International Journal of Science and Research (IJSR)
ISSN: 2319-7064
ResearchGate Impact Factor (2018): 0.28 | SJIF (2019): 7.583

Macroscopic World Examples of Collective Interferences Formed by Individual Deterministic Events

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Abstract: ***Unlocking the mystery of wave-particle duality is a compelling topic in modern physics. Instead of using the concept of probability, an explanation that can both speak out the individual events and interpret how they are together to construct wavelike complexities is always expected. In this paper, four specific examples are given to show that individual deterministic events are indeed able to build up collective interferences in our macroscopic world. In the first three examples, the collective behaviours appear due to two factors: one is the signals with varying intensities associated with the individual events; the other the involvement of the detector screen to receive the signals. The representative number of an event (that is the intensity of the signal associated the event) stops changing and is shown to be a definite value to the observer when the signal hits the detector screen. Regarding the fourth example, the collective behaviours are formed due to the employment of decimal system to measure the events.***

Keywords: wave-particle duality; collective interference; representative number; individual deterministic event; decimal system; macroscopic world

1. Introduction

Wave-particle duality is one of the most mysterious phenomenon in physics, and it has been continuously investigated by experimental and theoretical ways [1-2]. The double-slit experiments of microscopic entities such as electron and atom are thought to represent the central mystery of quantum mechanics [3]. The probability-based explanation has made great progress to interpret the collective complexities but it cannot speak out the individual events [2]. Fractal and deterministic chaos theories demonstrate how odds and irregularities can emerge inside deterministic systems [4]. Unification is highly regarded in searching for an ultimate theory to explain complex phenomena in the world. In physical world, the four fundamental interactions between matters including strong interaction, weak interaction, electromagnetism, and gravitation are believed to one kind of same interaction by many scientists. In mathematical world, the four fundamental interactions between numbers including summation, subtraction, multiplication, and division are conjectured to be one same interaction, and if abc conjecture [5] were proved, it would significantly enhance our understanding of the relationship between summation and multiplication. Wholeness is also affected by language, and the contribution of the subject-verb-object sentence structure to fragmentations is extensively discussed [6]. The possibility to incorporate Newton's second law and de Broglie's wave-particle duality into one framework was discussed [7]. Experimental investigations of the construction of collective interference by individual events of diffracted electrons were carried out [8]. Quantitative measures of wave and particle properties for multi-beam interferometers were conducted [9]. It was found that wave-particle duality relations can be described in terms of entropies [10]. It was guessed that wave-particle duality was illustrated by the polarization of the superposed light [11]. Wave-particle duality was shown to make it possible to relate the probabilities of winning the discrimination games [12]. Wave particle duality was interpreted as only the consequence of two complementary aspects that are continuity and discontinuity [13]. Fractal wave–particle duality was found to appear in the evolution of myoglobin and neuroglobin [14]. In a complete theory of quantum gravity, it was anticipated that quantum and gravity need to be assigned new meanings [15].

It is conjectured that both microscopic and macroscopic worlds have collective interference phenomena. And this paper shows the existence of collective interferences formed by individual deterministic events in macroscopic world. With the help of numerical simulations by computer programs to model many individual deterministic events, the collective interferences on the detector screens are found to be finally formed and then be visualized.

2. Macroscopic examples of collective interferences

Totally there are four examples to be given. The nature of the first five examples can be generalized that the collective interference formed on the detector screen is built up by signals sent from a single source or by the signals sent from two different sources. In the numerical simulations, the first step is to establish the Cartesian coordinate system. In the single signal source cases, the origin of the Cartesian coordinate system is the point of the single signal source, the horizontal axis (also called X axis) is along the straight line passing the signal source and perpendicular to the detector screen, and the vertical axis (also called Y axis) is along the straight line passing the detector screen. In figure 1, it is shown that the direction of the X axis is from the left to the right, and the direction of the Y axis is upward. In the cases of double signal sources, the origin of the coordinate system is the midpoint of the line segment connecting the two sources, as shown in figure 2.

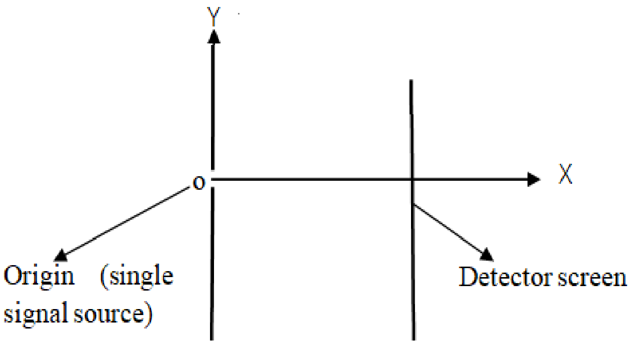

Figure 1: Cartesian coordinate for the cases of single signal source

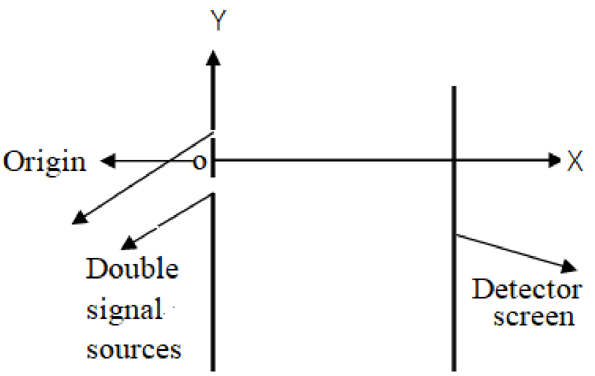

Figure 2: Cartesian coordinate system for the cases of double signal sources

2.1 Collective Interferences Formed by the Left-or-Right Motions of the Feet of the Persons Walking along Straight-line Trajectories

In this example, there are 5000 persons and every one walking along a straight line trajectory, as shown in figure 1-1. In figure 1-2, we see that a person walks to a detector screen from a single starting source and along a straight line trajectory. There is a diffraction angle between the straight line path and the horizontal direction. The distance covered by a single step of the person is 0.762 meters (it is about 30 inches, an average value of the distance covered by a single step of a walking adult). The distance between the starting source point and the detector screen is 100 meters. There are two possible situations when the person hitting the detector screen (assuming that the person does not use hand to hit the screen and only by one of the two feet): one is by the left foot; the other by the right, as shown in figure 1-3. If the left-foot-hitting happens, we assign the number 0 on the point being hit on the detector screen, and if the right-foot-hitting, the assigned number is 1.

