# Effect of Magnetic Field on Oscillatory Blood Flow in Multistenosed Artery

**Tanuj Kumar Rawat[1], Shivangi Kumari[2], S. P. Singh[3]**

Department of Mathematics, Dayalbagh Educational Institute, Deemed University Agra, India

**Abstract:** ***The present paper investigates the effect of magnetic field on oscillatory blood flow in multistenosed artery, and flow of blood is considered as laminar, incompressible and non-Newtonian. The results are obtained for oscillatory blood flow using shooting method. Numerical solution obtained for the axial velocity of fluid, flow rate, wall shear stress and impedance. Results show graphically. We observed that axial velocity of fluid decreases as the increase of the values of the Hartmann number and the frequency parameter. Axial velocity and flow rate increases as increase the Reynolds number, decreases as increase the value of Hartmann number. Wall shear stress increases as increase Reynolds number and Impedance to the flow increase due to increase of Reynolds number as well as Hartmann number.***

**Keywords:** magnetic field, oscillatory blood flow, multistenosed artery, shooting method, Hartmann number, Reynolds number

## 1. Introduction

The study of blood flow through arteries is of utmost importance in many cardiovascular diseases, particularly atherosclerosis. Atherosclerosis means the abnormal and unnatural growth in the lumen of an artery that develops at various locations in the cardiovascular system under unfavorable conditions. The actual cause of developmentof abnormal growth in the artery is not known but its effect over the cardiovascular system has been determined by studying blood flow in its vicinity. The flow of blood through an artery depends upon the pumping action of the heart that give rise to a pressure gradient which produces an oscillatory flow in the blood vessel Halder [1]. Womersley [2] studied the oscillatory motion of a viscous fluid in a rigid tube under a simple harmonic pressure gradient and examined the influence of frequency on the instantaneous flow rate. Misra and Shit [3] studied the effect of magnetic field on blood flow through an artery in unsteady situation and observed the effect of magnetic parameter. Sanyal [4] studied the effect of magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration. Bhuyan and Hazarika [5] studied the magnetic effect on flow through circular tube of non-uniform cross-section with permeable walls. Haldar [6] studied the oscillatory flow of blood in a stenosed artery under a simple harmonic pressure gradient and examined the effect of stenosis on the flow field by considering blood as a Newtonian fluid. Srivastava [7] investigated the MHD blood flow in a porous inclined stenotic artery under the influence of inclined magnetic field considering blood as electrically conducting Newtonian fluid. Gujral et.al., [11] investigate the effect of bypass in stenosed artery using Herschel - Bulkley fluid model. They [12] also studied of radial variation of viscosity on blood flow through overlapping stenosed artery, by taking a non- Newtonian fluid model. Parvin [13] studied the blood flow through arterial stenosis in presence of external magnetic field. An attempt has been made in this analysis to study the oscillatory flow of blood in a multistenosed artery under the influence of transverse magnetic field.

## 2. Formulation of the Problem

The geometry of the arterial segment having the multiple stenoses mathematically is given by [9]

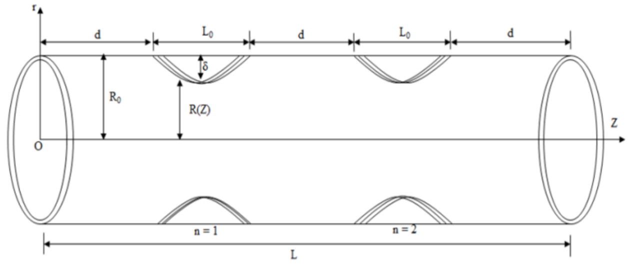


**Figure 4:** Geometry of multiple stenoses [9]

$$\frac{R(z)}{R_0} = \begin{cases} 1 - A[L_0^{s-1}(az - nd - (n-1)L_0) - (az - nd - (n-1)L_0)^s] & ; n(d + L_0) - L_0 \le az \le n(d + L_0) \\ 1 & ; \ otherwise \end{cases}$$

Where $h = \frac{\delta}{R_0}$, $A = \frac{h}{L_0^s} \frac{s^{s/(s-1)}}{(s-1)}$

$\delta(<< R_0)$ is the maximum height of the stenosis.

