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Linear Programming Problem for Maximization of Profit in Rubber Manufacturing and Molding Industry: A Case study of Shreyas Rubber Products

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Abstract: ***The goal of any manufacturing industry to maximize the profit subject to the availability of raw material, number of machinery and labors, man-hours, transportation of cost, processing time product on machines etc..In this paper we showed that how Linear programming problem technique is used to maximization of profit with respect to the given limited resources. The Shreyas Rubber Products industry is working in the area of manufacturing different types of rubber products and its molding as per the demand in the Pune district area. In this study we consider four samples of rubber products namely 1-inch starring,1.5-inch starring, O-Ring four piece set and O-Ring twenty-five piece set which is obtained by mixture of raw material EPDM. The Simplex method of Linear Programming Problem is used to optimize the profit objective function.***

Keywords: Rubber products, L.P.P, Simplex Method

1. Introduction

A Linear programming problem is a technique to optimize ie. Maximize or minimize the objective function. The objective function either profit or cost function. It is a mathematical technique for finding optimal solution when there are limited resources like machine, labor, raw material, man-hours, and other facilities. The technique of Linear Programming Problem is applicable to problem in which the total effectiveness can be expressed as a linear function. The limitations on resources expressed in term of linear inequalities or equalities. According to Samulson, Dorfman and Solow defines "A Linear programming is analysis of problems in which a linear function of a number of variables is to be maximized or minimized when these variables are subject to a number of constraints in the form of linear inequalities"[6]. Linear Programming Problem is extensively used to solve a variety of industrial problem in both public and private sectors for achieving maximum profit by utilizing the available resources in an optimal manner. The general objective is to determine a effective plan for production in the time period subject to requirement / limited resources without violating any of the constraints. It is most widely used technique of managerial decision making in number of various fields in Industrial applications as product mix-problem, production scheduling, blending problem, transportation problem, communication industry, Rail road industry, Management applications, Economics, Administration applications, Diet Problem, Non-industrial application like agriculture, environment protection etc. To this extent, linear programming has proved useful in modeling diverse types of problems in planning, routing, scheduling assignment and design. Many practical problems in operations problems in operations research can be expressed as linear programming problems. Certain special cases of linear programming such as network flow problems and multi commodity flow problems are considered important enough to have generated much research on specialized algorithm for their solution. Historically, ideas from linear programming have inspired many of the central concepts of optimization theory such as Duality, Decomposition and the importance of convexity and its generalizations. Standard form is the usual and most intuitive form of describing a linear programming problem.

2. Decision Variables

The unknown variables which are used to represent products, services, projects etc. These variables are usually inter-related in terms of utilization of resources and need simultaneous solution. It is denoted by $x_1, x_2, x_3, \ldots, x_n$

2.1 Terms in Linear Programming Problem:

2.1.1 Objective Function: Objective function is a linear function in terms of decision variables. The aim of the study to optimize it i.e. either maximization or minimization.

2.1.2 Constraints: The limitation on resources like production capacity, raw-material, labor, man-hours, machines. These resources being expressed as linear inequalities in terms of decision variables known as constraints.

2.1.3 Non-Negative Constraints/Restrictions: In a real situation the negative values of physical quantities are not possible, thus all decision variables must be non-negative known as non-negative constraints/restrictions. When the problem involves "n" decision-making variables and "m" constraints, the model can be represented mathematically in the form of either maximization or minimization of the object function.

Generally, Linear Programming problem can be written in a canonical form as:

$$\mathrm{Max}(Z) \text{ or } \mathrm{Min}(Z) = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n$$

Subject to the condition

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n \leq \; = \; \geq b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n \leq \; = \; \geq b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n \leq \; = \; \geq b_3$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n \leq \; = \; \geq b_m$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, \ldots, x_n \geq 0$$

Where $x_1, x_2, x_3, \ldots, x_n$ represent the variables (to be determined) while c_i are profit per unit product of i type for $i = 1,2,3, \ldots, n$.

The linear expression Z to be maximization for profit function and to be minimization for cost function is called the objective function. The m inequalities $\leq \; = \; \geq$ are the constraints which the objective function is to be optimized.