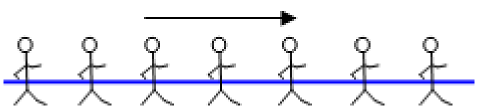

Figure 1-1: A person walking toward the right side along a straight line trajectory

As to the group of 5000 persons, they all start from a single starting source and go to the detector screen one by one. This means a person starts walking to the detector screen after the previous person hits the detector screen. For the diffraction angle θ (shown in figure 1-2), it is uniformly distributed between -30° and 30°. In the computer programs, θ is a random variable and its value is uniformly distributed between -30 and 30.

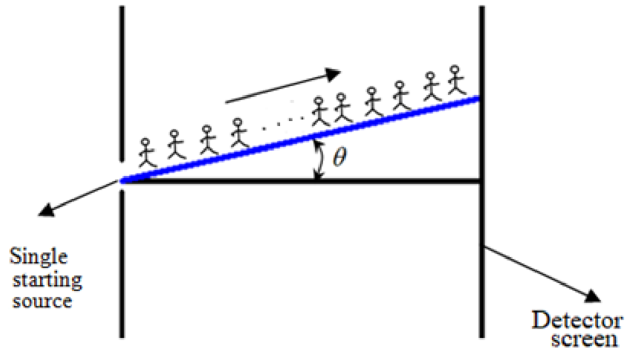

Figure 1-2: A person walks to a detector screen along a straight-line trajectory from a single starting source

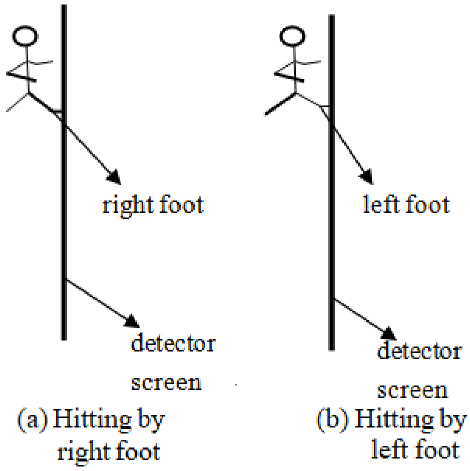

Figure 1-3: A person hitting the detector screen by right or left foot

Now we do a calculation for an individual event that is a person walks from the single starting source and along a straight line trajectory to hit the detector screen. The initial condition is that the person first moves the left foot. And the diffraction angle between the straight line trajectory and the horizontal axis is 15°. Based on this information, we can know which foot (the left or right) hit the detector screen finally. The X-coordinate of the position (on the detector screen) being hit is 100 meters, and the Y-coordinate is $100 \times \tan(15^{\circ}) = 26.79$ meters meters. So the distance between the starting source and hit position is 103.53 meters ($103.53 \approx \sqrt{100^2 + 26.79^2}$). From the calculation result of $103.53 \div 0.762 \approx 135.87$ and the initial condition (the person first steps forward by the left foot), we know that after 135 steps, the person needs to move forward by his right foot and hit the detector screen by his right foot finally, then at the hit position the assigned number is 1. The representative number of this individual event is 1.

After the all the events that are the 5000 persons hit the detector screen one by one, as to a detector box (on the detector screen) being hit by the persons, it has a representative number that is the sum of the representative numbers (0 or 1) of the individual events that happen in the detector box; the criterion for an individual event to happen in this box is that the position of the hit on the detector screen (by the foot left or right) falls in this box. In the simulation, the length of the detector box is about 1.15 meters and there are 100 detector boxes on the detector screen. Figure 1-4 shows the collective interference formed by these 5000 individual events, and the numbers on the X-axis are the central positions of the detector boxes.

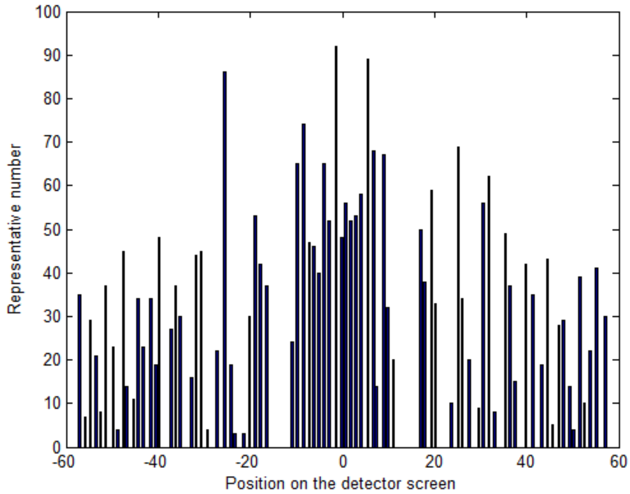

Figure 1-4: Collective interference formed by the left-or-right motions of the feet of the persons from a single starting source and travelling along straight line trajectories

Regarding the case of double starting sources, as shown in figure 1-5, the distance between the two sources is set as 1 meter in the simulation, and others parameters (such as the distance between the starting source and the detector screen, the distance covered by a single step, and the assigned number when being hit (on the detector screen) by the left or right foot) are same as that in the case of single starting source.