$$Z = \frac{1}{a}\left\{nd + (n-1)L_0 + \frac{L_0}{s^{1/(s-1)}}\right\}$$

Where R(z) and $R_0$ are the constricted and unconstricted radius of the artery respectively. Let L be the length of the artery, $L_0$ is te length of the stenosis, d is distance between equi spaced point, n is number of the stenosis, $a(\geq 1)$ is a parameter and $s(\geq 2)$ is the shape parameter. The governing equations may be written in the cylindrical coordinate system $(r, \theta, z)$ as-

**Equation of continuity-**

Consider the velocity along u ,0, w along r ,$\theta$ ,z respectively

$$\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + \frac{u}{r} = 0 \text{------ (2)}$$

**Equation of motion in axial direction-**

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) \quad (3)$$

**Equation of motion in radial direction-**

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \sigma B_0^2 w \quad \text{----(4)}$$

Where r and u are the radial coordinates and velocity; z and w are the axial coordinate and velocity; $B_0$ is the applied magnetic field in r direction.
As axially-symmetric flow in a rigid circular tube of radius $R_0$ is considered, for which
u=0, v=0, w=w(r, z, t), p = p(r, z, t).
So the equations (2), (3) and (4) becomes respectively as-

$$\frac{\partial p}{\partial r} = 0\,; \qquad \text{------(5)}$$

$$\frac{\partial w}{\partial z} = 0\,; \qquad \text{------ (6)}$$

$$\rho\left(\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \sigma B_0^2 w \text{ - (7)}$$

(7) Using (5) and (6), (7) becomes-

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \frac{\sigma}{\rho}B_0^{\,2} w \qquad \text{------- (8)}$$

The boundary conditions are-
w = 0 at r = R (z), no slip at the wall.

$\frac{\partial w}{\partial r} = 0$ at r = 0, symmetry about the axis.

## 3. Solution Procedure

It is convenient to write these equations in dimensionless form by means of the following transformation variables-

$$r' = \frac{r}{R_0},\quad z' = \frac{z}{R_0},\quad R' = \frac{R}{R_0},\quad w' = \frac{w}{w_0},\quad t' = \frac{t}{t_0},\quad p' = \frac{p}{\rho w_0^2},$$

Where $w_0$, $R_0$, p, $\mu$, and $B_0$ are the average velocity, radius in the unobstructed tube, pressure, density, viscosity of blood and the applied magnetic field in radial direction respectively.

Then the equation (8) reduces to the form

$$\frac{1}{R_t}\frac{\partial w'}{\partial t'} = -\frac{\partial p'}{\partial z'} + \frac{1}{R_e}\left(\frac{\partial^2 w'}{\partial r'^2} + \frac{1}{r'}\frac{\partial w'}{\partial r'}\right) - \frac{1}{R_e}M^2 w' \quad \text{----- (10)}$$

Where w′= w′ (r ′, t′) is the velocity in the axial direction.
The boundary conditions (dimensionless form) becomes-

$$\left.\begin{array}{l} \text{Where } w' = 0 \text{ at } R'(z'), \text{no slip at the wall.} \\ \frac{\partial w'}{\partial r'} = 0 \text{ at } r' = 0, \text{symmetry about the axis.} \end{array}\right\} \text{- (11)}$$

Let the solution for w and p be set in the form –

$$\left.\begin{array}{l} w(r,t) = W(r)e^{i\omega t} \\ -\frac{\partial p}{\partial z} = P(z)e^{i\omega t} \end{array}\right\} \qquad \text{-------------- (12)}$$

Also consider W(r) = U(r) + i V(r)

Where U(r) and (r) are respectively the real and imaginary parts of W(r) Substituting the dimensionless form of (12) in (10) and separating real and imaginary parts and dropping the prime signs we get

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - M^2 U = -PR_e R_0 - \beta^2 \qquad \text{------------- (13)}$$

$$\frac{d^2V}{dr^2} + \frac{1}{r}\frac{dV}{dr} - M^2 V = \beta^2 \qquad \text{----------- (14)}$$

The boundary conditions are-
U = 0, V = 0 at r = R(z), no slip at the wall.

