A Common Fixed Point Theorem For Weakly Compatible Maps In complex Valued-b-Metric Space

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Abstract : In this paper we proved a common fixed point theorems for pair of weakly compatible maps in complex valued b-metric space.The obtained results generalise and extend the result of some well known results [14].

Keywords : fixed point, Complex valued-b-metric spaces
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1. Introduction

In 1989, Backhtin [[2]]introduced the concept of b-metric space. In 1993, Czerwik[4] extended the results of b-metric spaces. Azam et al. [1] introduced new spaces called complex valued metric spaces and established the existence of fixed point theorems under the contraction condition.Later,the study of common fixed point theorems on complex valued metric spaces by several authors(see [6]-[12]).The Rao et.al[7] introduced concept of complex valued b- metric spaces . In this paper, we proved the common fixe point theorems in complex valued b-metric spaces for a pair of weakly compatible maps.The obtained results are extension of results proved by Sandeep Bhatt et.al.[14]

2. Preliminaries

Definition 2.1. [2] Let X be a nonempty set, $s \geq 1$ be a given real number and $d : X \times X \to R_+$ be a function.We say d is a b-metric on X if and only if for all $x, y, z \in X$ the following conditions are satisfied:

(1) $d(x, y) = 0$ if and only if $x = y$,
(2) $d(x, y) = d(y, x)$,

(3) $d(x,z) \leq s[d(x,y) + d(y,z)]$.
A pair (X,d) is called a b-metric space. If s =1, b-metric reduces to usual metric.

Let C be the set of complex numbers and let $z_1, z_2 \in C$.Define a partial order $\leq$ on C as:
$z_1 \leq z_2$ if and only if $Re(z_1) \leq Re(z_2)$,$Im(z_1) \leq Im(z_2)$.
It follows that $z_1 \leq z_2$ if one of the following conditions is satisfied:

(1) $Re(z_1) = Re(z_2), Im(z_1) < Im(z_2)$

(2) $Re(z_1) < Re(z_2), Im(z_1) = Im(z_2)$

(3) $Re(z_1) < Re(z_2), Im(z_1) < Im(z_2)$

(4) $Re(z_1) = Re(z_2), Im(z_1) = Im(z_2)$

In particular, we will write $z_1 \lesssim z_2$ if one of (1),(2) and (3) is satisfied and we will write $z_1 < z_2$ if only (3) is satisfied.

Definition 2.2. Let X be a non empty set and let $s \geq 1$ be a given real number.A function $d : X \times X \to C$ satisfies the following conditions

(1) $0 \leq d(x,y)$ for all $x, y \in X$ and $d(x,y) = 0$ if and only if $x = y$.

(2) $d(x,y) = d(y,x)$.

(3) $d(x,z) \leq s[d(x,y) + d(y,z)]$.
The pair (X,d) is called complex valued-b-metric space.

In 1996, Jungck introduced the concept of weakly compatible maps as

Definition 2.3. Two self maps f and g are said to be weak compatible if they commute at coincidence points..i.e.,if Su=Tu for some $u \in X$, then STu=TSu.

Definition 2.4. Let (x, d) be a complex valued b-metric space.

(1) A point $x \in X$ is called interior point of a set $A \subseteq X$ whenever $\exists\ 0 \leq r \in C$ such that $B(x,r) = \{y \in X : d(x,y) \leq r\} \subseteq A$.

(2) A subset $A \subseteq X$ is called open whenever each element of A is an interior point of A.

(3) A subset $A \subseteq X$ is called closed whenever each element of A is a point of A.

Definition 2.5. Let (X,d)be a complex valued b-metric space. Then a sequence $\{x_n\}_{n=1}^{\infty}$ in X is called a Cauchy's sequence if and only if for all $\varepsilon > 0$ there exist $n(\varepsilon) \in N$ such that for each $n, m \geq n(\varepsilon)$ we have $d(x_n, x_m) < \varepsilon$.