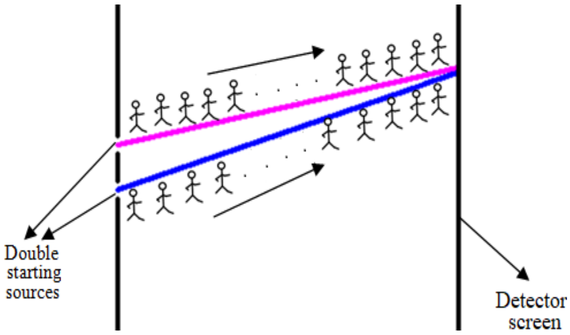

Figure 1-5: Two persons walk to a detector screen along two straight-line trajectories from two starting sources respectively

In figure 1-6, it can be seen that there is an uncertainty that which starting source does the person selects. In the simulation, equal probability is set. This means it is 50%-50% chance of starting from either source. In the computer program, there is a random variable uniformly distributed between 0 and 1, in each loop (each loop simulates an individual event), if the value of the random variable falls into the interval (0, 0.5), then the person selects the upper starting source; otherwise, the person selects the lower starting source.

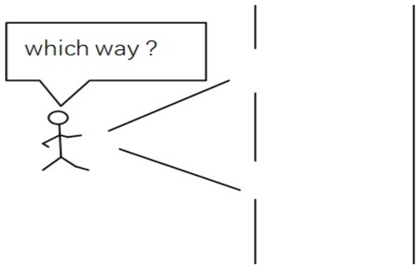

Figure 1-6: A person with two possible ways to take

In figure 1-7, it shows that the collective interference formed by the 5000 individual events, which correspond to 5000 loops in the computer program. In each individual event, the person first moves the person's left foot to step forward.

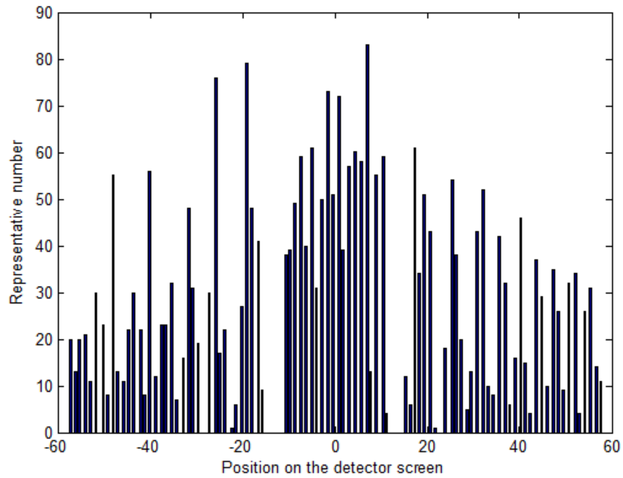

Figure 1-7: Collective interference formed by the left-or-right motions of the feet of the persons from two starting sources respectively and travelling along straight line trajectories

2.2 Collective Interferences Formed by the Motions of Persons Walking along Wavelike Trajectories

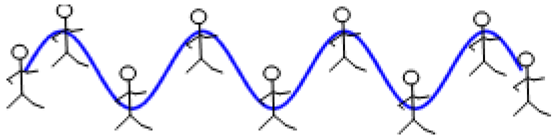

Figure 2-1: A person walking toward the right side along a wave trajectory

In this example, an individual person travels along a sine wave trajectory, as shown in figure 2-1. The wavelength of the sine wave is 1.524 meters (averagely, the distance covered by a step of a walking adult is 0.762 meters, and 1.524 = 2*0.762), and the amplitude of the sine wave is 3 meters. The distance between the single starting source (from which the individual person starts to walk) and the detector screen is 100 meters. An event is defined as that an individual person starts walking from the source with certain diffraction angle and to the detector screen. The initial phase of the sine wave at the starting source(s) is zero. The diffraction angle is between -30 degrees and 30 degrees. The diffraction angle is defined as the acute angle between the horizontal straight line and the axis straight line about which the sine wave oscillates, as shown in figure 2-2. Each event is assigned a number that is the position (Y coordinate) the individual person hits the detector screen. Totally there are 5000 persons (corresponding to 5000 individual events) and they walk to and finally hits the detector screen finally.

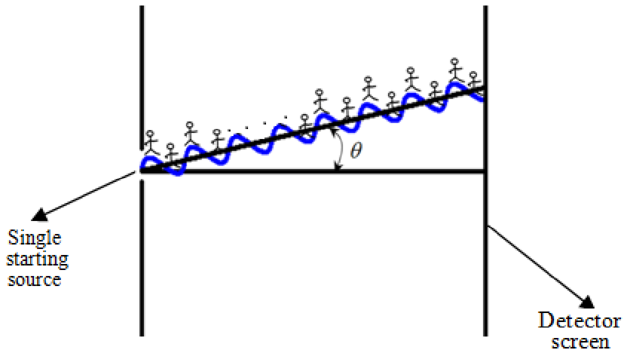

Figure 2-2: A person walks to a detector screen along a wavelike trajectory from a single starting source

A person travels along a wavelike trajectory, and the final position the person arrives at the detector screen is affected by the wavelength and the distance between the starting point and the detector screen. For an individual event, a person travels along a wave trajectory and arrives only at one definite place on the detector screen. In the simulation, finally after 5000 hitting the detector screen, the collective interference pattern emerges, as shown in figure 2-3.