$\frac{dU}{dr} = 0,\ \frac{dV}{dr} = 0 \ at\ r = 0$, symmetry about the axis.

Where

$R_t = \frac{t_0 w_0}{R_0}$ (non-dimensional time parameter)

$R_e = \frac{\rho R_0 w_0}{\mu}$ (Reynolds number)

M = $\sqrt{\frac{\sigma}{\mu}}B_0 R_0$ (Hartmann number)

$\beta^2 = \frac{\rho R_0^2 \omega}{\mu}$ (non-dimensional frequency parameter)

The volumetric flow rate Q is given by –

$$Q = 2\pi\int_0^{R(z)} wr dr$$

The real part of volumetric flow rate Q in dimensionless form (dropping the prime sign) is

$$Q = 2\pi\int_0^{R(z)} (Ur\sin\omega t - Vr\cos\omega t)\,dr$$

Where

$Q' = \frac{Q}{Q_0}$, $Q_0 = \frac{\pi C R_0^4}{8\mu}$ and $w_0 = \frac{C R_0^2}{4\mu}$

The shear stress at the wall r = R is defined as

$$\tau_R = -\mu\left(\frac{\partial w}{\partial r}\right)_{r=R(z)}$$

The real part of wall shear stress in dimensionless form (drop- ping the prime sign) is

$$\tau_R = -\frac{1}{2}\left[\frac{\partial}{\partial r}(Ur\sin\omega t - Vr\cos\omega t)\right]_{r=R(z)}$$

Where $\tau' = \frac{\tau_R}{\tau_0}$, and $\tau_R = \frac{CR_0}{2}$

The resistive impedance to the flow is defined by-

$$Z = \frac{-\frac{\partial p}{\partial z}}{Q}$$

In which the right hand side is known and can be obtained from equation.

## 4. Results and Discussion

The given problem is reduce to boundary value problem given by the equations (13) and (14). Solved these equation numerically using Shooting method (reduce boundary value problem to its initial value problem). Calculations have been done for various combinations of parameters i.e. the magnetic parameter (Hartmann numbers M), Reynolds number Re, Frequency parameter β under no slip conditions. Axial velocity profiles, flow rate, wall shear stress and flow impedance are computed for the various parameters.

Results are discuss graphically by using the following parameter values - ωt = 0.79, Lo = 6, d = .5, δ=.06, C=1.8, Ro=1. Values of Hartmann number, M=0,1,2,3,; Reynolds number Re = 1,2, 3,4,5 , 6 ,7 and frequency parameter β = 0,1,2,3 are used in this analysis.

In this problem see the effect of the parameters on the velocity field, flow rate, impedance as well as wall shear stress. The non-dimensional frequency parameter β plays an important role in this discussion and characterizes the dynamics flow patterns. The results for flow rate, impedance and the wall shear stress are explained with the help of this parameter.

Effect of axial velocity (u) with radial distance (r) varying magnetic parameter (Hartmann no M) .

Axial velocity (U) with radial direction (r) for different value of the magnetic parameter (Hartmann number M). The figure 2 shows that the axial velocity decrease with the intensity of magnetic field increases M (Hartmann number) with radial distance and the different values of M axial velocity decreases with the increase in radial distance. When magnetic field (Hartmann number M) increases fluid velocity decreases rapidly. It is also observed that fluid velocity is maximum at the axis of the tube and decreases towards the wall.

The axial velocity (U) with radial direction (r) for varying the values of β (frequency parameter). This figure 3, shows that the axial velocity decrease with increases the value of β with radial distance and the different values of β axial velocity decreases with the increase in radial distance. When value of β increases fluid velocity decreases rapidly. It is also observed that fluid velocity is maximum at the axis of the tube and decreases towards the wall.

In figure 4, we observe that the axial velocity for different values of Reynolds number Re. The axial velocity increases with increases the values of Reynolds number. Again for all the cases the fluid velocity is seen to be maximum at the axis of the tube and then decreases towards the walls.

