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Recent Developments on Probabilistic Graphical Model Applied in Data Analysis

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Abstract: ***Probabilistic graphical models (PGMs) are recognized to strongly trapping the dynamics of physical systems such as data-driven, communication, imaging, security and allied fields. These models are using to characterize mutually the physical properties of a distributed complex system. This has led to discovery of several PGM techniques of interests include Bayesian networks (BN), Gaussian graphical models (GGMs), graphical Markov models (GMMs) which are the straightforward components that establish the most sophisticated intuitive diagrams of complex conditional dependencies between a set of random variables as well as relationships between stochastic variables. Thereby, recent achievements in employing the graphical models, by purchasing new algorithms and concepts have been proposed to derive and enhance the efficiency of such models. In this perspective, this paper presents the latest progress of graphical models which has received considerable attention in broadband literature. It provides a rapid underlying and understanding of the graphical model and its potential applications with new insight in the massive interests involved in computer and information technologies. Therefore, this can be serves as pivotal guide that illustrate an update information such as research direction, suitability and limitation of the graphical model recently occur in computer science & information technology. So far, this review is attributed to provide a robust background to both industry and science community on ongoing fascinate applications in modern information technology. In addition, a simplified illustration of Bayesian and Markov approaches are depicted in Annex A and Annex B respectively.***

Keywords: Probabilistic graphical model; Bayesian networks; Gaussian graphical models; Graphical Markov models; Social data**.**

1. Probabilistic Graphical Models

Probabilistic graphical models (PGMs) are these of rational knowledge, cognitive, intellectual, and reasoning in the uncertain standpoint. The basic principle of probabilistic graphical models is to compactly encode a probability distribution on all the variables associated with the graph nodes as a product of local distributions of conditional probabilities. Bacha et al. [1] have introduced predictive model based on probabilistic graphical model (PGM) which use high-level visual features consisting of objects and scenes to recognize even in photo albums. This model integrate feature relevance to yields a more powerful inference and better discrimination between different event categories. The result prove an extensive experiments on the recently proposed PEC dataset containing over 61 000 images, and mitigate the influence of non-informative images usually contained in the albums. To define and represent the probabilistic conditional dependencies of a random variable, Hen et al. [2] introduced a framework based on Graphical models algorithm. This approach can measure the perturbation induced by a single element, to perform a classical diagnostic method diagnosis on observe data.

In computer system operations, Energy consumption may lead to slow execution, which can affect the simulation performance, especially when machine-learning framework are deploy. Manager and improve this impact could be a big challenge. Piatkowski et al. [3] have used structured discriminative to sample from high-dimensional generative models even on computational devices with slow floating-point units, based on a probabilistic graphical models. This framework relying on integer addition and binary bit shift operations, to approximates the full joint probability distribution of discrete multivariate random variables, and provide a qualitatively better result as their double precision counterparts. In order to evaluate parameters from data, an expectation maximization algorithm was developed by a novel probabilistic graphical model [4]. Demonstration on simulated as well as real data pointed out that the algorithm outstrips benchmark methods for separation and denoising strides. The method is used with high efficiency in noisy stimulus-evoked MEG and EEG sensor data that obtained in the occurrence of large contextual brain activity. The principle consists to clean the stimulus-evoked data by eliminating interference from noise artifacts and background features and finally splits the data into contributions from independent features. Neonatology and neuropediatric clinical diagnostic/prognostic have better monitoring process with Graphical modeling application. Sarishvili et al. [5] developed multivariate EEG pattern based on Probabilistic graphical model to evaluation of neonatal brain function. This model can build clusters of patients with pathological EEG to make a visual proof on a 3D multidimensional scaling coordinate system with a good performance.

The detecting deformable form problem in computer vision can be improved with the use of Probabilistic graphical model. Quiroz et al. [6] have proposed a new algorithm based on statistical pattern to detect the non-salient landmarks and dense set, which will provide an accurate model of landmark detection across multiple database.

