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Improved Ratio Type Estimators Using Auxiliary Attribute for Population Variance

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Abstract: ***This paper proposes a family of estimators based on the auxiliary information on a attribute. The bias and mean squared error are obtained up to the first order of approximation. The theoretical comparison are also supported by numerical examples based on the two natural populations, showing the superiority of the suggested family of estimators, both theoretically as well as empirically over estimators available in literature.***

Keywords: Ratio type estimator, bias, mean squared error, percent relative efficiency

1. Introduction

In sampling theory, the use of auxiliary information, is always beneficial in order to get more efficient estimates of the population parameters. Various authors have made the use of auxiliary attribute as a source of auxiliary information to increase the precision of the estimators, for the estimation of the population parameter under consideration. In recent years, many authors have also made use of various parameters associated with the auxiliary attribute f for e.g., standard deviation S_f, coefficient of variation C_f, coefficient of kurtosis $b_2(f)$ and correlation coefficient r of the population in estimation of the population variance. Bhushan (2013), Kalidar and Cingi (2003), Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh et al. (2008), Pandey and Dubey (1988), etc are some of the authors in the list. In this paper, a family of estimators have been proposed by adapting the estimator of Koyuncu (2012) and a class of log type estimators (Kumari et al. (2019)) using the auxiliary information on a attribute. Consider a finite population U = $(U_1,U_2,...,U_N)$ of size N from which a sample of size n is drawn according to simple random sampling without replacement (SRSWOR). Let y_i and x_i denotes the values of the study variable and auxiliary attribute for the i^{th} unit (i = 1,2,...,N), of the population. Further, let $\bar{y}$ and $\bar{f}$ be the sample means and $s_y^2 = \frac{\sum_{i=1}^{n}(yi-\bar{y})^2}{(n-1)}$ and $s_f^2 = \frac{\sum_{i=1}^{n}(f-\bar{f})^2}{(n-1)}$ be the sample variance of the study variable and auxiliary attribute respectively.

2. Estimators Available in Literature

2.1 Conventional variance estimator

$$t_0 = s_y^2$$

The bias and variance of t_0 to the first order of approximation, are given as

$$B(t_0) = 0$$
$$V(t_0) = S_y^4 \; I b_{2y}^*$$

2.2 Isaki ratio estimator

$$t_1 = s_y^2 \left[\frac{S_f^2}{s_f^2}\right]$$

The bias and MSE of t_1 to the first order of approximation, are given as

$$B(t_1) = S_y^2 \,\mathrm{I}\left[b_{2f}^* - I_{22yf}^* \right]$$
$$V(t_1) = S_y^4 \,\mathrm{I}\left[b_{2y}^* + b_{2f}^* - 2I_{22yf}^*\right]$$

2.3 Conventional Product Estimator

$$t_2 = s_y^2 \left[\frac{s_f^2}{S_f^2}\right]$$

The bias and MSE of t_2 to the first order of approximation, are given as

$$B(t_2) = S_y^2 \,\mathrm{I}\, b_{22}^*$$
$$V(t_2) = S_y^4 \,\mathrm{I}\left[b_{2y}^* + b_{2f}^* + 2I_{22yf}^*\right]$$

2.4 Isaki regression estimator

Isaki (1983) suggested the following regression estimator for population variance

$$t_3 = s_y^2 + b\left(S_f^2 - s_f^2\right)$$

where b is a sample regression coefficient whose population regression coefficient is β.

The bias and MSE of t_3 to the first order of approximation, are given as

$$B(t_3) = 0$$
$$V(t_3) = S_y^4 \,\mathrm{I}\left[b_{2y}^* - \frac{I_{22yf}^{*2}}{b_{2f}^*}\right]$$

2.5 Singh et al. estimator

Singh et al. (1973) considered the following estimator,

$$t_4 = \alpha_4 s_y^2$$

where α_4 is a Searl (1964) constant. The optimum value of Searls constant is α_4 = n/(n + b^*_{2y}) for which the mean squared error is minimum.

$$MSE(t_4)_{opt} = S_y^4 \, \mathrm{I}\left[\frac{n\, b^*_{2y}}{\mathrm{n} + b^*_{2y}}\right]$$