3. Purpose of the Study

The Sheyas Rubber Product Industry located in Pune, Maharashtra, India. The industry manufactures rubber from raw material on high pressure mill machine and various types of rubber like ring, gaskets etc. produced using on high pressure molding hydraulic machine as per the demand in the area of city. The main purpose of study to determine exact analysis of four products namely 1-inch Starring, 1.5-inch Starring, O-Ring four pieces set, O-Ring twenty-five pieces set. Our aim is to effectively estimate which of these products must be given more attention or produced maximum profit compare to other products subject to the available resources like raw material rubber compound, processing time, temperature, labor cost, finishing machine cost. In this study, linear programming technique will be used to make the best possible use of the total available productive resources of Shreyas Rubber Product. After obtaining the conclusion of this study, this work will assist the management of Shreyas Rubber Products to make valid decision regarding the production of these four material so as to make an objective and fruitful decision for reduction in wastage of resources like raw material, processing time, temperature, labor cost etc. may be avoided.

3.1 Simple Method

The simplex method which is used to solve linear programming problems was developed by George B. Dantzig in 1947 as a product of his research work during World War II when he was working in the Pentagon with the Mil. The method is applicable to any problem that can be formulated in terms of linear objective function subject to a set of linear constraints. The method allows us to evaluate the corner points in such a way that each successive corner point gives the same or better solution than the solution obtained in previous iteration. The solution is tested for optimality, if the improvement is possible in solution at the previous iteration then at a new corner point the solution is again tested for optimality. This iterative search for a better corner points is repeated until an optimum solution determined if it exists.

3.1.1 Slack Variable: A non-negative variable which is added to the left hand side of less or equal constraint to convert constraint into equations is called as slack variable.

3.1.2 Surplus Variable: A non-negative variable which is subtracted from left hand side of greater or equal constraint to convert constraint into equations is called as surplus variable.

3.2 Simplex Algorithm

This method or algorithm is used to obtain an optimum solution by applying following step one by one to linear programming problem.

Step-1: Standard Form of L.P.P:
A Linear programming problem in which all constraints are written as equations by using slack or surplus variable.

Step-2: Basic Solution:
A Linear programming problem having m equations with n variables. A basic solution obtained by setting (n-m) of the variables equal to zero and solving equations for the values of other m variables ,if unique solution exists is called basic solution and the variable corresponds to basic solution is basic variable and other variables are non-basic variable.

Step-3: Basic Feasible Solution:
A basic solution satisfies non-negative constraints of Linear programming problem called as basic feasible solution.

Step-4: To construct simplex table.

Step-5: Z_j row:
In the simplex table the row computed using $Z_j = \sum_{j=1}^{k} C_B a_{ij}$, where C_B the coefficients of basic variable are in the objective function, a_{ij} are elements in the j^{th} column and k is the total variables including slack or surplus. The elements in Z_j row are gross profit or loss.

Step-6: $C_j - Z_j$ row (Index Row / net evaluation):
The element in $C_j - Z_j$ row represents net profit or loss. Where C_j are the coefficients of variables in the objective function and $1 \leq j \leq k$.

Step-7: Pivot Column (Key column):
In the current simplex table choose most positive element in $C_j - Z_j$ row, if found that column is called pivot column or key column and variable corresponding to this column is entering variable i.e. this variable enter in the current solution in the next iteration.

Step-8: Pivot Row (Key Row):
First to evaluate ratio as $\frac{X_B}{a_{ip}}$, where X_B basic feasible solution in current simplex table and a_{ip} are the element in the pivot column or key column which is identified in step-7 for $i = 1,2,3, \ldots, n$ .Choose least positive element in the ratio, if found that row is called pivot row and variable corresponding to this row is leaving or outgoing variable i.e. this variable leave from the current solution in the next iteration.

Step-9: Pivot Element (Key Element):
The element at the intersection of pivot row and pivot column is pivot element. Convert this pivot element to unity

and remaining element in pivot column is to be zero by elementary row operation and apply step-4 repeatedly till all elements in $C_j - Z_j$ row are non-positive i.e. $C_j - Z_j \le 0$ go to step-10.