Study of volumetric flow rate Q with frequency parameter β for different values of Hartmann number M and Reynolds number Re. In figure 5 we observe that flow rate shows an increasing curve for range $0 \leq \beta \leq 3$, then decreases and in presence of stenosis flow rate increases due to narrowing artery radius decreases and pressure increases in stenosis region then flow rate increases, increase the value of β then flow rate decrease. Also observe that for a particular value of the frequency parameter β, the flow rate decreases with the increase in the values of Hartmann number M.

In figure 6 we observe that effect of Reynolds number Re on flow rate Q with frequency parameter β. It is observed that flow rate shows an increasing curve for range $0 \leq \beta \leq 3$, and in presence of stenosis flow rate increases due to narrowing artery radius decreases and pressure increases in stenosis region then flow rate increases, increases the value of β then flow rate decrease. Also observe that for a particular value of the frequency parameter β, the flow rate decreases. The flow rate increases with increases the different value Reynolds number Re.

In figure 7, the effect of Reynolds number Re on wall shear stress with frequency parameter β is shown. It is seen that as Reynolds number increases the shear stress also increases. Also the shear stress rises for $0 \leq \beta \leq 3$ and then decreases and in presence of stenosis flow rate increases due to narrowing artery radius decreases and pressure increases in stenosis region then flow rate increases, increases the value of β then flow rate decrease. Also observe that for a particular value of the frequency parameter β, the flow rate decreases. The flow rate increases with increases the different value Reynolds number Re.

It is observe that of resistive impedance Z with frequency parameter β is shown in figure 8, for different values of Hartmann number M. It is seen that for $0 \leq \beta \leq 2$, as the Hartmann number M increases, impedance to flow also increases and for $\beta > 4$, with the increase in Hartmann number, the impedance to flow decreases. Also for a particular value of Hartmann number, the impedance Z increases with the increase in the frequency parameter β.

In figure 9 shows the variation of impedance to the flow Z with frequency parameter β for different values of Reynolds number Re. As the Reynolds number increases, the impedance to flow Z decreases. Also for a particular value of Reynolds number, flow impedance Z increases with the increase in frequency parameter β.

## 5. Conclusions

In this analysis, a laminar, incompressible, flow of blood in a multi stenotic region in presence of a external magnetic field. The fluid velocity, flow rate, impedance and wall shear stress are examined graphically. From the above discussions the following observations have been made:

1) The fluid velocity decreases as the external magnetic field apply and as its intensity is increases fluid velocity decreases.
2) Fluid velocity decreases for the increase in the values of the frequency parameter β while increases for the increasing in the values of the Reynolds number.
3) Flow rate decreases with the increase of magnetic field intensity and increases with the increase of Reynolds number.
4) Wall shear stress shows higher values with the increase of Reynolds number.
5) Impedance shows higher values with the increase in the values of frequency parameter and shows decrease in value with the increase in the values of Reynolds number.

Thus the mathematical expressions may help medical practitioners to control the blood flow of a patient by applying a suitable magnetic field.

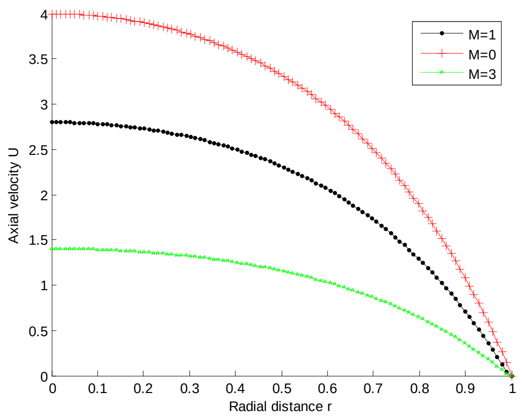


**Figure 2:** Variation of axial velocity with radial distance for different values of Hartmann number M at Re = 10 and $\beta = 0$ .

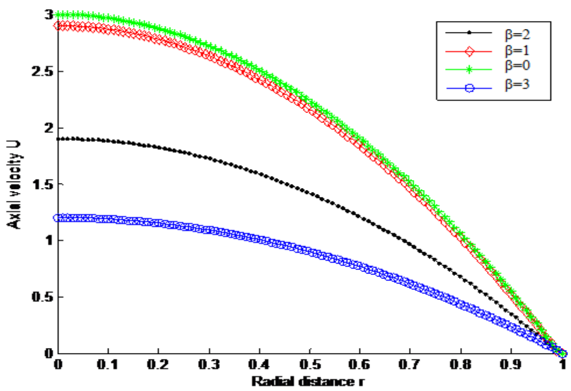

**Figure 3:** Variation of axial velocity with radial distance for different values of Frequency parameter β at Re= 10 and M=1.2 .

