Quintessence Model Calculations for Bulk Viscous Fluid and Low Value Predictions of the Coefficient of Bulk Viscosity

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Abstract: ***In this paper I have used the idea of bulk viscosity of fluid in FRW model and applied this idea into Quintessence to discuss about the cosmological inflation system. I have tried to derive the slow roll mechanism and to match it's result with the inflation condition. I have also mentioned the high viscosity and low viscosity conditions. At the very end I have proved the discrepancy came due to the high viscosity condition and tried to resolve this using the idea of well known time varying gravitational constant G(t).***

Keywords: Fluid mechanics; Quintessence model; cosmological inflation, Viscosity; Gravitational physics

1. Introduction

The cosmological constant corresponds to fluid with a constant equation of state w=-1, now the observations which constrain the value of w today to be close to that of the cosmological constant,these observations actually say relatively little about the time evolution of w, and so we can broaden our horizons and consider a situation in which the equation of state of dark energy changes with time, such as in inflationary cosmology. Scalar fields naturally arise in particle physics including string theory and these can act as candidates for dark energy. So far a wide variety of scalar-field dark energy models have been proposed. These include quintessence, phantoms, K-essence, tachyon, ghost condensates and dilatonic dark energy amongst many.

I have discussed only the quintessence model i.e the model with canonical lagrangian and kinetic energies. In section I ; I have discussed the quintessence model shortly for FRW model in perfect fluid in reference to the publication by Edmund J Copeland ; M.sami, and shinjiTsujikawa. In section II; I have given the quintessence model and inflationary calculations w.r.t the bulk viscous fluid and applied the inflation condition and shown that the coefficient of bulk viscosity should be low stabilization of universe. In section III; finally I have used the divergence less condition of energy-momentum tensor of bulk viscous fluid and proved that the gravitational constant should vary inversely with time to stabilize the universe with high value of coefficient of viscosity.

Section I

Quintessence is described by an ordinary scalar field φ which is a function of time, but we will see with particular potentials that lead to late time inflation. The action for Quintessence is given by

$$S = \int d^4x \sqrt{-g}[1/ 2(\nabla\varphi)^2 - V(\varphi)] \text{------------------ --(i)}$$

Where $(\nabla\varphi)^2 = g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ and V (φ) is the potential of the field. In a flat FRW spacetime the variation of the action with respect to φ gives

$$\ddot{\varphi} + 3H\dot{\varphi} + dV/d\varphi = 0 \text{ -----------------------------------(ii)}$$

The energy momentum tensor of the field is derived by the action in terms of $g^{\mu\nu}$:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \text{ ---(iii)}$$

We can write that $\delta\sqrt{-g} = -(1/2)\sqrt{-g}\, g_{\mu\nu}\delta g^{\mu\nu}$,then

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}\,[\,½\, g^{\alpha\beta}\,\partial_\alpha\varphi\partial_\beta\varphi + V(\varphi)\,] \text{ --- (iv)}$$

In the flat Friedmann background we obtain the energy density and pressure density of the scalar field:

$$\rho = -T_0^0 = ½\,\dot{\varphi}^2 + V(\varphi)\ ; p = T_1^1 = ½\,\dot{\varphi}^2 - V(\varphi) \text{ ---(va)}$$

Then we get ;

$$H^2 = 8\pi G/3\,[\,½\,\dot{\varphi}^2 + V(\varphi)\,] \text{ -------------------------- (via)}$$

$$\frac{\ddot{a}}{a} = -\,8\pi G/3\,[\,\dot{\varphi}^2 - V(\varphi)\,] \text{ ---------------------------- (vi b)}$$

From v we get

$$W = p/\rho = \frac{½\,\dot{\varphi}^2 - V(\varphi)}{½\,\dot{\varphi}^2 + V(\varphi)} \text{ ---------------------------------- (vb)}$$

From equation Vb if we get $V(\varphi) >> \dot{\varphi}^2$ then we can write

$$W = -1;\ \frac{w+1}{w-1} = 0 \quad \text{----------------(vii)}$$

This is the condition for cosmological inflation.

Section II

In the previous section the calculation has given for the perfect fluid. If I consider the universe as bulk viscous fluid , then we have to consider the viscous dissipation for energy momentum tensor which finally reduce the pressure.

Now consider due to bulk viscosity we have the coefficient of viscosity Ɛ . Now the apparent pressure will be P=p- ƐΘ where $\Theta = 3\frac{\dot{a}}{a}$.= expansion.