Step-10: Optimum Solution:
If all elements in $C_j - Z_j$ row are non-positive i.e. $C_j - Z_j \le 0$ then the current solution in the simplex table is optimal.

Problem: The Shreyas Rubber Product industry manufactures four type of rubber 1-inch Star Ring, 1.5- inch Star Ring, O-Ring four pieces set and O-Ring twenty-five pieces set. The raw material, processing time on molding machine, Finishing Time and profit per unit piece of each product per day is given in table below.

	Products				
	1-inch Star Ring	1.5-inch Star Ring	O-Ring four pieces set	O-Ring four pieces set	Availability
Raw Material EPDM	110 gm	30 gm	40 gm	21 gm	7500 gm
Processing Time on Molding Machine	8 min	5 min	5 min	6 min	420 min
Temperature in degree Celsius	200	200	200	200	200
Finishing Time	10 min	20 min	1 min	5 min	480 min
Profit	Rs.4	Rs.3.4	Rs.-1	Rs.0.15	

To find the optimum production of these four types of rubber products so as to maximize profit. The given problem converts into mathematical model as linear programming problem.

Let us define x_1 is the number of units manufactures of 1 inch starring , x_2 is the number of units manufactures of 1.5-inch starring. x_3 is the number of units manufactures of O-Ring ring four piece set, x_4 is the number of units manufactures of O-Ring ring twenty-five piece set.

4. Formulation of Linear Programming Problem

4.1 Objective Function

Profit for 1 unit of product1-inch straring,1.5-inch starring, O-Ring four piece set, O-Ring twenty-five piece set is Rs.4.00,Rs.3.40,Rs.-1.00 and Rs.0.15 respectively. Therefore profit for x_1, x_2, x_3, x_4 units of these four products is $\text{Rs.}4.00x_1$, $\text{Rs.}3.40x_2$, $\text{Rs.}-1.00x_3$, $\text{Rs.}0.15x_4$ respectively. Therefore total profit of firm is $Z = 4.00x_1 + 3.40x_2 - 1.00x_3 + 0.15x_4$

4.2 Constraint-1(Raw material EPDM)

EPDM required for 1 unit of product1-inch starring, 1.5-inch starring ,O-Ring four piece set, O-Ring twenty-five piece set is 110 gm ,30 gm ,40 gm and 21 gm respectively. Therefore for x_1, x_2, x_3, x_4 units of these four products is $110x_1$ gm , $30x_2$ gm , $40x_3$ gm , $21x_4$ gm respectively. Therefore total raw material required $110x_1 + 30x_2 + 40x_3 + 21x_4$ and available is 7500 gm. Therefore $110x_1 + 30x_2 + 40x_3 + 21x_4 \le 7500$

4.3 Constraint-2(Processing Time) : Processing time required for 1 unit of product1-inch starring, 1.5-inch starring, O-Ring four piece set, O-Ring twenty-five piece set is 8 min, 5 min, 5 min and 6 min respectively. Therefore time required for x_1, x_2, x_3, x_4 units of these four products is $8x_1$ min , $5x_2$ min, $5x_3$ min, $6x_4$min respectively. Therefore total processing time required $8x_1 + 5x_2 + 5x_3 + 6x_4$ minutes and available time is 420 min.
Therefore $8x_1 + 5x_2 + 5x_3 + 6x_4 \le 420$

4.4 Constraint-3(Finishing Time):

Finishing time for 1 unit of product1-inch starring, 1.5-inch starring, O-Ring four piece set, O-Ring twenty-five piece set is 10 min, 20 min, 1 min and 5 min respectively. Therefore for time required for x_1, x_2, x_3, x_4 units of these four products is $10x_1$ min , $20x_2$ min , $1x_3$ min, $5x_4$ min respectively. Therefore total finishing time is $10x_1 + 20x_2 + 1x_3 + 5x_4$ minutes and available time is 480 minutes. Therefore $10x_1 + 20x_2 + 1x_3 + 5x_4 \le 480$
Hence the given problem as L.P.P is

$$\text{Max } Z = 4.00x_1 + 3.40x_2 - 1.00x_3 + 0.15x_4$$
Subject to the condition
$$110x_1 + 30x_2 + 40x_3 + 0.15x_4 \le 7500$$
$$8x_1 + 5x_2 + 5x_3 + 6x_4 \le 420$$
$$10x_1 + 20x_2 + 1x_3 + 5x_4 \le 480$$
$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, x_4 \ge 0$$