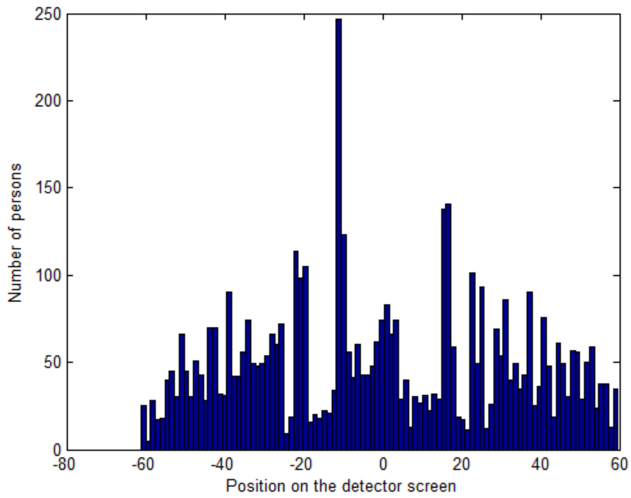

Figure 2-3: Collective interference formed by persons from a single starting source and walking along wavelike trajectories

As to an individual event with the diffraction angle of 15°, the X-coordinate of the final position at which the person hits is 100 meters, and the Y-coordinate of the final position(Y_{wave}) is calculated according to the formula below:

$$Y_{wave} = Y_{straightline} + A \cdot \wp(\varphi) / \cos(\theta) \quad (2\text{-}1)$$

where A is the amplitude of the wave, φ is the phase of the wave at the arrival position on the screen, θ is the diffraction angle, $\wp$ is a function that depends on the specific type of the wave, such as sine wave, triangle wave or square wave or saw tooth wave, $\wp(\varphi) = \sin(\varphi)$ if it is a sine wave path along which the person travels.

$Y_{straightline}$ is the Y coordinate of the position hit on the detector screen if the person travels along a straight line trajectory. In this case, $Y_{straightline} = 100 \times \tan(15^\circ) = 26.79$, and the amplitude of the sine wave is $A = 3$, and the diffraction $\theta = 15^\circ$. The phase of the sine wave when it hits the detector screen is

$$\varphi = \text{mod}(\sqrt{100^2 + 26.79^2}, 1.524) / 1.524 \times 2\pi = 5.85$$

where mod is the function to calculate the remainder. Finally we have $Y_{wave} = 26.79 + 3 \times \sin(5.85) / \cos(15^\circ) = 25.49$, which is the Y coordinate of the position hit on the detector screen by the person travelling along the sine wave trajectory. The representative number of this individual event is 25.49.

After the all the events that are the 5000 persons hit the detector screen one by one, as to a detector box (on the detector screen) being hit by the persons, it has a representative number that is the sum of the representative numbers of the individual events that happen in the box; the criterion for an individual event to happen in this box is that the position of the hit on the detector screen falls in this box. In the simulation, the length of the box is about 1.21 meters and there are 100 boxes on the detector screen. Figure 2-3 shows the collective interference formed by these 5000 individual events, and the numbers on the X-axis are the central positions of the detector boxes.

In the situation of double starting sources as shown in figure 2-4, the distance between the two starting sources is 1 meter. it is 50%-50% chance of starting from either source. The finally formed collective interference is shown in figure 3-5.

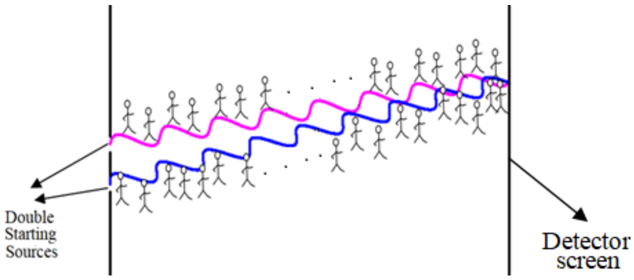

Figure 2-4: Two persons walk to a detector screen along two wavelike trajectories from two starting sources respectively

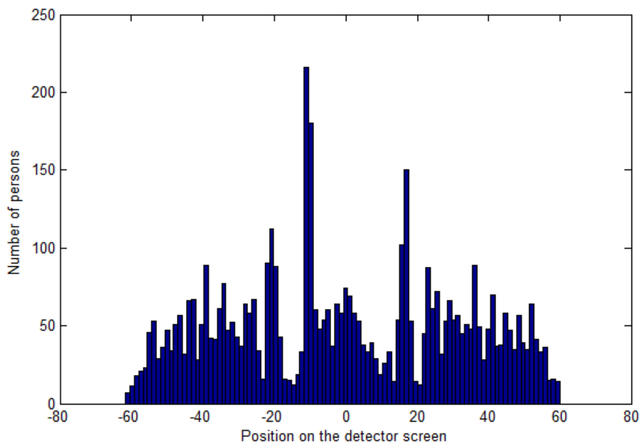

Figure 2-5: Collective interference formed by persons from two starting sources and walking along wavelike trajectories

2.3 Collective Interference Formed by Writing π

In this example, the irrational number Pi (3.1415912653...) is employed to build up the collective interference pattern. In the computer program to do the simulation, each digit of Pi occupies a unit length, and the decimal point also occupies a unit length. For example, the number 3.14159265 occupy the length of 10 units. The distance between the source and the detector screen is 100. The diffraction angle is between -30 degrees and 30 degrees. In the case of double starting sources, the distance between the two sources is 6. In figure 3-1, it shows writing Pi from a single source. For an individual event, the position on the detector screen receives the last digit is assigned the number equal to the digit. There are 50, 000 individual events and each event corresponds to a loop in the computer programs.

In figure 3-1, it shows the Pi is written from s single starting source. The collective interference formed by writing Pi with different diffraction angles and from a single starting source is shown in figure 3-2.

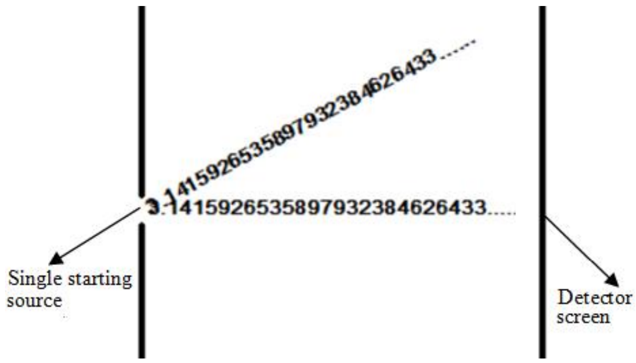

Figure 3-1: Writing Pi from a single starting source

As to an individual event with the diffraction angle of 13 degrees, since the distance between the source and the detector screen is 100, the X-coordinate of the the position (on the detector screen) hit by the letter is 100, the distance between the starting source and the position hit (on the detector screen) is $(102.63 \approx 100 \div \cos(13^{\circ}))$. Then we determine which of the digits of Pi finally hits the detector screen. The number 102.63 indicates that finally the 101th digit after the decimal point finally hits the detector screen. The reason is that the number 102.63 tells us that the 103th element of Pi finally hits and the first two elements of Pi is 3 and the decimal point '.'. It is known that the 101th digit after the decimal point of Pi is 8. The representative number of this individual event is this number 8.