So from the equation (Va) I can get

$$\rho = -T_0^0 = ½\varphi^2 + V(\varphi) \quad ; p = T_1^1 = ½\varphi^2 - V(\varphi) + \varepsilon\Theta \text{ ----------------(viii)}$$

Now as we know

$$\Theta = 3\frac{\dot{a}}{a}.\ ; \text{and } H = \frac{\dot{a}}{a}. \quad \text{And so } \Theta = 3H \text{ -----------(ix)}$$

So we get ;

$$\rho = -T_0^0 = ½\varphi^2 + V(\varphi);\ p = T_1^1 = ½\varphi^2 - V(\varphi) + 3\varepsilon H \text{ (x)}$$

Now from VIa we get

$$H= \sqrt{8\pi G/3[½\varphi^2 + V(\varphi)]} \text{-------------- (xi)}$$

So from Vb ; X and XI I can get

$$W=\frac{½\varphi^2 - V(\varphi) + 3\mathcal{E}\sqrt{8\pi G/3[½\varphi^2 + V(\varphi)]}}{½\varphi^2 + V(\varphi)} \text{-----------------(XIIa)}$$

Now if we get $V(\varphi) \gg \varphi^2$ then from the equation XXII we can get ;

$$W= -1 + 3\mathcal{E}K/V > -1 \text{ --------------------------------(XIIb)}$$

where $K=\sqrt{8\pi G/3}$

so if Ɛ -> 0 then only we can get w=-1 otherwise it will be greater than -1 and thus it can not produce the inflation condition and that's why I can say that for high viscosity the universe will not face the cosmological inflation condition.

Section III

From the FRW model using the divergence less condition of energy momentum tensor

$T_{\mu\nu} = \rho U_\mu U_\nu + p\ h_{\mu\nu}$

We can write

$$\dot{\rho} + 3(\rho + p)H = 0 \text{------------------------------------ (xiii)}$$

So using the definition VIII we can get

$$\varphi^{\cdot}\varphi^{\cdot\cdot} + 3(½\dot{\varphi}^2 + V(\varphi) + \dot{\varphi}^2 - V(\varphi) + 3\mathcal{E}H)H + (\frac{\partial V}{\partial\varphi})\varphi^{\cdot}=0$$

Or ; using the definition of H from Via we get

$$\varphi^{\cdot\cdot} + 3H\varphi^{\cdot} + \frac{\partial V}{\partial\varphi} + (9\mathcal{E}K^2/2)\dot{\varphi}^2 + 9\mathcal{E}K^2 V(\varphi) = 0 \text{ ----(xiv)}$$

Now during inflation as $^{\cdot\cdot}$is very high so we can get

$$3H\varphi^{\cdot} + \frac{\partial V}{\partial\varphi} + (9\mathcal{E}K^2/2)\dot{\varphi}^2 + 9\mathcal{E}K^2 V(\varphi) = 0$$

$$\text{Or ; } {}^{\cdot} = \frac{-3H \pm \sqrt{[9H^2 - 2K_2(K_2 V + \frac{\partial V}{\partial\varphi})]}}{K_2} \text{ ---------------(XV)}$$

Now if $V(\varphi) >> \varphi 2$ then slow roll mechanism for cosmological inflation should follow and $V(\varphi)$ remains almost constant. So we can assume that

$\frac{\partial V}{\partial\varphi} = 0$ and $H^2 = V/3$

So using this in the equation XV we get

$$\varphi^{\cdot} = \frac{-3H \pm \sqrt{[9V - 2K_2^2 V]}}{K_2} = -3HK_2^{-1} \pm \text{constant------(XVi)}$$

(as the term under the square root is very high and denominator is also very high as$K_2^{-1}$is very high) Now asK_2 is directly proportional to G and the value of G is of the order of 10^{-11} so$K_2$is very low in it's value.On the other hand $K_2^{-1}$is too high.

So we see that from equation XVI the value of damping term $\varphi^{\cdot}$ is high but this we got using the condition $\varphi^{\cdot}$ is small. Show viscosity creates ambiguity. Now if we consider the value of G was very high at the time of inflation then only we can resolve the ambiguity of that problem. Otherwise the slow roll model break down. On the other hand the measurement shows the low value of G. So We have to consider time varying G idea and we can say that G varies inversely as time.

2. Conclusion

From the above calculation in the three section I can get the following conclusions.

- The bulk viscosity of the universe should be low in value to get the cosmic inflation condition. So during inflation the coefficient Ɛ -> 0.
- If we consider the high viscosity , then the flatness problem , horizon problem and monopole problem can not be resolved with cosmological inflation concept.So we can say that universe can not be highly viscous.
- The gravitational constant should vary with time in inverse order in viscous universe i.e the value of this constant should be high during inflation and it should reduce with time increases to reach present time.
- If we consider the gravitational constant don not change with time inverse orderly, the slow roll model will fail to discuss the cosmological inflation and that is impossible. So gravitational constant term G should vary with time in case the universe is viscous.

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