\right|-\left|H_d\left(e^{j\omega}\right)\right|=\left|\sum_{n=0}^{N-1} h_d(n) w(n) e^{-j\omega n}\right|-\left|H_d\left(e^{j\omega}\right)\right| \tag{3}$$

Where $h_d(n)$ is the impulse response of the ideal filter, and $w(n)$ is the window function. According to the error function E, the fitness function $f(w)$ is established as

$$f(w)=\max(|E|) \tag{4}$$

The fitness function $f(w)$ is a non-linear function of $w(n)$, the window function $w(n)$ is a discrete variable in the frequency domain, takes a value in $[0,1]$, and $w(n)$

determines the performance of designed filter. During the optimization process, N is a fixed value to optimize the filter coefficients and improve the filter ripple.

The ant colony algorithm is used to optimize the window function, that is, the value of the window function $w(n)$ is selected to minimize the fitness function $f(w)$. Construct an optimization function and use the formula (2) to find the frequency response $H(e^{j\omega})$ of designed filter.

2.2 Ant colony algorithm optimization Chebyshev approximation

If $h(n)$ in (2) is a real sequence, then $H(e^{j\omega})$ is

$$H(e^{j\omega}) = H(\omega)e^{j\theta(\omega)} \quad (5)$$

Among them, $H(\omega)$ is the amplitude function and $\theta(\omega)$ is the phase function. Under linear phase characteristic

$$h(n) = \pm h(N-1-n) \quad (6)$$

Transfer function $H(z)$ is

$$H(z) = \frac{1}{2}\left[H(z) \pm z^{-(N-1)}\right]H(z^{-1}) = \frac{1}{2} z^{-\left(\frac{N-1}{2}\right)} \sum_{n=0}^{N-1} h(n)\left[z^{\left(\frac{N-1}{2}-n\right)} \pm z^{-\left(\frac{N-1}{2}-n\right)} \right] \quad (7)$$

Design type II filter, get the amplitude function $H(\omega)$

$$H(\omega) = \sum_{n=0}^{N-1} h(n)\cos\left[\left(n - \frac{N-1}{2}\right)\omega\right] \quad (8)$$

Because of the even symmetry, the merger is available

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} 2h(n)\cos\left[\left(\frac{N-1}{2} - n\right)\omega\right] \quad (9)$$

Set $m = \frac{N}{2} - n$, and use n instead of m, then

$$H(\omega) = \sum_{n=1}^{\frac{N}{2}} b(n)\cos\left[\left(n - \frac{1}{2}\right)\omega\right] \quad (10)$$

Where the filter pulse coefficient $b(n) = 2h\left(\frac{N}{2} - n\right)$, $n = 1, 2, \cdots, \frac{N}{2}$.

Ideal frequency response $H_d(e^{j\omega})$, at discrete point $\{\omega_i | i = 1, 2, \cdots, M\}$, there is an error function E for the designed filter amplitude $|H(e^{j\omega})|$ and ideal filter amplitude $|H_d(e^{j\omega})|$, which is expressed as

$$E = \sum_{i=1}^{M} \left(\left|H(e^{j\omega_i})\right| - \left|H_d(e^{j\omega_i})\right|\right)^2 \quad (11)$$

Substituting equation (10) into equation (11), we get

$$E = \sum_{i=1}^{M} \left(\left| \sum_{n=1}^{\frac{N}{2}} b(n)\cos\left[\left(n - \frac{1}{2}\right)\omega_i\right] e^{-j\left(\frac{N-1}{2}\right)\omega_i} \right| - \left|H_d(e^{j\omega_i})\right| \right)^2 \quad (12)$$

The error function E has $\frac{N}{2}$ unknowns, and it can be seen here that it is a nonlinear function of the filter coefficient $h(n)$. The Chebyshev approximation method needs to minimize the maximum absolute error between the actual filter frequency and the ideal filter, so here we need to select the filter coefficient $b(n)$ to minimize E.

The ant colony algorithm is used for optimization, the objective function is determined as formula (12), the optimization constraint condition is set to $|h(n)| \le 1$, $|b(n)| \le 2$, and the filter frequency response $H(e^{j\omega})$ is calculated using formula (2).

2.3 Optimization of ant colony algorithm

The specific steps of using ant colony algorithm to optimize the filter design are:

(1) Initialize the parameters
Before optimization starts, related parameters need to be initialized, including ant colony size m, pheromone volatilization coefficient ρ, transfer probability constant P_0, maximum iteration number $iter\_max$, and initial value of iteration number $iter = 1$.

(2) Construct the required solution space
Randomly generate an ant colony, and randomly place ants in different starting places. For Ant k $(k = 1, 2, 3, \cdots, m)$, the fitness function value is calculated according to the objective function formula until the calculation is completed.

(3) Update pheromone
As the number of iterations progresses, find the best fitness value and record the optimal solution (best fitness value) in the current number of iterations. At the same time, update the calculation information.

(4) Determine whether the exercise has stopped
If the number of iterations , then let , and then clear the ant's fitness function value table and return to the above step (2); otherwise, stop the calculation, at this time the optimal solution is obtained, and the optimal solution is output.

(5) Find the best fitness value
Record the optimal solution in the current number of iterations, compare all the optimization coefficients after the iteration together, and then select the best optimization coefficient from the middle and use it as the optimized FIR digital filter coefficient.