Computer vision is an important area where graphical model has large application, especially with the exponential complexity of probabilistic inference or SK's inability to manage uncertainty. Sarmiento et al. [7] have provided a hybrid system based on the combine probabilistic inference and Semantic Knowledge for more efficiency to the mobile robotic agents applicability. To addressing the uncertainty quantification problem in multiscale deformation processes especially on disk forging of poly-crystal materials, Chen and al. [8] have used probabilistic graphical model technique to develop an accuracy and efficiency framework. Comparing to surrogate model, this framework can be used to predict the mechanical response fields for any microstructure realization input. Graphical model nowadays, can be used to model the relationship between stochastic input and model responses for efficiently perform uncertainty quantifications. Wan and al. [9] have proposed a probabilistic Graphical model, which consider stochastic input and model responses as random variable, and based on this relationship, provide factorization of a high dimensional joint probabilistic distribution. The framework can use the belief propagation algorithm to make efficient prediction. Sustain wine industry production, it important predict and control grape berry maturation and climate variability. Many experiments based on Graphical model have been proposed to provide better answer to face this challenge. Baudrit et al. [10] have implemented probabilistic graphical approaches with as robust mathematical model to experiment the dependency between climate and grape berry maturation, and capitalize the fragment heterogeneous knowledge include in collected data. This approach can predict sugar, acidity, and anthocyanin over the grape maturity. Discover any satisfactory granularity of topics over today's large-scale multimedia data on web is becoming more challenging, especially the exploitation of textual description on the video content. Liu et al. [11] proposed Preference-Topic Model (PTM) framework based on generative probabilistic model to enhance the insufficient textual information. This method comparing to LDA in discovering informative topics, combine user preference discovery and document topic-mining task to better extract information and preferences from sparse real-life video application data. Improving object recognition field, require reliable and advance method to capture the uncertainty inherent in the era. Graphical Models defined as convenient tools used to masters the uncertainty inherent of varieties problem may provide some suitable approach. Sarmiento et al. [12] have proposed conditional Random Fields (CRFs) approaches which is based on an improved Probabilistic Graphical Models (PGMs) particularly Undirected Graphical Models (UGMs). This approach, set-up a working-system as successful as possible for better result in object recognition study, and improve efficiently the used of parallelization techniques. The challenge of improving the way collective behavior of agent emerges on game theory system is reels. Based on probabilistic graphical model generation, a framework of how collective behavior emerges from each individual action on game theory system was studied by Qin et al. [13]. This method decomposes the generative process into independent micro-level games, analyze the collective data to learn new paradigm and infer the most likely parameter to make prediction. Knowledge acquisition in data requires some advance technique that helps improve this knowledge expressions and some managing expertise. Liu et al. [14] have introduced probabilistic graphical model framework as logical predicate formulas to combine logical and probabilistic knowledge. This model combine algorithm of predicate graph (PG), which evaluate the dependence relations among predicate formulas, and qualitative probabilistic network (QPN) as the underlying framework of probabilistic knowledge; to express the ultimate result of knowledge fusion. Form machine learning to deep learning tool, many method or framework have been develop to reduce the number of connections and nodes, redundancy problems in data while preserving the classification accuracy. Zhang et al. [15] have proposed deep probabilistic models, a framework based on Probabilistic Graphical Models (PGMs), which is a result of deep Probabilistic Graphical Networks (PGNs and deep compression techniques combination. The model provide accurate result of solving parameter redundancy, and compressions improvement.

2. Expansion of Bayesian networks in graphical model

A novel method in modeling & predicting the spatial distribution of vegetation types in specific region (case of Swaziland tested on a random sample of mapped vegetation types) based on bioclimatic and physiographic variables was developed [16]. The method applied a data mining concept to extrapolative vegetation mapping via probabilistic graphical models. In achieving this, Bayesian networks involved the classification and the parameterization which using the expectation maximization algorithm were suitably implemented. The result promises high comprehensive of identification and classification of the main environmental and geology variables to response for an example the climatic mitigation. Data mining and data analytics technique as wild as their application eras can cover, may require some improvement with the large hierarchical data to handle, especially the late of scalability features present in direct Graphical model algorithm. An extension to Bayesian network to handle those hierarchical data in an appropriate time was introduced [17]. With multi-label classification for the annotation, this method can achieves high precision and be able to predict more accurately latent semantic relationships. To evaluate accurately the evidential value of a matching Y-chromosomal DNA profile defined by biological stain associated, Andersen et al. [18] have proposed a method based on the combine directed Graphical Model (Bayesian networks) and the Chow-Liu algorithm, which consider the population frequency estimation, and provide dependency between loci. The Resolution of particular class of the inference problems application form the hidden Markov process required both low and high dimension of dynamic system model. Lee al. [19] improve this efficiency of uncertainty quantification framework with the use of parallel probabilistic graphical model. This approach yields a non-parametric Gaussian mixture description to facilitate a synergetic application together with multiple graphs in addressing the Bayesian data assimilation. To provide an accurate prediction of building façade structures, Loch-Dehbi al. [20] have developed graphical model based a combine Bayesian networks to involves the discrete and continuous variable, and the logic programs. This model required a small number of most likely hypotheses on Probability density functions of model parameters in order to be able to scan huge model spaces avoiding the pitfalls of approximate reasoning and to exploit the potential of both observations and models.