2.6 Das and Tripathi estimator

$$t_5 = s_y^2 \left[\frac{S_f^2}{S_f^2 + \alpha_5\left(s_f^2 - S_f^2\right)}\right]$$

where α_5 is a constant. The bias and MSE of t_5 to the first degree of approximation is given as

$$B(t_5) = S_y^2 \, \mathrm{I}\left[\alpha_5 b^*_{2y} - \alpha_5 I^*_{22yf}\right]$$
$$V(t_5) = S_y^4 \, \mathrm{I}\left[b^*_{2y} + \alpha_5 b^*_{2f} - 2\alpha_5 I^*_{22yf}\right]$$

The MSE of t_5 is optimum for $\alpha_5 = \frac{I^*_{22yf}}{b^*_{2f}}$ and is given by

$$MSE(t_5)_{opt} = S_y^4 I\left[b^*_{2y} - {I^{*2}_{22yf}}\Big/{b^*_{2f}}\right]$$

2.7 Prasad and Singh estimator

Prasad and Singh (1992) introduced the following estimator

$$t_6 = \alpha_6 \, s_y^2 \left[\frac{S_f^2}{s_f^2}\right]$$

where α_6 is a constant. The bias and MSE of t_6 to the first degree of approximation is given as

$$B(t_6) = S_y^2 \, \mathrm{I}\left[\alpha_6\left(n + b^*_{2f} - I^*_{22yf}\right) - n\right]$$
$$V(t_6) = S_y^4 \, \mathrm{I}\left[\alpha_6^2\left(n + b^*_{2y} + 3b^*_{2f} - 4I^*_{22yf}\right) - \right.$$
$2\alpha 6n+b2f*-I22yf*-n$

The MSE of t_6 is optimum for

$$\alpha_6 = {\left(n + b^*_{2f} - I^*_{22yf}\right)}\Big/{\left(n + b^*_{2y} + 3b^*_{2f} - 4I^*_{22yf}\right)}$$

and is given by

$$MSE(t_6)_{opt} = S_y^4 I\left[n - \frac{\left(n + b^*_{2f} - I^*_{22yf}\right)^2}{\left(n + b^*_{2y} + 3b^*_{2f} - 4I^*_{22yf}\right)}\right]$$

2.8 Garcia and Cebrian estimator

Garcia and Cebrian (1996) introduced the following estimator

$$t_7 = s_y^2 \left[\frac{S_f^2}{S_f^2}\right]^{\alpha_7}$$

where α_7 is a constant. The bias and MSE of t_6 to the first degree of approximation is given as

$$B(t_7) = S_y^2 \, \mathrm{I}\left[\alpha_7 \frac{(\alpha_7 + 1)}{2} b^*_{2f} - \alpha_7 I^*_{22yf}\right]$$
$$V(t_7) = S_y^4 \, \mathrm{I}\left[b^*_{2y} + \alpha_7^2 b^*_{2f} - 2\alpha_7 I^*_{22yf}\right]$$

The MSE of t_7 is optimum for $\alpha_7 = \frac{I^*_{22}}{b^*_{2f}}$ and is given by

$$MSE(t_7)_{opt} = S_y^4 I\left[b^*_{2y} - {I^{*2}_{22yf}}\Big/{b^*_{2f}}\right]$$

2.9 Upadhaya and Singh estimator

Upadhaya and Singh (2001) suggested following estimator

$$t_8 = s_y^2 + \alpha_8\left(S_f^2 - s_f^2\right)$$

where α_8 is a constant. The MSE of t_8 is optimum for

$\alpha_8 = \frac{S_y^2 I^*_{22yf}}{S_f^2 b^*_{2f}}$ and is given by

$$MSE(t_8)_{opt} = S_y^4 I\left[b^*_{2y} - {I^{*2}_{22yf}}\Big/{b^*_{2f}}\right]$$