Definition 2.6. Let (X,d)be a complex valued b-metric space. Then a sequence $\{x_n\}_{n=1}^{\infty}$ in X is called convergent sequence if and only if there exists $x \in X$ such that for all $n \in N$ for all $n > n(\varepsilon)$ we have $d(x_n, x) < \varepsilon$,then we write $lim_{n\to\infty} x_n = x$.

Definition 2.7. The complex valued b-metric space is complete if every Cauchy sequence convergent.

3. MAIN RESULT

Theorem 3.1. *Let (X, d) be a complex valued -b metric space and let f,g,S and T are four self maps of X such that*

(1) $T(X) \subseteq f(X)$ *and* $S(X) \subseteq g(X)$.
(2) $\{f, S\}$ *and* $\{g, T\}$ *are weakly compatible.*

and

$$d(Sx, Ty) \leq ad(fx, gy) + b[d(fx, Sx) + d(gy, Ty)] + c[d(fx, Ty) + d(gy, Sx)]. \tag{1}$$

$\forall x, y \in X$ *where a,b,c are non negative and satisfy* $a + 2b + 2c \leq 1$, $s > 1$ *then S, f, g and T have a unique common fixed point in X.*

Proof. Let $x_0 \in X$ and $\{y_n\}_{n=1}^{\infty}$ be a sequence in X such that

$$y_{2n} = Sx_{2n} = gx_{2n+1} \, and \, y_{2n+1} = Tx_{2n+1} = fx_{2n+2} \tag{2}$$

for $n = 0, 1, 2, ...$
Consider

$$\begin{aligned} d(y_{2n}, y_{2n+1}) &= d(Sx_{2n}, Tx_{2n+1}) \\ &\leq ad(fx_{2n}, gx_{2n+1}) + b[d(fx_{2n}, Sx_{2n}) + d(gx_{2n+1}, Tx_{2n+1})] \\ &\qquad + c[d(fx_{2n}, Tx_{2n+1}) + d(gx_{2n+1}, Sx_{2n})] \\ &\leq ad(y_{2n-1}, y_{2n}) + b[d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})] \\ &\qquad + c[d(y_{2n-1}, y_{2n+1}) + d(y_{2n}, y_{2n})] \\ &\leq (a + b + c)d(y_{2n-1}, y_{2n}) + (b + c)d(y2n, y_{2n+1}) \end{aligned} \tag{3}$$

This implies that

$$d(y_{2n}, y_{2n+1}) \leq \frac{a + b + c}{1 - b - c} d(y_{2n-1}, y_{2n})$$
$$d(y_{2n}, y_{2n+1}) \leq kd(y_{2n-1}, y_{2n})$$

where $k = \frac{a+b+c}{1-b-c} < 1$ Similarly,we can show that
$d(y_{2n+1}, y_{2n+2}) \leq kd(y_{2n}, y_{2n+1})$
Continuing this process, we get
$d(y_{2n+1}, y_{2n+2}) \leq kd(y_{2n}, y_{2n+1}) \leq .... \leq k^{n+1}d(y_0, y_1)$.
Now we show that $\{y_n\}$is Cauchy sequence in X. Let $m, n \in N$ and $m > n$.

$$\begin{aligned} d(y_n, y_m) &= s[d(y_n, y_{n+1}) + d(y_{n+1}, y_m)] \\ &\leq s[d(y_n, y_{n+1})] + s[d(y_{n+1}, y_m)] \\ &\leq s[d(y_n, y_{n+1})] + s[s\{d(y_{n+1}, y_{n+2}) + d(y_{n+2}, y_m)\}] \\ &\leq s[d(y_n, y_{n+1})] + s[s\{d(y_{n+1}, y_{n+2}) + d(y_{n+2}, y_m)\}] \\ &\leq sk^n d(y_0, y_1) + s^2 k^{n+1} d(y_0, y_1) + ... \\ &\leq sk^n d(y_0, y_1)[1 + sk + (sk)^2 + ...] \\ &\leq \frac{sk^n}{1 - sk} d(y_0, y_1) \end{aligned} \tag{4}$$