Indeed, Bayesian network modelling in some case do not required background knowledge of relationships. This suitable approach comparing to the limited regression modeling techniques analysis on multivariate data had being applied on collected dataset form 200 horse manager's about the 2007 Australia equine influenza outbreak [21]. This analysis required computationally intensive techniques with an optimal graphical statistical model to provide new insights on different biosecurity measures and it relationships perceptions.

To resolve the complex and incomplete scenes, uneven point density and noises challenge that are facing Mobile Light Detection and Ranging (LiDAR) technology, it important to exploit some others techniques contribution such as the graphic model, Kang and al. [22] introduced a framework an automatic classification of mobile LiDAR point clouds based on probabilistic graphical model. The effectiveness and the robustness this method comparing to the existing approaches come from the combines Bayesian network and Markov random technique, and help obtain locally continuous and globally optimal classification results. The application of conditional independence relationships in some field on incompletely categorical data are mostly represent by some advance technique of Graphical models. Geng et al. [23] by using incomplete data one at a time, proposed a recursive formula to describes how posterior means are updated. To apply Bayesian learning easily, this recursive formula can be used to explore expectation–maximization (EM) algorithm and Calculation this approximate posterior means. Ehrmann et al. [24] proposed graphical model approach by combining it with other statistical concepts to define a health systems hypothesis on people living with Spinal Cord Injuries (SCI) in Switzerland. This approach is an empirical foundation hypothesis that could inform health systems about people's health needs.

In energy sector, probabilistic graphical model with data driven approach proposed by O'Neill et al. [25] to improve building energy management. This method is mainly base on Bayesian Networks (BNs) model to define the probabilistic dependencies among corresponding variables and predict HVAC (Heating, Ventilation and Air-conditioning) hot water energy consumption in an office building. Bouri et al. [26] developed Bayesian Graphical Structural Vector Autoregressive (BGSVAR) model to imply the commodity volatility and stock market development prediction in individual BRICS stock markets. Although this predictability differs across each BRICS countries, this method can define the uncertainty in commodity and provide the predictability of global implied volatility indices evidence of these countries. Ramazzotti et al. [27] proposed use of a Suppes-Bayes Causal Networks based on probabilistic graphical model to investigate the structure learning of Bayesian Networks as NP-hard problem. This method is based on a specific structural constraints specially Suppes' probabilistic causation, to model efficiently the cumulative phenomena. The experimentation result proves the improvement of inference accuracy with the use of this model, comparing to the existing genetic Algorithm and local search techniques. The inference accuracy implies temporal ordering on the variable, and space solution reduction.

3. Expansion of Gaussian Graphical Models

Gaussian graphical model (GGM) is widespread as a means of sightseeing network structures, such as social and gene monitoring networks. Rapid growth in the demand of data analysis has paid more attention in improvement of numerical methods including new GGM. The later was proposed based on the robustified against possible outliers [28] . Focusing on this, an L1 penalized maximum likelihood concept is regularly used to perform high-dimensional graphical models. Nevertheless, this approach remains sensitive to outliers. Thus y-divergence method has been introduced as a robust estimation procedure to overcome this issue [29]. The method contains a redescending property that required feature in robust statistics. However, the parameter estimation procedure is built based on the Majorize-Minimization algorithm, which promises that the objective function is uniformly decreases at each iteration and agree well with extensive simulation studies. This new process in GGM achieves much better contamination ratio than over existing ordinary GGM particularly in two real data analyses.

In learning GGM dependency structure, Leppä-aho et al. [30] proposed a Bayesian approximate inference method. This method derives an analytic expression to approximate the marginal likelihood based on pseudo-likelihood. To work for high-dimentional data, the model require easy and fast applicable scoring function based on combine sparsity inducing prior for the graph structure.

After introduced the trek separation, that gives an essential & sufficient condition in terms of the graph at zero sub-determinant for all covariance matrices that fit to the GGM, Draisma [31] extended such result to provide an explicit cancellation-free formulas for the expansions of non-zero sub-determinants. In order to derive a network model of brain data using the graphical modeling, it is interesting to estimate connectivity between different regions of interest, and then evaluate the statistical significance. This procedure includes aggregating results through several patients & frequency ranges, in order to attain a complete result that can serve as active parameters in the graph construction. Improving scientist's complex data sets analysis in biological field, regardless to probable interaction topologies and the clustering of the associated peculiar data problem, probabilistic relationships between nodes or variables are require. With the use of Graphical model, Hassen et al. [32] introduced Graphical Expectation Maximization (GEM) parameters estimation and data clustering algorithm to model biological interactions with a mixture of multivariate Gaussian distributions. This model provides accurate result on signal transduction network of the epidermal growth factor (EGFR) protein.