After the all the events that are the 50, 000 persons hit the detector screen one by one, as to a detector box (on the detector screen) being hit by the persons, it has a representative number that is the sum of the representative numbers of the individual events that happen in the box; the criterion for an individual event to happen in this box is that the position of the hit on the detector screen falls in this box. In the simulation, the length of the box is about 0.23 meters and there are 500 boxes on the detector screen. Figure 3-2 shows the collective interference formed by these 50, 000 individual events, and the numbers on the X-axis are the central positions of the boxes.

In the case of double starting sources, the distance between the two sources is set as 6. It is 50%-50% chance of starting from either source to write Pi. In figure 3-4, it shows the Pi is written from two starting sources. The collective interference pattern formed by writing Pi from the two sources is shown in figure 3-5.

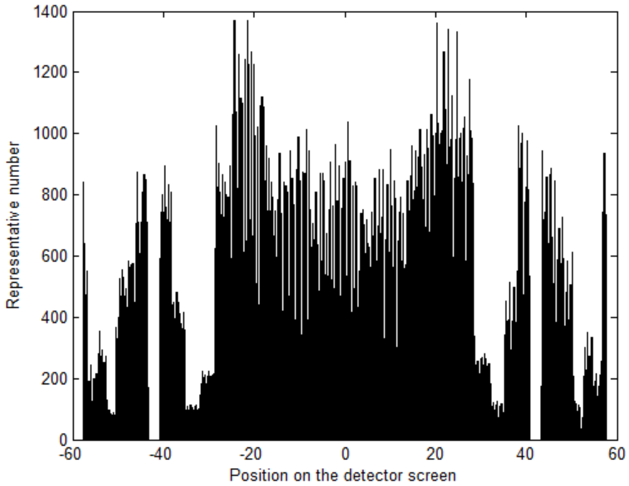

Figure 5-2: Collective Interference formed by Writing Pi from a single starting source

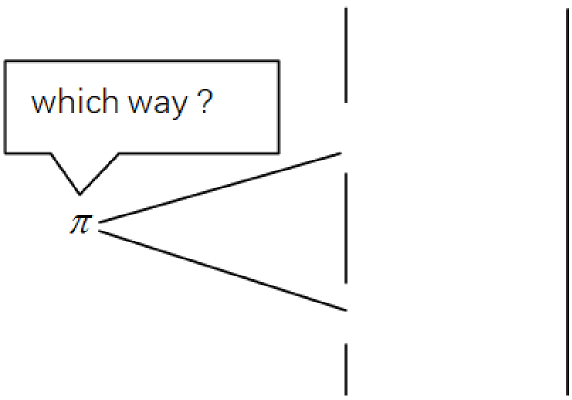

Figure 3-3: Two Possible Starting Sources to Write Pi

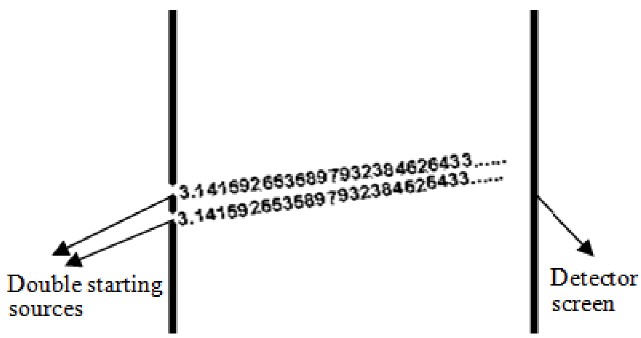

Figure 3-4: Writing Pi from two starting sources

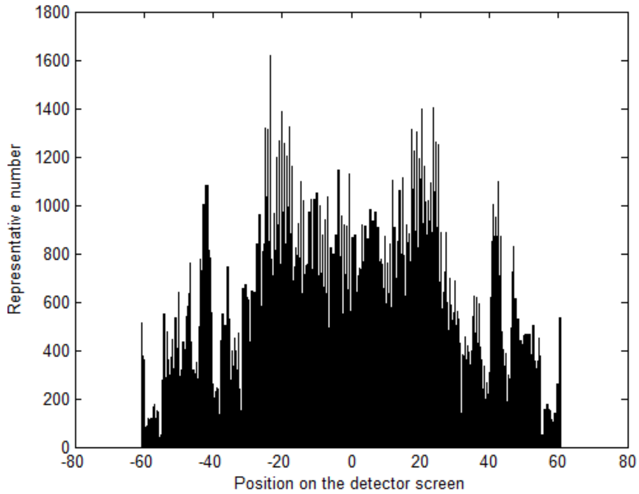

Figure 3-5: Collective Interference formed by Writing Pi from Two Starting Sources

2.4 Collective Interference formed by Persons' Continuous Walkings (forth and back) Inside a Space Interval

Image that a group people walk in a space interval one by one, and every individual person moves forth and back inside the interval. The length of the space interval is 9 and it is composed of 9 subintervals; each subinterval has a length of 1, and the length of one step every individual person moves is 1. The distance between the starting position and the left side of the first subinterval is half step. These conditions secure that after the person moves one step, the person is inside the next subinterval. The number of step(s) every person moves is according to the sequence {1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024…}. This means that the first person moves 1 step, the second person 2 steps, the third person 4 steps, and so on… The following shows how the collective interference emerges.

Event_1: The first person moves 1 step. In this event, the first person moves from a starting position (as shown in the subfigure (a) in figure 4-1) and then moves forward 1 step toward the right side and finally settles down in the 1st subinterval (as shown in the subfigure (b) in figure 6-1). The representative number of this event is 1.