Taking the limit $n, m \to \infty$, we get
$\lim_{n\to\infty} \frac{sk^n}{1-sk} d(y_0, y_1) = 0$ This implies $|d(y_n, y_m)| \to\to 0$
Hence $\{y_n\}$ is Cauchy sequence. Since X is complete , so there exist a point z in X such that
$\lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} gx_{2n+1} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} fx_{2n+2} = z.$

Since $T(X) \subseteq f(X)$,there exist a point u in X such that $z = fu$.
From (1) we have

$$\begin{aligned} d(Su, z) &\leq s[d(Su, Tx_{2n+1}) + d(Tx_{2n+1}, z)] \\ &\leq s[ad(fu, gx_{2n+1}) + b\{d(fu, Su) + d(gx_{2n+1}, Tx_{2n+1})\} \\ &\qquad + c\{d(fu, Tx_{2n+1}) + d(gx_{2n+1}, Su)\}] + d(Tx_{2n+1}, z) \\ &\leq s[ad(z, z) + b\{d(z, Su) + d(z, z)\} + c\{d(z, z) + d(z, Su)\}] + d(z, z) \\ &\leq s(b + c)d(su, z) \end{aligned} \tag{5}$$

which is contradiction,therefore $Su = fu = z$
Since$S(X) \subseteq g(X)$ there exist a point v in X such that$gv = z$.
From(1) we have

$$\begin{aligned} d(z, Tv) &\leq s[d(z, Sx_{2n}) + d(Sx_{2n}, Tv)] \\ &= s[d(Sx_{2n}, z) + d(Sx_{2n}, Tv)] \\ &\leq s[d(Sx_{2n}, z) + ad(fx_{2n}, gv) + b\{d(fx_{2n}, Sx_{2n}) + d(gv, Tv)\} \\ &\qquad + c\{d(fx_{2n}, Tv) + d(gv, Sx_{2n})\}] \\ &\leq s[d(z, z) + ad(z, z) + b\{d(z, z) + d(z, Tv)\} + c\{d(z, Tv) + d(z, z)\}] \\ &\leq s(b + c)d(z, Tv) \end{aligned} \tag{6}$$

which is a contradiction.Therefore $Tv = gv = z$ and $Su = fu = Tv = gv = Z$
Since f and S are weakly compatible maps, then Sfu=fSu. Therefore Sz=fz.
Now we show that z is a fixed point of S. If not $d(Sz, z) \geq 0$.
From(1),we have

$$\begin{aligned} d(Sz, z) &= d(Sz, Tv) \\ &\leq [ad(fz, gv) + b\{d(fz, Sz) + d(gv, Tv)\} + c\{d(fz, Tv) + d(gv, Sz)\}] \\ &\leq [ad(Sz, z) + b\{d(Sz, Sz) + d(z, z)\} + c\{d(Sz, z) + d(z, Sz)\}] \\ &\leq (a + 2c)d(Sz, z) \end{aligned} \tag{7}$$

which is a contradiction.Therefore $Sz = z$and $Sz = fz = z$
Similarly g and T are weakly compatible maps, then Tz=gz.
Now we show that z is a fixed point of T. If not $d(Tz, z) \geq 0$.
From (1), we have

$$\begin{aligned} d(z, Tz) &= d(Sz, Tz) \\ &\leq [ad(fz, gz) + b\{d(fz, Sz) + d(gz, Tz)\} + c\{d(fz, Tz) + d(gz, Sz)\}] \\ &\leq [ad(z, Tz) + b\{d(z, z) + d(Tz, Tz)\} + c\{d(z, Tz) + d(Tz, z)\}] \\ &\leq (a + 2c)d(z, Tz) \end{aligned} \tag{8}$$

which is a contradiction.Therefore $Tz = z$and this implies $Tz = gz = z$
So we have proved $Sz = Tz = fz = gz = z$ and it follows z is common fixed point of f,g,S and T