2.10 Shabbir and Gupta (2006) estimator

Sabbir and Gupta (2006) proposed the following estimator
$t_9 = \lambda \, t_m$

where λ is a Searls (1964) contant whose value is to be determined later. Here t_m is a combination of Singh et al. (1973), Prasad and Singh (1992) and is defined as

$$t_m = K_1 \, s_y^2 + K_2 \, s_y^2 \left(\frac{S_f^2}{s_f^2}\right)$$

where K_1 and K_2 are the weights such that $K_1 + K_2 = 1$

The optimum MSE of t_9 is given by

$$MSE(t_9)_{opt} = S_y^4 I\left[n - \frac{\left(n + I^*_{22yf} - \frac{I^{*2}_{22yf}}{b^*_{2f}}\right)^2}{\left(n + b^*_{2y} + 2\, I^*_{22yf} - 3\frac{I^{*2}_{22yf}}{b^*_{2f}}\right)}\right]$$

2.11 Shabbir and Gupta (2007) estimator

$$t_{10} = K_1 \, s_y^2 + K_2 \left(S_f^2 - s_f^2\right) exp\left(\frac{S_f^2 - s_f^2}{S_f^2 + s_f^2}\right)$$

where K_1 and K_2 are suitably chosen coantants.

Situation 1. k1 + K_2 = 1 The bias and MSE of t_{10} the first degree of approximation are given as

The optimum MSE of t_{10} is given by

$$MSE(t_{10})_{opt} = S_y^4 I\left[A_1 - \frac{(A_1+A_3)^2}{(A_1+A_3+2A_3)}\right]$$

Situation 2. Unconstrained choice of K_1 and K_2 The bias and MSE of t_{10} the first degree of approximation are given as

$$MSE(t_{10}^*)_{opt} = S_y^4 I\left[\frac{Var(\hat{S}_{reg})}{1+\frac{Var(\hat{S}_{reg})}{S_y^4}}\right]$$

2.12 Kadilar and Cingi estimator

Kadilar and Cingi (2006) suggested the following ratio type estimator

$$t_{11} = w_1 s_y^2 + K_2\left(s_y^2 \frac{S_f^2}{s_f^2}\right)\upsilon$$

where w_1 and w_2 are the weights such that $w_1 + w_2 = 1$

The optimum MSE of t_{11} is given by

$$MSE(t_{11})_{opt} = S_y^4 I\left[n - \frac{\left(n + I_{22yf}^* - \frac{I_{22yf}^{*2}}{b_{2f}^*}\right)^2}{\left(n + b_{2y}^* + 2\, I_{22yf}^* - 3\frac{I_{22yf}^{*2}}{b_{2f}^*}\right)}\right]$$

2.13 Yadav and Kalidar (2013) estimator

Yadav and Kalidar (2013) introduced the following estimator

$$t_{12} = s_y^2 + exp\left(1 - \frac{\alpha_{12} s_f^2}{S_f^2 + (\alpha_{12}-1)s_f^2}\right)$$

Where α_{12} is a constant. The optimum MSE of t_{12} is given by

$$MSE(t_{12})_{opt} = S_y^4 I\left[b_{2y}^* - {I_{22yf}^{*2}}/{b_{2f}^*}\right]$$

2.14 Yadav and Kadilar (2014) estimator

Yadav and Kadilar (2014) introduced the following ratio-product-ratio estimator

$$t_{\alpha,\beta_{12}} = s_y^2\, \alpha\left(\frac{(1-\beta)s_f^2+\beta S_f^2}{\beta s_f^2+(1-\beta)S_f^2}\right) + s_y^2\,(1-\alpha)\left(\frac{(1-\beta)S_f^2+\beta s_f^2}{\beta S_f^2+(1-\beta)s_f^2}\right)$$

Where α *and* β is a constant. The optimum MSE of $t_{\alpha,\beta_{12}}$ is given by

$$MSE(t_{12})_{opt} = S_y^4 I\, b_{2y}^*$$

3. Suggested Family of Estimators

In this paper, the following family of estimator has been proposed for the estimation of the parameter under consideration i.e., the population variance of the study variable y using auxiliary information on a attribute.

$$T_c = \left[w_1 s_y^2 + w_2\left(\frac{S_f^2}{s_f^2}\right)\right]\left[1 + alog\left(\frac{S_f^{*2}}{s_f^{*2}}\right)\right]$$

where a is the characterizing scalar.

$$s_f^{*2} = as_f^2 + b\,,\ S_f^{*2} = aS_f^{*2} + b$$

such that a($\neq 0$) and b are either real numbers or functions of the known parameters of the auxiliary attribute *f* such as the standard deviations S_f, coefficient of variation C_f, coefficient of kurtosis b_{2f} , coefficient of skewness b_{1f_i} and correlation coefficient *r* of the population.