GGM has being widely apply in many ears especially genes variables representation for dependencies description. To improve the initial single level Gaussian graphical model used for this representation, Cheng et al. [33] have developed multipath way analysis model based on multilevel Gaussian graphical model (MGGM). This method considered the dependency among pathways to describe the genes network on one side, and the genes within each pathway for other side, to sparser on the gene level and estimate the network on the pathway level more accurately.

Abbruzzo et al. [34] used the class of GGM on limited sample size of Italian airports national and regional airports financial data to provide relationship evidence on within financial set and operational indicators. Their method provides visualize conditional independence structures of the variables with which can lead to better financial performance and increase opportunities to expand the business. The application of high-dimensional graphical models especially with GGM, involves in some case the maximization of Gaussian log likelihood subject to reach its full potentiality. In addition, Tan et al. [35] proposed the cluster graphical lasso with two-step procedure on the variable subset: the single linkage hierarchical clustering to identify connected components, and the penalized log likelihood to maximized within each connected component.

Most Gaussian theorem application based on Graphical model are only focus on the ambiguous partial correlation especial pairwise conditional independence test. By constructing a uniformly and a powerful unbiased test of Neyman structure [36] stat that partial correlation test still uniformly most powerful unbiased one. Model selection as process of conditional independences corresponding to missing edges in the graph, is equivalent to zero values of corresponding partial correlation coefficients, and be improve based on graphical model techniques. Drton et al. [37] have introduced and extended to Gaussian graphical models based on directed graphs and chain graphs (CGs) named SINful procedure to hence tends to select sparser graphs than previous procedures such as backward stepwise selection. This method combine Fisher's z-transform, Šidák's correlation inequality, and Holm's step-down procedure to controls the overall error rate for incorrect edge inclusion, and evaluate the multiple hypotheses specified by these zero values.

The possibility of mimics Gaussian graphical models based on skew distribution within graphical models itself is challenging process in multivariate statistic field. This require skew-Gaussian decomposable graphical model (SGDG) accommodation as proposed by Zareifard et al. [38] based on a novel decomposable graphical model, which can reflects conditional independencies among multivariate components. Better Manage systemic risks in banking and finance require some in financial network model. Considering jointly market data, Cerchiello et al. [39] have proposed a novel graphical Gaussian model by decomposes the conditional dependencies between financial institutions and estimates their risks based on countries and institutions correlations. This model improves the estimation of network models and interprets systemic risks.

Using Gaussian methods for High dimensional data modeling required some specified method that can burden and yield parsimonious for strong assumption on the data. Müller et al. [40] have improved this method by adapting vine copulas such as marginal distributions and bivariate copulas with the Gaussian graphical model. This approach increase computational effort as data dimension increase and provide more flexibility and feasibility when modeling high dimensional data. By considering, the multivariate dependencies between continuous random variables, Bagher et al. [41] proposed a method based probabilistic graphical model to avoid learning complex approximations and estimate new distribution algorithm. This method use Gaussian Mixture adding optimize algorithm to evaluate the joint distribution with the dependencies among selected random variable which lead to find the best propose permutation variables as a result.

4. Expansion of Graphical Markov Models

Based on 2008-2009 financial crisis experience, today's financial market required new methodology to predict any unexpected shock. Bianchi et al. [42] proposed Markov Switching Graphical Seemingly Unrelated Regression (MS-GSUR) model based on indirect Graphical Model. This model is define to; investigate time-varying systemic risk with a range of multi-factor asset pricing models, the no correlation evidence between firm-level centrality with market values but linked financial losses. The combination of Graphical model application with other computational method can provide better accurate result especially for fault diagnosis process. Kim et al. [43] proposed the application of two-process models methodology with, Markov random fields modelling with graphical lasso method for modeled extensively, and KBP algorithm to obtain the conditional marginal probability on non-parametric values. This method will lead to effectively isolate the fault variable and reasonable deduction of the fault cause.

Sequential decision-making problems can be improve with framework or method based on graphical models. It may require an approximated dynamic programming scheme define form an optimal selection strategy that incorporates a chosen utility function. Martinelli et al. [44] proposed a computational properties exploration by simulating an approximation model, from naive and myopic heuristics to more complex look-ahead schemes. By using directed acyclic graph this simulation, explore oil exploration strategies to over multiple prospects modeled and with Markov random field to a reservoir drilling decision problem modeled, with the improvement of the naive or myopic constructions used in petroleum industry today.