Uniqueness :
If z and v are two distinct common fixed point of f,g,S and T. we have

$$\begin{aligned} d(z,v) &= d(Sz,Tv) \\ &\leq ad(fz,gv) + b\{d(fz,Sz) + d(gv,Tv)\} + c\{d(fz,Tv) + d(gv,Sz)\}] \\ &\leq ad(z,v) + b\{d(z,z) + d(v,v)\} + c\{d(z,v) + d(v,z)\}] \\ &\leq s(a+2c)d(z,v) \end{aligned} \tag{9}$$

which is a contradiction.Therefore $z = v$ and this implies z is the unique common fixed point of f,g,s and T. □

Corollary:Let (X,d) be a complex valued -b metric space and let f,S and T are three self maps of X such that

(1) $T(X) \subseteq f(X)$ and $S(X) \subseteq f(X)$.
(2) $\{f,S\}$ and $\{g,T\}$ are weakly compatible.

and

$$d(Sx,Ty) \leq ad(fx,fy) + b[d(fx,Sx) + d(fy,Ty)] + c[d(fx,Ty) + d(fy,Sx)]. \tag{10}$$

$\forall x,y \in X$ where a,b,c are non negative reals and satisfy $a+2b+2c \leq 1$, $s > 1$ then S, f and T have a unique common fixed point in X.

Proof. Put $f = g$ in theorem 3.1 we get the result. □

Theorem 3.2. *Let (X,d) be a complex valued -b metric space and let f,g,S and T are four self maps of X such that*

(1) $T(X) \subseteq f(X)$ *and* $S(X) \subseteq g(X)$.
(2)

$$d(Sx,Ty) \leq h\max\{d(fx,gy), d(fx,Sx), d(gy,Ty), d(fx,Ty), d(gy,Sx)\}. \tag{11}$$

.

(3) $\{f,S\}$ *and* $\{g,T\}$ *are weakly compatible.*

$\forall x,y \in X$ *where a,b,c are non negative and satisfy* $1+2hs \leq 1$, $s > 1$,$0 < h < 1$ *then S,f,g and T have a unique common fixed point in X.*

Proof. Let $x_0 \in X$ and $\{y_n\}_{n=1}^{\infty}$ be a sequence in X such that

$$y_{2n} = Sx_{2n} = gx_{2n+1}\, and\, y_{2n+1} = Tx_{2n+1} = fx_{2n+2} \tag{12}$$

for $n = 0,1,2,...$
Consider

$$\begin{aligned} d(y_{2n},y_{2n+1}) &= d(Sx_{2n},Tx_{2n+1}) \\ &\leq h\max\{d(fx_{2n},gx_{2n+1}), d(fx_{2n},Sx_{2n}), d(gx_{2n+1},Tx_{2n+1}) \\ &\qquad d(fx_{2n},Tx_{2n+1}), d(gx_{2n+1},Sx_{2n})\} \\ &\leq h\max\{d(y_{2n-1},y_{2n}), d(y_{2n-1},y_{2n}), d(y_{2n},y_{2n+1}) \\ &\qquad d(y_{2n-1},y_{2n+1}), d(y_{2n},y_{2n})\} \\ &\leq h\max\{d(y_{2n-1},y_{2n}), d(y_{2n},y_{2n+1}), d(y_{2n-1},y_{2n+1})\} \end{aligned} \tag{13}$$

Now we discuss the different cases.

Case 1
If $\max\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), d(y_{2n-1}, y_{2n+1})\} = d(y_{2n-1}, y_{2n})$
then from (13) we have
$d(y_{2n}, y_{2n+1}) \leq hd(y_{2n-1}, y_{2n})$
continuing the process, we get $d(y_{2n}, y_{2n+1}) \leq h^{2n+1}d(y_0, y_1)$.