It is noteworthy that, if a = b = 0, then the proposed estimator becomes the usual variance estimator $\hat{S}_y^2$. If a = b = +1, then the proposed class of estimators become a ratio type estimator and when a = b = −1, then the proposed class of estimators become a product type estimator

4. Properties of the Suggested Classes of Log-Type Estimators

In order to obtain the bias and mean square error (MSE), let us consider

$E(\epsilon_0) = E(\epsilon_1) = 0, E(\epsilon_0)^2 = I\, b_{2y}^*, E(\epsilon_1)^2 = I\, b_{2f}^*, E(\epsilon_0\epsilon_1) = I\, I_{22yf}^*,$

Where $b_{2f}^* = b_{2f} - 1$, $b_{2y}^* = b_{2y} - 1$ and, $I_{22yf}^* = I_{22fy} - 1$, $I = \frac{1}{N}$, $I_{pq} = {m_{pq}}/{m_{20}^{\frac{p}{2}} m_{02}^{\frac{q}{2}}}$,

$m_{pq} = {\sum_{i=1}^{N}(Y_i-\bar{Y})^p(f_i-\bar{f})^q}/{N}$, $b_{2y} = {m_{40}}/{m_{20}^2}$, $b_{2f} = {m_{04}}/{m_{02}^2}$ are the coefficient of kurtosis of *y* and *f* respectively.

Theorem 1. The bias and mean squared error of the proposed estimators are given by

$$Bias(T) = S_y^2\left[w_1\left(1 + I a\eta r_{yf}\sqrt{b_{2y}^* b_{2f}^*} - \frac{a\eta^2}{2}\, I b_{2f}^*\right) - 1\right] + w_2\left(1 + I a\eta b_{2f}^* - \frac{a\eta^2}{2}\, I b_{2f}^*\right)$$

$$MSE(T) = S_y^4\, w_1^2\, A + w_2^2\, B + S_y^4\, w_1\, D + S_y^2\, w_2\, G + S_y^2\, w_1\, w_2\, F + S_y^4$$ where

$$A = \left[1 + I\left(b_{2y}^* + a^2\eta^2 b_{2f}^* + 4a\eta r_{yf}\sqrt{b_{2y}^* b_{2f}^*} - a\eta^2 b_{2f}^*\right)\right]$$

$$B = \left[1 + I\left(b_{2f}^* + a^2\eta^2 b_{2f}^* + 4a\eta r_{yf}\, b_{2f}^* - a\eta^2 b_{2f}^*\right)\right]$$

$$D = I\left(a\eta^2 b_{2f}^* - 2a\eta r_{yf}\sqrt{b_{2y}^* b_{2f}^*}\right) - 2$$

$$G = I\left(a\eta^2 b_{2f}^* - 2a\eta b_{2f}^*\right) - 2$$

$$F = 2 + 2I\left(2a\eta b_{2f}^* + 2a\eta r_{yf}\sqrt{b_{2y}^* b_{2f}^*} + r_{yf}\sqrt{b_{2y}^* b_{2f}^*} - a\eta^2 b_{2f}^* + a^2\eta^2 b_{2f}^*\right)$$

$$\eta = \frac{a\,S_f^2}{a\,S_f^2 + b},\; r_{yf} = \frac{I_{22yf}^*}{\sqrt{b_{2y}^* b_{2f}^*}}$$

respectively,

Corollary 1- The mean square error of the proposed class of estimator T_c will be minimum for the optimum value of the characterizing parameters, given

$$w_{1opt} = \frac{GF - 2BD}{4AB - F^2}$$

$$w_{2opt} = S_y^2\left[\frac{DF - 2GA}{4AB - F^2}\right]$$

and the minimum value of the mean square error within the proposed class of estimator is

$$MSE(T_c)_{opt} = S_y^4\left[1 - \frac{BD^2 - DFG + G^2A}{4AB - F^2}\right]$$

5. Multivariate extension of the suggested classes of estimators using multiple auxiliary information

Let there are k auxiliary attributes then we can use the attributes by taking a linear combination of these k estimators of the form given in section 2, calculated for every auxiliary attribute separately, for estimating the population variance. Then the estimators for population variance will be defined as

$$T_c = \left[w_1 s_y^2 + w_2\left(\frac{S_f^2}{s_f^2}\right)\right]\left[\prod_{i=1}^{n}\left[1 + a_i log\left(\frac{S_{f_i}^{*2}}{s_{f_i}^{*2}}\right)\right]\right]$$

where $a_i{}'s$ are the optimizing scalar, i= 1, 2, . . . n.