Standard Form of L.P.P. using slack variable s_1, s_2, s_3 is
$$\text{MaxZ} = 4.00x_1 + 3.40x_2 - 1.00x_3 + 0.15x_4 + 0s_1 + 0s_2 + 0s_3$$
Subject to the condition
$$110x_1 + 30x_2 + 40x_3 + 21x_4 + s_1 = 7500$$
$$8x_1 + 5x_2 + 5x_3 + 6x_4 + s_2 = 420$$
$$10x_1 + 20x_2 + 1x_3 + 5x_4 + s_3 = 480$$

Here number of variables $n = 7$ and number of constraints $m = 3$.For basic solution set $(n - m) = 7 - 3 = 4$ of the variable equal to be zero. Let $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, $x_4 = 0$, by solving constraints value of other variables are $s_1 = 7500, s_2 = 420, s_3 = 480$ are initial basic solution.

Table 1: Initial Simplex Table

cb	C_j	4	3.4	-1	0.15	0	0	0		
v	B.V.	x_1	x_2	x_3	x_4	s_1	s_2	s_3	X_B	Ratio
0	s_1	110	30	40	21	1	0	0	7500	68.2
0	s_2	8	5	5	6	0	1	0	420	52.5
0	s_3	10	20	1	5	0	0	1	480	48
	Z_j	0	0	0	0	0	0	0		
	$C_j - Z_j$	4	3.4	-1	0.15	0	0	0		

In the table-1,Most positive element in $C_j - Z_j$ row is 4 therefore corresponding column is pivot element and variable x_1 is entering variable and along row least positive ratio is 48,thus row is pivot row and corresponding variable s_3 is leaving variable.

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Table 2: Second Simplex Table

cb	C_j	4	3.4	-1	0.15	0	0	0	
v	**B.V.**	x_1	x_2	x_3	x_4	s_1	s_2	s_3	X_B
0	s_1	0	-190	29	-34	1	0	-11	2220
0	s_2	0	-11	4.2	2	0	1	-0.8	36
4	x_1	1	2	0.1	0.5	0	0	0.1	48
	Z_j	4	8	0.4	2	0	0	0.4	
	$C_j - Z_j$	0	-4.6	-1.4	-1.85	0	0	-0.4	

In table-2, all elements in $C_j - Z_j$ row are non-positive i.e. $C_j - Z_j \leq 0$ thus the current solution in the second simplex table is optimal. Finally solution of problem is

$$x_1 = 48,\ x_2 = 0,\ x_3 = 0,$$
$$x_4 = 0, s_1 = 2220,\ s_2 = 36, s_3 = 0$$

5. Conclusion

The data of four samples of rubber products collected from Shreyas Rubber Product Industry namely 1-inch Starring, 1.5 inch Starring, O-Ring four piece set and O-Ring twenty-five piece set and after finding the solution using Simplex method It was observe that, if Shreyas Rubber Products industry manufactures first type of product i.e. 1-inch Starring only with number of units 48 per day and no other three products manufacture i.e. zero production then firm has a total maximum profit Rs. is

$$\max(Z) = 4.00x_1 + 3.40x_2 - 1.00x_3 + 0.15x_4$$
$$\max(Z) = 4 * 48 + 3.40 * 0 - 1.00 * 0 + 0.15 * 0$$
$$\max(Z) = 192$$

6. Recommendation

After completing research work and observation, We thereby strongly recommended to the management of Shreyas Rubber Product Industry is that they only manufactures 1-inch starring rubber product and no other three products will produced then firm achieve highest profit subject to the available limitation of resources .

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