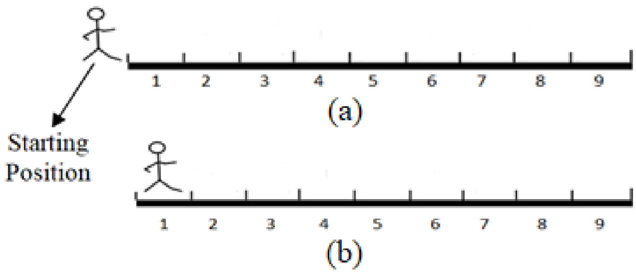

Figure 4-1: Event 1

Event_2: The second person moves 2 steps. In this event, the second person moves from a starting position (as shown in the subfigure (a) in figure 4-2) and then moves forward 2 steps toward the right side and finally settles down in the 2nd subinterval (as shown in the subfigure (b) in figure 4-2). The representative number of this event is 2.

International Journal of Science and Research (IJSR)
ISSN: 2319-7064
ResearchGate Impact Factor (2018): 0.28 | SJIF (2019): 7.583

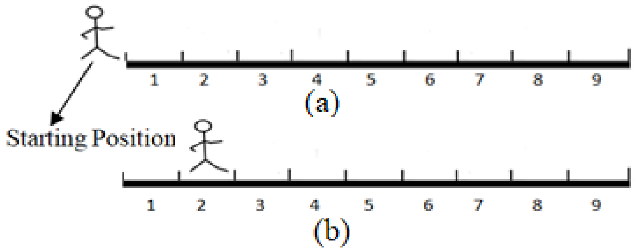

Figure 4-2: Event 2

Event_3: The third person moves 4 steps.. In this event, the third person moves from a starting position (as shown in the subfigure (a) in figure 4-3) and then moves 4 steps forward toward the right side and finally settles down in the 4th subinterval (as shown in the subfigure (b) in figure 4-3). The representative number of this event is 4.

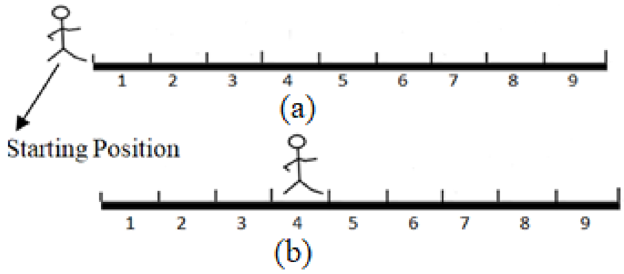

Figure 4-3: Event 3

Event_4: The fourth person moves 8 steps. In this event, the fourth person moves from a starting position (as shown in the subfigure (a) in figure 4-4) and then moves forward 8 steps toward the right side and finally settles down in the 8th subinterval (as shown in the subfigure (b) in figure 4-4). The representative number of this event is 8.

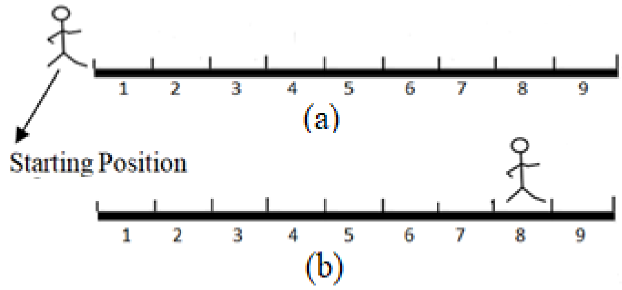

Figure 4-4: Event 4

Event_5: The fifth person moves 16 steps. In this event, the fifth person moves from a starting position (as shown in the subfigure (a) in figure 4-5), and then moves forward 9 steps toward the right side (to be in the 9th subinterval, as shown in the subfigure (b) in figure 4-5), and then moves backward 7 steps toward the left side and finally settles down in the 2nd subinterval (to be in the 2nd subinterval, as shown in the subfigure (c) in figure 4-5). The representative number of this event is 2.

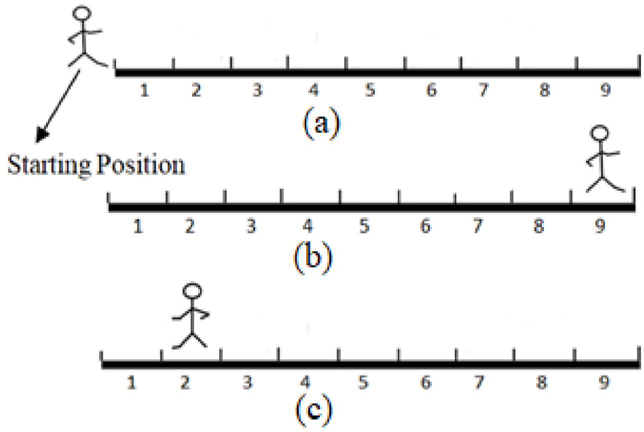

Figure 4-5: Event 5

Event_6: The sixth person moves 32 steps. In this event, the sixth person moves from a starting position (as shown in the subfigure (a) in figure 4-6), and then moves forward 9 steps toward the right side 9 steps (to be in the 9th subinterval, as shown in the subfigure (b) in figure 4-6), and then moves backward 9 steps toward the left side (to be in the 1st subinterval, as shown in the subfigure (c) in figure 4-6), and then moves forward 9 steps toward the right side (to be in the 9th subinterval, as shown in the subfigure (d) in figure 4-6), and then moves backward 5 steps toward the left side and finally settles down in the 4^{th} subinterval(to be in the 4th subinterval, as shown in the subfigure (e) in figure 4-6). The representative number of this event is 4.

Table 4-1 shows that numbers of step(s) and the final positions (in which subinterval) the individual person settles down. It can be seen that a periodical wavelike pattern emerges (2, 4, 8, 2, 4, 8, 2, 4, 8…). Figure 4-7 shows the representative wave for the collective interference formed by the persons walking in the space interval one by one and according to the sequence {1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024…}.