6. Properties of the suggested classes of estimators using multiple auxiliary information

Theorem 3*. The bias of the proposed estimators are given as*

$$Bias(T) = S_y^2\left[w_1\left(1 + I\sum_{i=1}^{n} a_i\eta^2 b_{2f_i}^* - I\sum_{i=1}^{n} a_i\eta r_{yfi}\sqrt{b_{2y}^* b_{2f_i}^*} - I\sum_{i=1}^{n}\frac{a_i{}^2\eta^2}{2}\, b_{2f_i}^*\right) - 1\right] + w_2\left(1 + I b_{2f_i}^* + I\sum_{i=1}^{n} a_i\eta b_{2f_i}^* + I\sum_{i=1}^{n}\frac{a_i{}^2\eta^2}{2}\, b_{2f_i}^*\right)$$

$$MSE(T) = S_y^4\, w_1^2\, A + w_2^2\, B + S_y^4\, w_1\, D + S_y^2\, w_2\, G + S_y^2\, w_1\, w_2\, F + S_y^4$$

where

$$A = \left[1 + I\left(b_{2y}^* + \sum_{i=1}^{n} a_i^2\eta^2 b_{2f_i}^* - 4\sum_{i=1}^{n} a_i\eta r_{yf_i}\sqrt{b_{2y}^* b_{2f_i}^*} - \sum_{i=1}^{n} a_i\eta^2 b_{2f_i}^*\right)\right]$$

$$B = \left[1 + I\left(b^*_{2f_i} + \sum_{i=1}^{n} a_i^2\eta^2 b^*_{2f_i} + 4\sum_{i=1}^{n} a_i\eta r_{yf_i} b^*_{2f_i} + \sum_{i=1}^{n} a_i\eta^2 b^*_{2f_i}\right)\right]$$

$$D = I\left(2\sum_{i=1}^{n} a_i\eta r_{yf_i}\sqrt{b^*_{2y} b^*_{2f_i}} - \sum_{i=1}^{n} a_i\eta^2 b^*_{2f_i}\right) - 2$$

$$G = I\left(-\sum_{i=1}^{n} a_i\eta^2 b^*_{2f_i} - 2\sum_{i=1}^{n} a_i\eta b^*_{2f_i}\right) - 2$$

$$F = 2 + 2I\left(2\sum_{i=1}^{n} a_i\eta b^*_{2f_i} - 2\sum_{i=1}^{n} a_i\eta r_{yf_i}\sqrt{b^*_{2y} b^*_{2f_i}} - r_{yf_i}\sqrt{b^*_{2y} b^*_{2f_i}} + \sum_{i=1}^{n} a_i\eta^2 b^*_{2f_i} + \sum_{i=1}^{n} a_i\eta^2 b^*_{2f_i}\right)$$

$\eta = \frac{a\, S^2_{f_i}}{a\, S^2_{f_i} + b}$, $r = \frac{I^*_{22yf_i}}{\sqrt{b^*_{2y} b^*_{2f_i}}}$ respectively.

7. Some members of the class of estimators T_c

It can be easily seen that the proposed class T_c is a generalized form of class of estimators for the $a_1(\neq 0)$, b1 and $a_2(\neq 0)$, b_2 are either real numbers or functions of the known parameters of the auxiliary attribute f such as the standard deviations S_f, coefficient of variation C_f, coefficient of kurtosis $b_2(f)$ coefficient of skewness $b_1(f)$ and correlation coefficient r of the population. Therefore, a wide variety of estimators can be designed using the above known population parameters. Some of them are listed below.