3. Numerical Results

Design a low-pass filter with order 35, technical specifications are as follows

$$H_d\left(e^{j\omega}\right)=\begin{cases}1 & \omega\le 0.4\pi\\ 0 & 0.5\pi\le\omega\le\pi\end{cases} \tag{13}$$

In the design process, the number of ants $m=500$, the pheromone volatilization coefficient $\rho=1$, the transfer probability constant $P_0=0.4$, and the maximum number of iterations $iter\_\max=200$.

The comparison of the amplitude-frequency response of the low-pass filter designed by the window function method and the ant colony optimization algorithm is shown in Figure 1. The dotted line is the amplitude-frequency response of the window function low-pass filter optimized by the ant colony algorithm, and the solid line is the amplitude-frequency response of the low-pass filter designed by Hanning window. It can be found that the amplitude-frequency response of the filter optimized by the ant colony algorithm is relatively close to the result of the window function method design. Figure 2 shows the amplitude-frequency response of the low-pass filter. From the figure, it can be seen that the filter designed by the ant colony algorithm has a more pronounced attenuation around the cut-off frequency of the stop band.

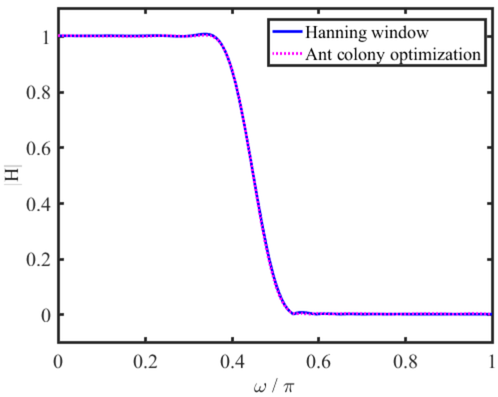

Figure 1: Low-pass filter amplitude-frequency response

Then use the ant colony optimization algorithm combined with Chebyshev approximation to design a low-pass filter of order 35. The technical specifications are as follows

$$H_d\left(e^{j\omega}\right)=\begin{cases}1 & \omega\le 0.3\pi\\ 0 & 0.4\pi\le\omega\le\pi\end{cases} \tag{14}$$

In the design process, the parameter setting of the ant colony optimization algorithm is the same as the previous example.

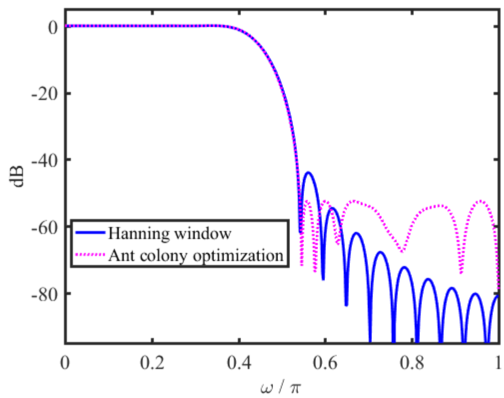

Figure 2: Low-pass filter logarithmic amplitude-frequency response

Using Parks-McClellan algorithm to achieve Chebyshev approximation design and ant colony optimization algorithm designed low-pass filter amplitude-frequency response comparison shown in Figure 3. The solid line is Chebyshev design low-pass filter amplitude frequency response, and the dotted line is the ant colony algorithm to optimize the amplitude-frequency response of the low-pass filter. It can be seen that the ant colony algorithm can effectively suppress the pass band ripple. Figure 4 shows the logarithmic amplitude-frequency response of the low-pass filter designed by the Parks-McClellan algorithm and the ant colony optimization algorithm. The ant colony algorithm achieves a slightly larger stop band attenuation than the Parks-McClellan algorithm. Therefore, the ant colony optimization algorithm design is closer to the amplitude response of the ideal FIR digital filter than the Chebyshev approximation.

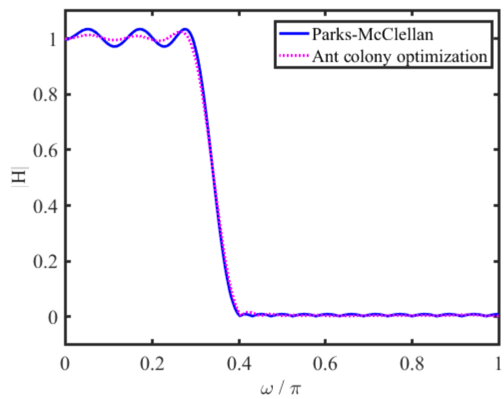

Figure 3: Low-pass filter amplitude-frequency response

4. Conclusion

In this paper, the ant colony algorithm is used to optimize the design of the FIR digital filter. The window function method and the Chebyshev approximation method are respectively studied. The method described the design steps to realize the optimization of FIR digital filter using ant colony algorithm. Given the low-pass filter design index, the proposed method was used to complete the simulation design. The experimental results show that the optimization of the ant colony algorithm results in a smaller error between the designed filter and the ideal filter, which meets the design index requirements.

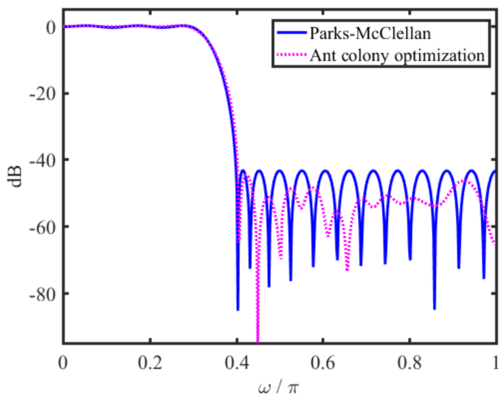

Figure 4: Low-pass filter logarithmic amplitude-frequency response

5. Acknowledgements

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