Human action recognition form a stream of unsegmented sensory as challenge it can be may requires some improve method to empower Human-computer interaction and surveillance applications used the define them. Natarajan et al. [45] have proposed generative hidden Markov models (HMMs) based on conditional random fields (CRFs) and their discriminative counterpart. Based on graphical model, this method show it effectiveness by helping define an efficient learning and inference algorithm that could apply in automatic sign language (ASL) recognition, and for gesture and action recognition in videos.

5. Graphical Model in Real Life Applications

The increasing number of methodological achievements (most especially in Bayesian networks) over the last twenty years attests to their interest and consistency. Thus the fields of their applications are numerous and assorted such as ecology, imaging, bioinformatics, robotics, secure sensor activation [46], learning in speech [47] and so on. Usually, conditional independence assessments based on directed graphs are well requested for learning graphical models from recognized data. Currently innovative algorithm based on undirected graphical models is available [48]. It derived from directed graph and formerly converting it into an undirected one trashes assets including computation time. It gives advantage over existing directed models by minimizing the conditional independence tests with best exactness. A secure sensor activation issue was studied under simulations patterns and new proposed algorithms were suitably established [46]. This is handled to challenge and tackle the internet attackers in interfering with private data (in purchasing information about the battleground). However, the efficiency of the proposed approach was demonstrated and extensively discussed. In high-throughput experimental molecular data, the modeling of associations has brought exceptional insights into signalling mechanisms and biological pathways. In this concern networks and Graphical models have mainly established to be suitable abstractions. In order to determine significant associations Ad hoc thresholds are regularly used in conjunction with structure learning algorithms. Based on these conditions, a statistically inspired approach in identifying significant associations in a network has been presented to hold this limitation [49].

Indeed, network abstractions and Graphical models have relished major attention across the biological and medical communities. These abstractions mechanisms are exclusively handy in decrypting the interactions between the entities of interest from high-throughput data observed. Existing classical techniques to identify significant edges in the ensuing graph rely on ad hoc thresholding of the edge assurance predicted in multiple independent realisations of networks learned from the known data. For example large ad hoc threshold values are essentially mutual, and are selected in an effort by minimising noisy edges in the concerning network. Although useful to minimise false positives, such a choice can emphasize false negatives by prominent effect on the topology of the network which can be applied to several classes of graphical models learned from any nature of data. Family of variance reduction schemes that generalize in graphical models the sample mean from the conventional OR search space to the AND/OR search space was introduced by Gogate and Dechter [50]. Such new AND/OR sample means let trading space and time with variance. By theoretically performing this authors show the smaller variance in the AND/OR space due to the AND/OR sample mean which is defined over a larger virtual sample size when it is compared with the OR sample mean. Empirically, they also demonstrated the far closer AND/OR sample mean to the true mean than the OR sample mean. Several alternative parametrizations have been identified in a multinomial decomposable graphical model; particularly by considering the conditional probabilities of clique-residuals assumed separators, and generalized log-odds-ratios. Thus consistent reference prior for appropriate groupings of the parameters is created for each parametrization and each of these reference priors is conjugate to the likelihood and the result is suitably achieve by change of variable [51].

Here a new method based on p-value combiners used for the first time in the applications of EEG data analysis was proposed [52]. The method split into two features: group-wide tests and frequency-wide tests. This requited to be effectively adjusted in controling for the false detection rate. The investigation was applied to EEG data collected from distinctive mental health patients groups, to draw the corresponding graphical models for each group and also highlight the structural connectivity differences. As the results the suggested approach brings significant enhancements over the step down procedure in terms of applicability, false negative rate across the network models and error rate. Heckerman [53] introduced two satisfactory assumptions, named component independence and mechanism independence in applying the methods to learn a causal networks. This is based on combination with parameter modularity, likelihood equivalence & parameter independence. Kim [54] introduced posterior preference probabilities that satisfying a strong stochastic transitivity condition to paired comparison rankings based on a graphical model with the optimal ranking criterion. This has given rise to achieve efficient ranking in products of independent normal means and in generalized variances scalar functions of K multivariate normal populations.

Graph-based Multi-modal Parcellation (GraMPa) was proposed by Parisot et al. [55] for automatic parcellation of the cortex using various bases of information. It is an iterative frame pattern to handle the massive variety of accessible input modalities to hold the multi-modal parcellation assignment. It revealed that multimodal parcellations harvest more accurate brain delineations with respect to well established atlases and provides a background to compare parcellations by other modalities quantitatively in setting coarse parcellations as a potential reference.