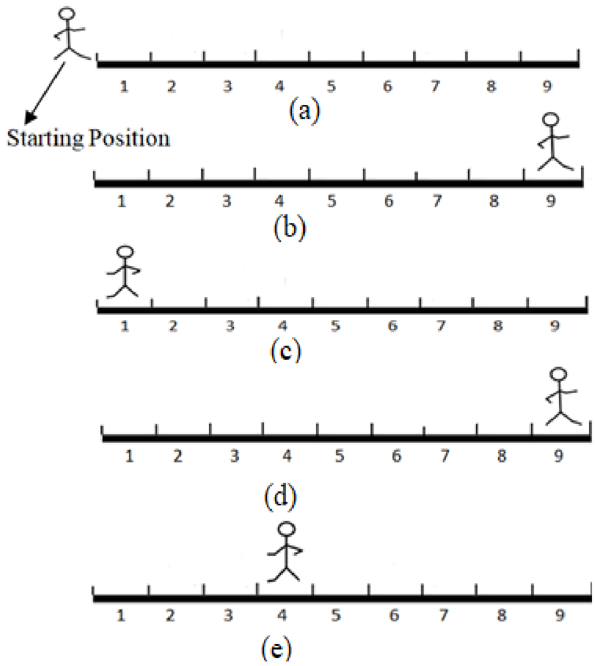

Figure 4-6: Event 6

Table 4-1: Numbers of Step(s) and Final Positions in the Subinterval

Number of step	Final position
1	1
2	2
4	4
8	8
16	2
32	4
64	8
128	2
256	4
512	8
1024	2
2048	4
4096	8
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.	.

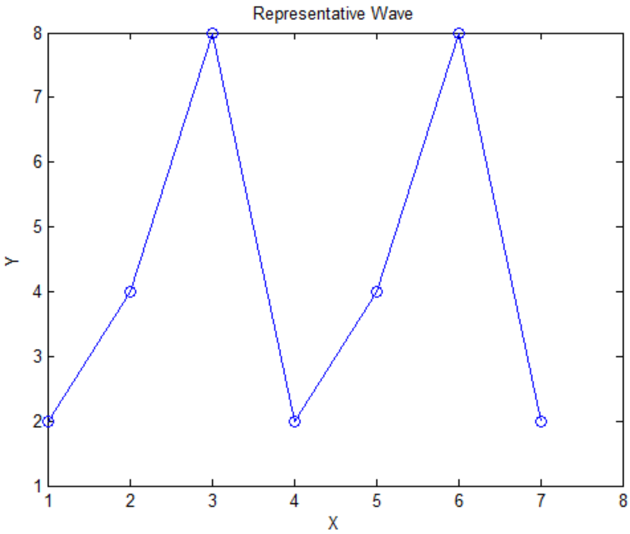

Figure 4-7: Representative wave of the collective interference built up by persons' continuous walkings (forth and back) in the space interval and the walkings are according to the sequence {1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024…}.

By the method above we can go further to find that collective interferences will occur if the group of persons walking one by one in the space interval according to the sequences including {1, 4, 16, 64, 256…} (in which the next element is four times of the previous one), {1, 5, 25, 125, 625, 3125…}(in which the next element is five times of the previous one), {1, 7, 49, 343, 2401, 16807…} (in which the next element is seven times of the previous one), {1, 8, 64, 512, 4096, 32768…}(in which the next element is eight times of the previous one). The figures (4-8 to 4-11) show the collective interferences manifested by the representative waves by theses sequences respectively.

However, the sequences including {1, 3, 9, 27, 81…} (in which the next element is three times of the previous one), {1, 6, 36, 216, 1296, 7776…} (in which the next element is six times of the previous one), and {1, 9, 81, 729, 6561, 59049…}(in which the next element is nine times of the previous one). The reason is that the individual person will finally continue to settle down in the 9th subinterval if walking according to these sequences.

Moreover, it is found that collective interference pattern will occur if the group of the persons walks in the space interval according to Fibonacci sequence {1 1, 2, 3, 5, 8, 13, 21, 34…} (in which the element is equal to the sum of the previous two elements), as shown in figure 4-12.

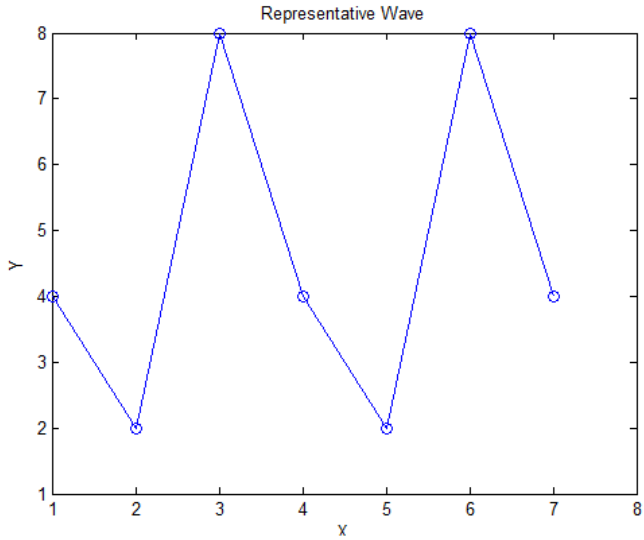

Figure 4-8: Representative wave of the collective interference built up by persons' continuous walkings (forth and back) in the space interval and the walkings are according to the sequence {1, 4, 16, 64, 256…}