Table 1: Some generalized members of the proposed class of estimators T_c

Log type estimator T_c	a_1	b1
$T_{c1} = \left[w_1 s_y^2 + w_2\left(\frac{S_f^2}{s_f^2}\right)\right]^a \left[1 + log\left(\frac{S_f^2}{s_f^2}\right)\right]^b$	1	0
$T_{c2} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{S_f^2 + C_f}{s_f^2 + C_f}\right)\right]^a \left[1 + log\left(\frac{S_f^2 + C_f}{s_f^2 + C_f}\right)\right]^b$	1	C_f
$T_{c3} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{b_2(f)S_f^2 + C_f}{b_2(f)s_f^2 + C_f}\right)\right]^a \left[1 + log\left(\frac{b_2(f)S_f^2 + C_f}{b_2(f)s_f^2 + C_f}\right)\right]^b$	b_{2f}	C_f
$T_{c4} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{C_f S_f^2 + b_2(f)}{C_f s_f^2 + b_2(f)}\right)\right]^a \left[1 + log\left(\frac{C_f S_f^2 + b_2(f)}{C_f s_f^2 + b_2(f)}\right)\right]^b$	C_f	b_{2f}
$T_{c5} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{S_f^2 + S_f}{s_f^2 + S_f}\right)\right]^a \left[1 + log\left(\frac{S_f^2 + S_f}{s_f^2 + S_f}\right)\right]^b$	1	S_f
$T_{c6} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{b_1(f)S_f^2 + S_f}{b_1(f)s_f^2 + S_f}\right)\right]^a \left[1 + log\left(\frac{b_1(f)S_f^2 + S_f}{b_1(f)s_f^2 + S_f}\right)\right]^b$	b_{1f}	S_f
$T_{c7} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{b_2(f)S_f^2 + S_f}{b_2(f)s_f^2 + S_f}\right)\right]^a \left[1 + log\left(\frac{b_2(f)S_f^2 + S_f}{b_2(f)s_f^2 + S_f}\right)\right]^b$	b_{2f}	S_f
$T_{c8} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{S_f^2 + r}{s_f^2 + r}\right)\right]^a \left[1 + log\left(\frac{S_f^2 + r}{s_f^2 + r}\right)\right]^b$	1	r
$T_{c9} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{S_f^2 + b_2(f)}{s_f^2 + b_2(f)}\right)\right]^a \left[1 + log\left(\frac{S_f^2 + b_2(f)}{s_f^2 + b_2(f)}\right)\right]^b$	1	$b_2(f)$
$T_{c10} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{C_f S_f^2 + r}{C_f s_f^2 + r}\right)\right]^a \left[1 + log\left(\frac{C_f S_f^2 + r}{C_f s_f^2 + r}\right)\right]^b$	C_f	r
$T_{c11} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{r\, S_f^2 + C_f}{r\, s_f^2 + C_f}\right)\right]^a \left[1 + log\left(\frac{r\, S_f^2 + C_f}{r\, s_f^2 + C_f}\right)\right]^b$	r	C_f
$T_{c12} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{b_2(f)S_f^2 + r}{b_2(f)s_f^2 + r}\right)\right]^a \left[1 + log\left(\frac{b_2(f)S_f^2 + r}{b_2(f)s_f^2 + r}\right)\right]^b$	$b_2(f)$	r
$T_{c13} = s_y^2\left[w_1 s_y^2 + w_2\left(\frac{r\, S_f^2 + b_2(f)}{r\, s_f^2 + b_2(f)}\right)\right]^a \left[1 + log\left(\frac{r\, S_f^2 + b_2(f)}{r\, s_f^2 + b_2(f)}\right)\right]^b$	r	$b_2(f)$

8. Comparison of estimators

In this section, we compare the proposed classes of estimators with some important estimators. The comparison will be in terms of their MSEs up to the order of n^{-1}. Let us define

$C_1 = b_{2y}^* + b_{2f}^*$ -$2I_{22}^*$, $C_2 = b_{2y}^* + b_{2f}^*$ -$2I_{22}^*$,