Facial feature tracker in applications such as facial expression analysis systems was proposed based on a graphical model framework through video streams processing, the method affords robustness in real-world conditions such as occlusions & arbitrary head motions [56]. The validation was done on real video data under different suitable conditions. Besides, comparison was made to demonstrate the best accuracy of the proposed method through both quantitative analysis and visual displays.

Chen et al. [57] have demonstrated that a graphical method can be applied for modeling individual driving comportments, and assist a safety analysis in driving behaviors. The proposed method put accents on drivers' major driving behaviors and involve by ranking and extracting their representative driving patterns. This can directly elucidate a driver's driving practices by the behavior graphs. Likewise, the result brings a quantitative analysis comparison between drivers in the driving conduct landscapes and distinguishes. Therefore, these contribute to the notion of a technique that can describe drivers' crucial driving patterns, which can be considered as the primary gadgets in driving conducts. Also, this technique affords any other point of view on the analysis of driving conducts. In the evaluation of driving safety, it can be appropriately applied to different research areas, for example in the analysis of the link between behaviors & fuel consumption, the effect of the external environment through examination the modifications in the graphs.

The characterization of relationships between physical entities with Network modeling are becoming widely, and more complex especially in term of learning form large network data with linear or nonlinear relationship. Liu et al. [58] proposed an efficient regression-based algorithm based on novel graphical model to uncover both linear and nonlinear relationships from a large number of variables. This algorithm comparing to the traditional network learning method, will learning the sparse tree-embedded graphical model (STGM) from data to improve nonlinear relationship in practice. For complex high-dimensional problems, despite the relationship between variable identification performances observe on a discrete graphical model, it is difficult to estimate the parameter by computing the maximum likelihood. Massam and Wang [59] provided, through simple averaging of the MLE of the parameters of local likelihoods, improvement with some detailed study of the maximum composite likelihood estimate of the parameter in a discrete graphical model. While the application of automated tools for the diagnosis of defects or damage in the mechanical and structural systems become less accurate, new adequate methods are needed. Bornn et al. [60] have introduced a framework which have the autoregressive model of all the previous work, and mainly based on direct graphical model. By providing a natural inclusion of structural knowledge through the form of prior distributions on the model parameters, this framework allows clear indications of structural change and damage. To handle inference and dependencies problem in presence high spatial resolution data from structures through the camera-based measurement techniques, Ghazi et al. [61] proposed a novel approach based on graphical models. This approach considers the spatial dependencies between sensor measurements in dense sensor networks and improves damage localization accuracy in structural health monitoring (SHM) application. The Graphical model with the dependency relationship among objects has a major representation in real-time object recognition. To prove this statement, Yun et al. [62] provided an experimental of transition matrix in the graphical model based on architectural diversity with a uniform implementation level combination. His result provides a representation of the conditional probability of object existence and confirm how fast and efficiency way objects recognition can be. Based on graph theory, Chalmond [63] has provided some innovation of Spatio-temporal graphical modeling proving the similarity between graphs that the kernel can be able to capture not just based on the topological relationships between their individual vertices, but also the topological relationships between subgraphs. To do so, he adopts a paradigm in which the intrinsic fluctuations correspond to a latent diffusion process on the graph resulting stochastic interactions within the system, while the extrinsic fluctuations correspond to a time drift reflecting the effects of the environment on the system. Bhushan et al. [64] provided another case of Gaphical model application especially in hoursehold energy saving with graphical causal models method. In his article "Studying the effects of intervention programmes on household energy saving behaviours using graphical causal models", the graphical causal models is developed to provide better understanding the impact of household behaviors inter of energy saving. Graphical models as a powerful tool in computational convenience can be use efficiently to defining conditional independence and covariance matrix regularization. Fitch al. [65] provided an accurate proof by examined three ways of estimating the variance of the inverse covariance elements to expected that these elements would be similar across all the models, which was the case. Shan and Kim [66] have proposed joint adaptive graphical lasso based graphical model to optimize accurately the unbalanced multi-class problem if a joint estimation. This approach combines information across classes during the estimation process to performs better accurately in term of false positive and discovery rate, and Mathew's correlation coefficient. Rieger and Kuchen [67] have proposed an approach of model- driven framework based on declarative graphical model named Münster App Modeling Language (MAML), to describe mobile apps in a platform-agnostic fashion for process modelers, domain experts and software developers. This framework comparing to existing graphical notations, is define for user-oriented specification of business app functionality, and support data model inference to create semantically correct models and support automated processing. Improving Fed-batch fermentation process required an advance way to simulating the operational equations of kinetics-based model. Wang et al. [68] have used graphical model method to determine the conditional independence method low order and the correction between variable on the fed-batch fermentation process. This method proceeds by multivariate auto regressive to learn the variables parameters to effectively approximate the ideal result. To provide an improve classification of bird breed, Huang et al. [69] proposed a framework based on Graphical model named saliency based graphical model (GMS). This framework use SVM algorithm to over-segment, extracts and classifies the objects in the image based on local or global contest, and saliency of each region. The classification result on this model is improved by employ Posterior probability distribution in addition. To improve the graphical model learning with discrete variable; Peyhardi et al. [70] proposed a framework based the generalization of the Rao–Rubin condition and all relied characterizations. This method enables the characterization of graphical models for convolution splitting distributions. To resolve features extraction and body joints learning reliability on Depth-images-based human He et al. [71] proposed a framework by incorporating a graphical model into regression forests and extracting from human body silhouette a novel 3D Local Shape Context feature. This model can learn efficiently the local body structures, and localize joints. It helps to characterize the local structure of body joints and exploit structural constrains. Duarte et al. [72] have experimented the staged tree models form the conditional independence between events depicted in the tree graph, to model invariants and algebraic properties. As graphical models are not always regular exponential families but decomposable, this model provide sufficient conditions for staged trees with the rise to toric variety. The result proved the existing geometric characterization of decomposable directed graphical model could be generalize by a straightforward combinatorial of staged trees. Improving software modelling may require some new design process that could include documenting information responsible and those responsible for understanding. Graphical modeling approach may provide graphical notation for better methodologies development. Considering the trade-offs between different dimensions, Sousa et al. [73] proposed a standard evaluation of the elements and diagrams of the notation based on considering cognitive dimensions. This proposed framework supports the ISO/IEC 24744 methodology metamodeling standard and improves the interaction between user's policy and developers. Wermuth et al. [74] provided a wield view and applicable reviews of graphical model especially Marckov model. This model have tree essential concept had been developed independent more than account. The observe result by tracing development pathways and analyzing longitudinal data simulation are been define with the sequence of regressions. Improving the direct and indirect analysis and effects of tourist- and trip-related characteristics on multi-destination trip behavior becoming a challenge process. Ferrante et al. [75] proposed a combine framework based on a multinomial logistic regression model and graphical models to analysis the main determinants of multi-destination trip behavior. The simulation result prove that motivation for the trip, the number of previous visits, the length of stay are positively impact and directly depend on the proposed multi-destination.