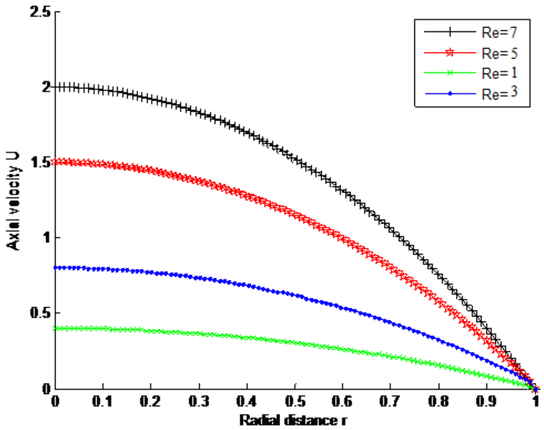


**Figure 4:** Variation of axial velocity U with radial distance r for different values of Reynolds number Re at M= 1.2. and β=1 .

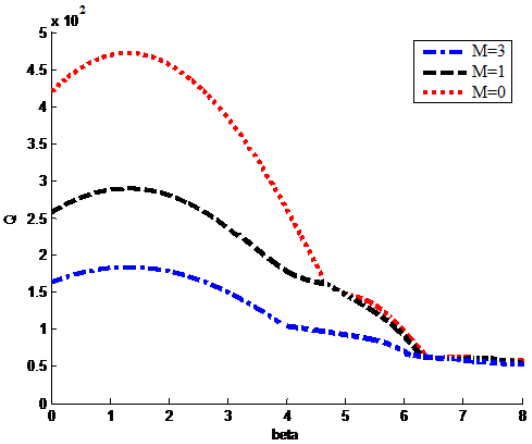


**Figure 5:** Variation of flow rate Q with frequency for different values of Hartmann number M at Re=10

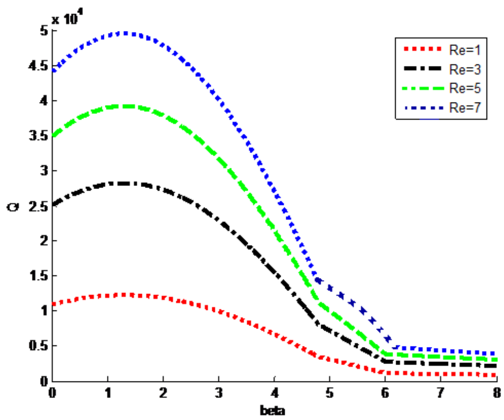


**Figure 6:** Variation of flow rate Q with frequency for different values of Reynolds number Re and M=0

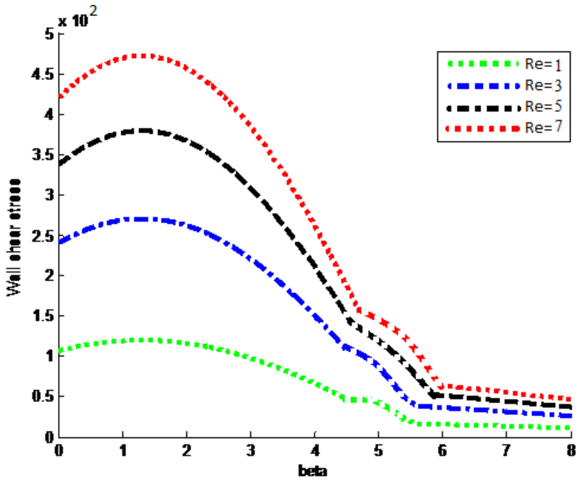


**Figure 7:** Variation of wall shear stress with frequency for different values of Reynolds number Re .

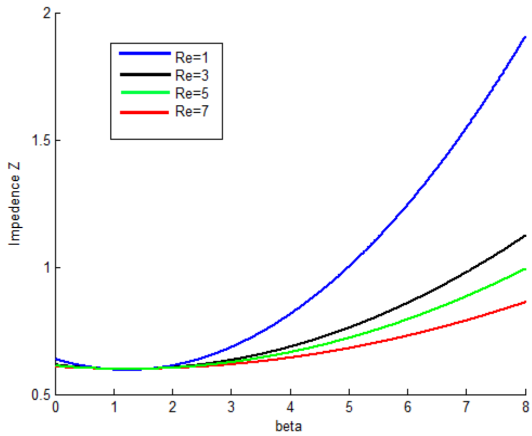


**Figure 8:** Variation of impedance Z with frequency for different values of Reynolds number Re.

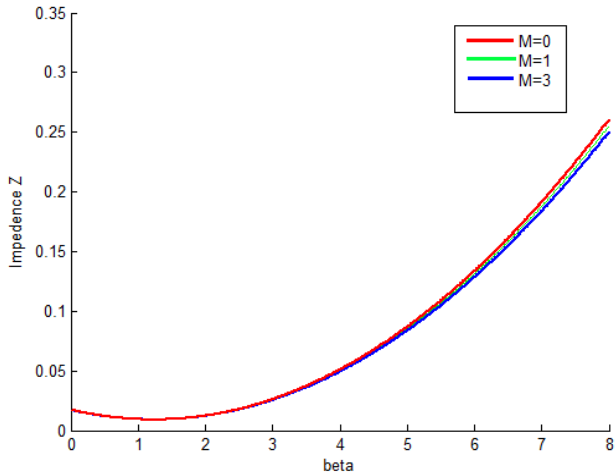


**Figure 9:** Variation of impedance Z with frequency for different values of Hartmann number M.

## References

[1] Haldar, K., "Effect of the shape of stenosis on the resistance to blood flow through an artery," Bulletin of Mathematical Biology, 47(4), 545-550.(1985)

[2] Womersley, J.R."Oscillatory motion of a viscous liquid in a thin-walled elastic tube" Phil. Mag. 46, 199-221(1955).

[3] Misra, J.C. and Shit, G.C, "Effect of magnetic field on blood flow through an artery: A numerical model," 12(4), 3-16 (2007).

[4] D. C. Sanyal , "effect of magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration, Journal of Physical Sciences", Vol., 43-56 43 (2007).