D = $b_{2y}^* b_{2f}^* - I_{22}^*$, E = $\frac{n b_{2y}^*}{n+b_{2y}^*}$

F = $\left[n - \frac{\left(n+b_{2f}^*-I_{22}^*\right)^2}{n+b_{2y}^*+3b_{2f}^*-4I_{22}^*}\right]$, G = $\left[n - \frac{\left(n+I_{22}^*-\frac{I_{22}^{*2}}{b_{2f}^*}\right)^2}{n+b_{2y}^*+2I_{22}^*-3\frac{I_{22}^*}{b_{2f}^*}}\right]$,

H = $\left[A_1 - \frac{(A_1+A_3)^2}{A_1+A_2+2A_3}\right]$

$MSE(t_0) > MSE(T_1^*)_{opt}$ if $b_{2y}^* + \frac{B^2}{A}$ - n > 0

$MSE(t_1) > MSE(T_1^*)_{opt}$ if $C_1 + \frac{B^2}{A}$ - n > 0

$MSE(t_2) > MSE(T_1^*)_{opt}$ if $C_2 + \frac{B^2}{A}$ - n > 0

$MSE(t_3) > MSE(T_1^*)_{opt}$ if $D - \left(n - \frac{B^2}{A}\right) b_{2f}^* > 0$

$MSE(t_4) > MSE(T_1^*)_{opt}$ if $E - \frac{B^2}{A}$ - n > 0

$MSE(t_5) > MSE(T_1^*)_{opt}$ if $D - \left(n - \frac{B^2}{A}\right) b_{2f}^* > 0$

$MSE(t_6) > MSE(T_1^*)_{opt}$ if $F + \frac{B^2}{A}$ - n > 0

$MSE(t_7) > MSE(T_1^*)_{opt}$ if $D - \left(n - \frac{B^2}{A}\right) b_{2f}^* > 0$

$MSE(t_8) > MSE(T_1^*)_{opt}$ if $D - \left(n - \frac{B^2}{A}\right) b_{2f}^* > 0$

$MSE(t_9) > MSE(T_1^*)_{opt}$ if $G + \frac{B^2}{A}$ - n > 0

$MSE(t_{10}) > MSE(T_1^*)_{opt}$ if $H + \frac{B^2}{A}$ - n > 0

$MSE(t_{11}) > MSE(T_1^*)_{opt}$ if $G + \frac{B^2}{A}$ - n > 0

$MSE(t_{12}) > MSE(T_1^*)_{opt}$ if $D - \left(n - \frac{B^2}{A}\right) b_{2f}^* > 0$

9. Empirical Study

To compare the efficiency of the suggested class of estimator numerically, we considered nine natural data sets. The description of the population is given below.

Population 1. (Cochran (1977), Pg. no. 107)
y : number of persons per block
f : number of rooms per block
S_y^2= 214.69, S_f^2 = 56.76, b_{2y}^* = 1.2387, b_{2f}^* = 1.3523, I_{22}^*= 0.5432, C_f = 0.1450, $\bar{f}$ = 58.8,
ρ = 0.6515, n =10.

Population 2. (Cochran (1977), Pg. no. 203)
y : actual weight of peaches on each tree
f : eye estimate of weight of peaches on each tree.
S_y^2 = 99.81, S_f^2 = 85.09, b_{2y}^* =0.9249, b_{2f}^* = 1.2932, I_{22}^* = 1.1149, C_f = 0.1621, $\bar{f}$ = 56.9,
ρ = 0.9937, n = 10.

Population 3. (Sukhatme P. V. (1970), Pg. no. 185)
y : wheat acreage in 1937
f : wheat acreage in 1936
S_y^2= 26456.99, S_f^2 = 22355.76, b_{2y}^* = 2.1842, b_{2f}^*= 1.2030, I_{22}^*= 1.5597, C_f = 0.5625, $\bar{f}$ = 265.8, ρ = 0.977, n = 10.

Population 4. (Singh D and Chaudhary F. S., Pg. no. 107).
y : number of boats landing at a particular centre
f : catch of fish in quintals.
S_y^2=201324.4 , S_f^2 = 396.8889, b_{2y}^* = 0.9462, b_{2f}^*= 0.6078 , I_{22}^*= 0.6333, C_f = 0.7288,
$\bar{f}$ = 27.3333 , ρ = 0.9308 , n=4.