Case 2
If $\max\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), d(y_{2n-1}, y_{2n+1})\} = d(y_{2n}, y_{2n+1})$
then from (13)we have
$d(y_{2n}, y_{2n+1}) \leq hd(y_{2n}, y_{2n+1})$, a contradiction, since $0 < h < 1$
$d(y_{2n}, y_{2n+1}) \leq h^{2n+1}d(y_0, y_1)$.

Case 3
If $\max\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), d(y_{2n-1}, y_{2n+1})\} = d(y_{2n-1}, y_{2n+1})$
then from (13) we have

$$\begin{aligned} d(y_{2n}, y_{2n+1}) &\leq hd(y_{2n-1}, y_{2n+1}) \\ &\leq hs[d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})] \\ &\leq \frac{hs}{1-hs}d(y_{2n-1}, y_{2n}) \\ &\leq kd(y_{2n-1}, y_{2n}) \end{aligned} \tag{14}$$

where $k = \frac{hs}{1-hs} < 1$, continuing this process we get
$d(y_{2n}, y_{2n+1}) \leq k^{2n+1}d(y_0, y_1)$.
Now we show that $\{y_n\}$is Cauchy sequence in X. Let $m, n \in N$ and $m > n$

$$\begin{aligned} d(y_n, y_m) & s[d(y_n, y_{n+1}) + d(y_{n+1}, y_m)] \\ &\leq s[d(y_n, y_{n+1})] + s[d(y_{n+1}, y_m)] \\ &\leq s[d(y_n, y_{n+1})] + s[s\{d(y_{n+1}, y_{n+2}) + d(y_{n+2}, y_m)\}] \\ &\leq s[d(y_n, y_{n+1})] + s[s\{d(y_{n+1}, y_{n+2}) + d(y_{n+2}, y_m)\}] \\ &\leq sk^n d(y_0, y_1) + s^2k^{n+1}d(y_0, y_1) + ... \\ &\leq sk^n d(y_0, y_1)[1 + sk + (sk)^2 + ...] \\ &\leq \frac{sk^n}{1-sk}d(y_0, y_1) \end{aligned} \tag{15}$$

Taking the limit $n, m \to \infty$, we get
$\lim_{n\to\infty} \frac{sk^n}{1-sk}d(y_0, y_1)$
Hence $\{y_n\}$ is Cauchy sequence. Since X is complete , there exist a point z in X such that $\lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} gx_{2n+1} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} fx_{2n+2} = z$.

since$T(X) \subseteq f(X)$,there exist a point u in X such that $z = fu$ we have

$$\begin{aligned} d(Su, z) &\le s[d(Su, Tx_{2n+1}) + d(Tx_{2n+1}, z)] \\ &\le s[h \max\{d(fu, gx_{2n+1}), d(fu, Su), d(gx_{2n+1}, Tx_{2n+1}), \\ &\qquad d(fu, Tx_{2n+1}), d(gx_{2n+1}, Su)\}] + sd(Tx_{2n+1}, z) \\ &\le sh \max\{d(z, z), d(z, Su), d(z, z), d(z, z), d(z, Su)\} + sd(z, z) \\ &\le shd(Su, z) \end{aligned} \tag{16}$$

which is contradiction .therefore $Su = fu = z$ Since$S(X) \subseteq g(X)$ there exist a point v in X such that$gv = z$
This implies that