Social network as wildly platforms used for information getting purpose, had being facing semantic sparsity problem leading to low precision and low-quality semantic representation. Based on Graphical model, Kou et al. [76] have proposed multi-feature probabilistic graphical model (MFPGM) which overcome the semantic sparsity problem. This method exploits social media data features to and generate high-quality semantic representations. MFPGM comparing to the existing method, is more accurate and allow searching time and topics relation analysis.

6. Conclusion

This short review summarizes the latest developments in probabilistic graphical models (PGMs). As aforementioned, these models present an approach to establish and model relationships between random variables. It revealed that, recently, less attention has been paid to PGMs owing to the high computational performance achieved by neural networks; which is driving as a prodigious revolutionary technology in computer and information science. Despite, a lot of perspectives surrounding the PGMs in modern computer technology have been achieved with fascinating real life applications and show that PGMs are still expected to open a promising way in broad range technology.

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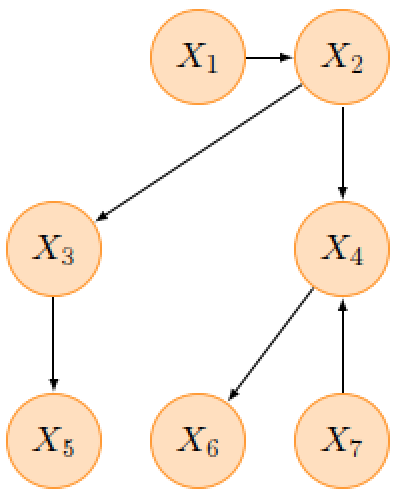
Annex A. Simplified approach of Bayesian implementation

More details can be found in [77,78]