Population 5. (Singh D and Chaudhary F. S., Pg. no. 141).
y :number of bearing lime trees
f :area under lime (in acres)
S_y^2=6564586.45, S_f^2 =1092.1024, b_{2y}^* =12.2574, b_{2f}^*=4.5788, I_{22}^*=6.7126, C_f =1.4273 ,
$\bar{f}$ =22.6209 , ρ = 0.9021, n=9.

Population 6. (Choudhary F. S. and Singh D., Pg. no. 176).
y : number of cows in milk enumerated
f : number of cows in milk in the previous year.
S_y^2 =332721.2079, S_f^2 =281472.7868, b_{2y}^* =6.2079, b_{2f}^*=5.0043, I_{22}^*=4.9528, C_f = 0.8276,
$\bar{f}$ = 641.05, ρ=0.8933, n=8.

Population 7. (Singh S., Pg. no. 324-325).
y : approximate duration of sleep (in minutes)
f : age in years of the persons.
S_y^2 =3582.579, S_f^2 =85.2367, b_{2y}^* =1.6678, b_{2f}^* =1.2389, I_{22}^*=0.9961, C_f = 0.1349, $\bar{f}$ = 67.2667,
ρ = -0.8552, n=9.

Population 8. (Singh S., Pg. no. 1114).
y : appropriate duration of sleep (in minutes)
f : age in years of the persons.
S_y^2 =0.0073, S_f^2 =0.0063, b_{2y}^* =2.6323, b_{2f}^* =2.4016, I_{22}^*=1.8351, C_f=1.2352, $\bar{f}$ = 0.1831, ρ=0.7789, n=11.

By using the above data set, the percent relative efficiency of the different estimator are given in Table 2.

Est.	Pop 1	Pop 2	Pop 3	Pop 4	Pop 5	Pop 6	Pop 7	Pop 8
t_0	100	100	100	100	100	100	100	100
t_1	121.38	320.81	815.60	329.16	359.35	475.29	182.39	193.03
t_2	33.68	19.48	33.56	33.54	40.50	29.39	34.04	30.24
t_3	121.38	639.14	1347.98	330.39	507.23	475.29	192.40	214.00
t_4	112.38	639.14	121.42	30.91	236.19	177.59	118.53	123.93
t_5	121.38	109.23	1347.98	330.96	507.23	475.29	192.40	214.00
t_6	112.95	639.14	818.13	348.83	381.77	558.96	212.28	242.02
t_7	121.38	639.14	1347.98	330.39	507.23	475.29	192.40	214.00
t_8	121.38	639.14	1347.98	330.39	507.23	475.29	192.40	214.03
t_9	143.14	749.18	1434.48	349.88	528.70	559.08	220.35	258.35
t_{10}	121.38	489.41	306.84	227.25	319.46	457.77	188.82	228.88
t_{10}^*	82.34	648.39	1369.81	354.05	643.42	552.89	210.93	237.93
t_{11}	82.53	322.84	815.55	329.05	358.77	269.13	182.54	102.02
t_{12}	121.38	639.14	1347.98	330.39	507.23	475.29	192.40	214.00
$t_{\alpha,\beta_{12}}$	100	100	100	100	100	100	100	100
$\boldsymbol{T_c}$	**147.02**	**1163.41**	**1863.95**	**439.47**	**991.93**	**2120.43**	**247.66**	**311.67**

In the above table, the relative efficiency of the proposed estimator is much better as compared to other estimators for all the data sets given here.

10. Conclusion

The present study extends the idea of Kumari et al. (2019) regarding the effective use of auxiliary information if the relationship between the study variable and the auxiliary attribute is of logarithmic type. Further, the efficiency of the proposed estimators are compared with some conventional estimators and some recent estimators of Singh et al. (1973), Das and Tripathi (1978), Sisodia and Dwivedi (1981), Isaki (1983), Bahl and Tuteja (1991), Prasad and Singh (1992), Swain (1994), Garcia and Cebrian (1996), Upadhaya and Singh (2001), Kalidar and Cingi (2006a, 2006b); Gupta and Shabbir (2006, 2007), Yadav and Kadilar (2013, 2014). The proposed estimator is most efficient than all the estimators. This study is also supported through an empirical sudy and the result of this study is quite encouraging.

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