$$\begin{aligned} d(z, Tv) &\le s[d(z, Sx_{2n}) + d(Sx_{2n}, Tv)] \\ &= s[d(Sx_{2n}, z) + d(Sx_{2n}, Tv)] \\ &\le s[d(Sx_{2n}, z) + h \max\{d(fx_{2n}, gv), d(fx_{2n}, Sx_{2n}), \\ &\qquad d(gv, Tv), d(fx_{2n}, Tv), d(gv, Sx_{2n})\} \\ &\le s[d(z, z) + h \max\{d(z, z), d(z, z), d(z, Tv), d(z, Tv), d(z, z)\}] \\ &\le shd(z, Tv) \end{aligned} \tag{17}$$

which is a contradiction .Therefore $Tv = gv = z$ and $Su = fu = Tv = gv = Z$
Since f and S are weakly compatible maps, then Sfu=fSu. Therefore Sz=fz.
Now we show that z is a fixed point of S. If not $d(Sz, z) \ge 0$.
we have

$$\begin{aligned} d(Sz, z) &= d(Sz, Tv) \\ &\le h \max\{d(fz, gv), d(fz, Sz), d(gv, Tv), d(fz, Tv), d(gv, Sz)\} \\ &\le h \max\{d(Sz, z), d(Sz, Sz), d(z, z), d(Sz, z), d(z, Sz)\} \\ &\le hd(Sz, z) \end{aligned} \tag{18}$$

which is a contradiction.Therefore $Sz = z$and $Sz = fz = z$.
Similarly g and T are weakly compatible maps, then Tz=gz.
Now we show that z is a fixed point of T. If not $d(Tz, z) \ge 0$.
we have

$$\begin{aligned} d(z, Tz) &= d(Sz, Tz) \\ &\le h \max\{d(fz, gz), d(fz, Sz), d(gz, Tz), d(fz, Tz), d(gz, Sz)\} \\ &\le h \max\{d(z, Tz), d(z, z), d(Tz, Tz), d(z, Tz), d(Tz, z)\} \\ &\le hd(z, Tz) \end{aligned} \tag{19}$$

which is a contradiction.Therefore $Tz = z$and this implies $Tz = gz = z$
So we have proved $Sz = Tz = fz = gz = z$ and it follows z is common fixed point of f,g,S and T
Uniqueness :
If z and v are two distinct common fixed point of f,g,S and T. we have

$$\begin{aligned} d(z,v) &= d(Sz,Tv) \\ &\leq h\max\{d(fz,gv),d(fz,Sz),d(gv,Tv),d(fz,Tv),d(gv,Sz)\} \\ &\leq h\max\{d(z,v),d(z,z),d(v,v),d(z,v),d(v,z)\} \\ &\leq hd(z,v) \end{aligned} \tag{20}$$

which is a contradiction.Therefore $z = v$ and this implies z is the unique common fixed point of f,g,s and T. □

Corollary: Let (X, d) be a complex valued -b metric space and let f,S and T are three self maps of X such that

(1) $T(X) \subseteq f(X)$ and $S(X) \subseteq g(X)$.
(2)

$$d(Sx,Ty) \leq h\max\{d(fx,fy),d(fx,Sx),d(fy,Ty),d(fx,Ty),d(fy,Sx)\}. \tag{21}$$

.

(3) $\{f, S\}$ and $\{g, T\}$ are weakly compatible.

$\forall x, y \in X$ where a,b,c are non negative and satisfy $1 + 2hs \leq 1$, $s > 1$,$0 < h < 1$ then S,f and T have a unique common fixed point in X.

Proof. Put $f = g$ in theorem 3.2 we get the result. □

Example: Let $X = [0, 1]$ and $d : X \times X \to X$ defined by $d(x, y) = |x - y|^2 + i|x - y|^2$ $\forall x, y \in X$
then (X, d) is complex valued b-metric spaces with s=2 . Now we define self maps $S, T fandg : X \to X$ such that $Sx = \frac{x}{9}$, $Tx = \frac{x^2}{18}$, $fx = x$ and $gx = \frac{x^2}{2}$ satisfies the contractive conditions of theorem 3.1 and theorem 3.2 and $x = 0$ is unique common fixed point of S,T,f and g.

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