Let $\mathcal{G} = (X, E)$ be a graph, $X_i \in X$, and let P be a directed path from X_i to X_i. Then P is denoted as a directed cycle. A graph $\mathcal{G} = (X, E)$ is a directed acyclic graph,(DAG) if it contains no directed cycles. A Bayesian network B is a tuple$\left(\mathcal{G}, \{P_{X_1}, \dots, P_{X_N}\}\right)$ where $\mathcal{G} = (X, E)$ is a G , each node X_i corresponds to an RV, and $P_{X_1}, \dots, P_{X_N}$ are conditional probability distributions $(CPDs)$ associated with the nodes of the graph. Each of these $CPDs$ has the form

$$P_{X_1}\left(X_i | P_{a_{\mathcal{G}}}(X_i)\right).$$

The Bayesian Network $\mathcal{B}$ defines the joint probability distribution $P_{\mathcal{B}}(X_1, \dots, X_N)$ according to

$$P_{\mathcal{B}}(X_1, \dots, X_N) = \prod_{i=1}^{N} P_{X_1}\left(X_i | P_{a_{\mathcal{G}}}(X_i)\right).$$

$$P_{\mathcal{B}}(X_1, \dots, X_N) = P(X_1) \prod_{i=2}^{N} P(X_i | X_{i-1}).$$

In the Bayesian approach to inference, we specify a sampling model $Pr(X|\theta)$ (density or probability mass function) for our data given the parameters and a prior distribution for the parameters $Pr(\theta)$ reflecting our knowledge about θ before we see the data, we then computer the posterior distribution

$$Pr(\theta|X) = \frac{Pr(X|\theta).\ Pr(\theta).}{\int Pr(X|\theta)\ .\ Pr(X|\theta) d\theta'}$$

The form of a posterior distribution. The posterior distribution also provides the basis for predicting the values of a future observation x^{new}, via the predictive distribution:

$$Pr(x^{new}|X) = \int Pr(x^{new}|\theta)\ .\ Pr(\theta|X) d\theta.$$

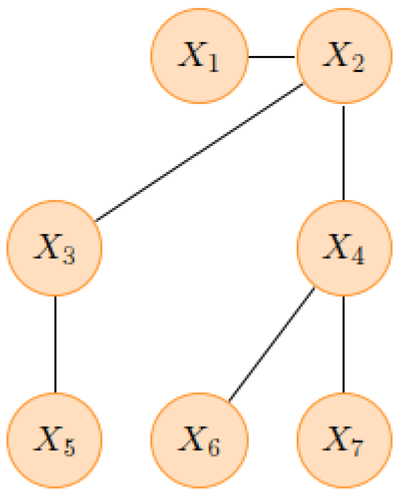
Annex B. Simplified approach of Markov implementation

More details can be found in [77,78]

The undirected graph $\mathcal{G} = (X, E)$ is a complete if every pair distinct nodes is connected by sn edge, i.e. if

$$\forall\, X_i, X_j \in X, i \neq j: (X_i - X_j) \in E.$$

A Markov network is a tuple $\mathcal{M} = \left(\mathcal{G}, \{\psi_{C_1}, \dots, \psi_{C_L}\}\right)$, where $\mathcal{G} = (X, E)$ is an undirected graph with maximal clique $C_1, \dots, C_L$. The nodes X correspond the RVs and $\psi_{C_1} : VAL(C_l) \to \mathbb{R}_+$ are nonnegative functions, called factors or potentials, overs the maximal cliques of the graph$\mathcal{G}$.



The MN defines a probability distribution according to

$$P_{\mathcal{M}}(X_1, \dots, X_N) = \frac{1}{Z} \prod_{l=1}^{L} \psi_{C_l}(C_l),$$

Where Z is a normalization constant given as

$$Z = \sum_{X \in VAL(X)} \prod_{l=1}^{L} \psi_{C_l}(X(C_l))$$

Let $X_1, \dots, X_N$ be a sequence of RVs. There RVs form a Markov chain if they satisfy:
$P(X_i|X_1, \dots, X_{i-1}) = P(X_i|X_{i-1}), \forall i.$ Which is call Markov Property.

The Markov network $\mathcal{M}$ to represent the Markov Chain, has the maximal cliques $C_i = \{X_1, X_{i+1}\}$ for $i = 1, \dots, N-1$. If we define the factor ψ_{C_i} to represent the function $P(X_{i+1}|X_i)$ for $i = 2, \dots, N-1$ and $\psi_{C_i} = P(X_1)P(X_2|X_1)$, then the Markov Network specifies the joint probability distribution of an Markov Chain, i.e.

$$P_{\mathcal{M}}(X_1, \dots, X_N) = \prod_{l=1}^{N-1} \psi_{C_l}(C_l)$$

$$P_{\mathcal{M}}(X_1, \dots, X_N) = P(X_1) \prod_{i=2}^{N} P(X_i | X_{i-1})$$

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