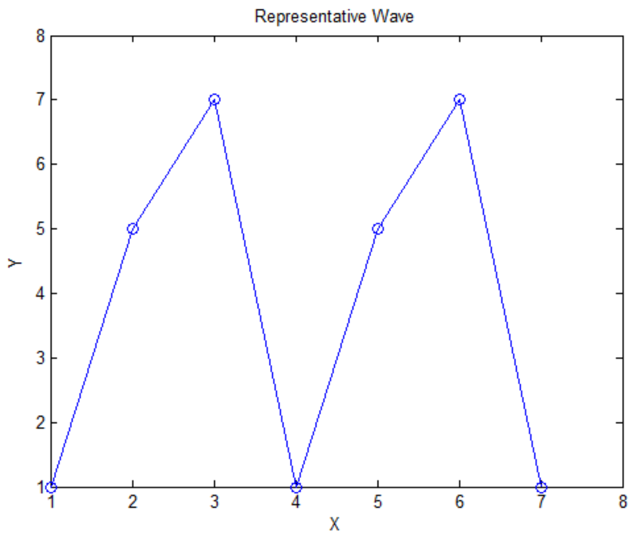

Figure 4-9: Representative wave of the collective interference built up by persons' continuous walkings (forth and back) in the space interval and the walkings are according to the sequence {1, 5, 25, 125, 625, 3125…}

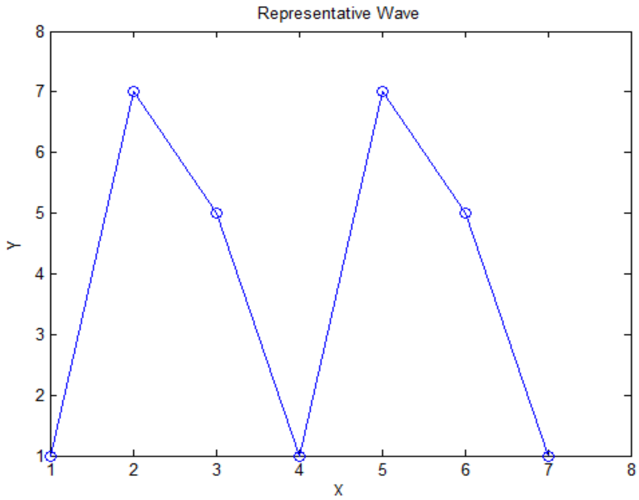

Figure 4-10: Representative wave of the collective interference built up by persons' continuous walkings (forth and back) in the space interval and the walkings are according to the sequence {1, 7, 49, 343, 2401, 16807…}

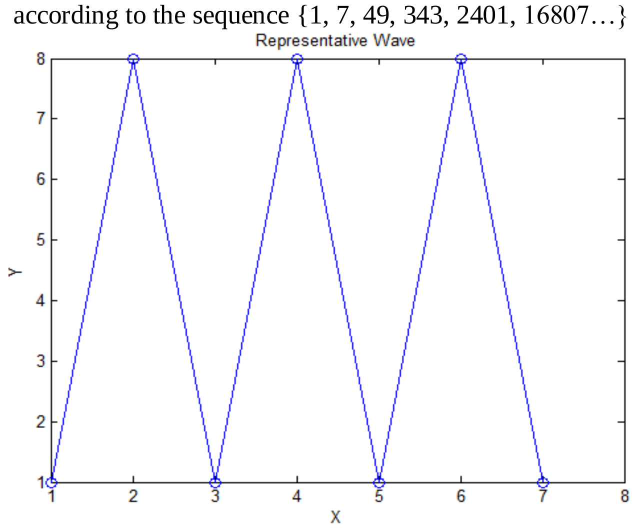

Figure 4-11: Representative wave of the collective interference built up by persons' continuous walkings (forth and back) in the space interval and the walkings are according to the sequence {1, 8, 64, 512, 4096, 32768…}

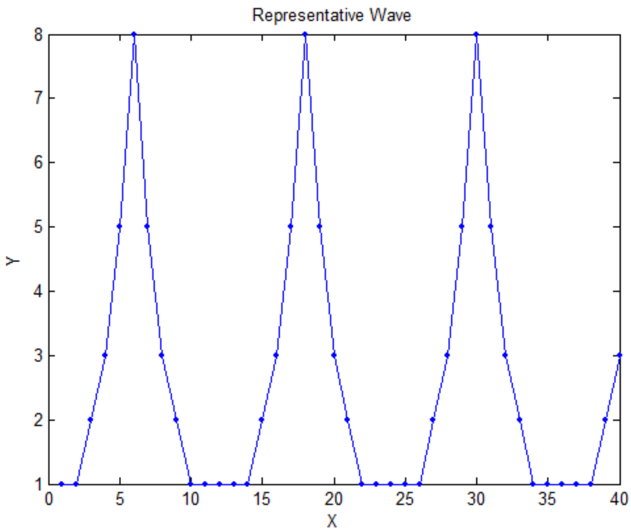

Figure 4-12: Representative wave of the collective interference built up by persons' continuous walkings (forth and back) in the space interval and the walkings are according to the Fibonacci sequence {1, 1, 2, 3, 5, 8…}

3. Conclusion

Collective interferences patterns are shown in macroscopic world. Totally there are seven specific examples provided to illustrate how the individual deterministic events are spoken out and then they form the collective interferences. As to the first three examples, the underlying mechanism can be generalized that a group of individual signals with varying intensities it a detector screen, and the differences of the paths (from the stating source to the hit position on the detector screen) cause the interferences. For the fourth example, the underlying mechanism for this happening is the employment of decimal system. Every numeration system has its own base, and the base is equal to the number of unique numeric symbols employed in the system. Decimal system (for which the base is 10) is widely used probably because of the fact that human beings have 10 fingers and it is convenient to count by use of this system. The numbers of 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 serve as the 'elementary particles' in a decimal system and work together to represent any number in this system. Based on the methods of the sixth and seventh examples, different collective interferences behaviour will be observed if other numerous systems (such as octal and hexadecimal systems) are employed. In an octal system, the 'elementary particles' are 0, 1, 2, 3, 4, 5, 6, 7, and the length of the space interval needs to be 7 to build up the collective interference patterns. In a hexadecimal system, the 'elementary particles' are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and the length of the space interval needs to be 15 to build up the collective interference patterns.

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