[5] Bhuyan, B.C. and Hazarika, G.C. , "Magnetic effect on flow through circular tube of non-uniform cross-section with permeable walls," Applied Science Periodical,5(1), 20-27.(2003)

[6] K. Haldar, "Oscillatory flow of blood in a stenosed artery," Bulletin of Mathematical Biology, 49(3), 279-287.(1987)

[7] LM., Srivastava, "Analysis of flow characteristics of the blood flowing through an inclined tapered porous artery with mild stenosis under the influence of an inclined magnetic field". *Journal of Biomechanics"*:18(7):479–485(2014).

[8] J.N Kapur, Mathematical Models in Biology and Medicine, Affiliated East-West Press Private Limited(1992).

[9] S. Gupta,(2014) Ph.D.:"Modeling and analysis of blood flow through a stenosed artery" Thesis, Department of mathematics, Dayalbagh Educational Institute Agra .

[10] G.Hazarika and B.Sharma, "Magnetic field effect on oscillatory flow of blood in a stenosed artery" Proceedings of International Conference on Frontiers in Mathematics (2015).

[11] K.Gujral and S.P.Singh "Modeling and analysis of the effect of bypass in stenosed artery using Herschel - Bulkley fluid model" publication in Pacific Journal of Applied MathematicV ol.9 No. 2 (2017).

[12] K.Gujral and S.P.," Analytical study of radial variation of viscosity on blood flow through overlapping stenosed artery: A non- Newtonian fluid model" Aligarh Bulletin of Mathematics Vol. 36 No. 1-2 pp 35-55 (2017).

[13] S. Parvin, " Numerical analysis of a blood flow model for arterial stenosis in presence of external magnetic field" AIP conference proceedings Vol.2121(2019).