International Journal of Science and Research (IJSR)
ISSN: 2319-7064
ResearchGate Impact Factor (2018): 0.28 | SJIF (2018): 7.426

Relation Between the Last Terms and Common Ratios of Two Different Geometric Progressions [S_1 and S_2] [For Finite Number of Terms]

Chirag Gupta

Abstract: ***The new relation $X_n/Y_n = r_1/r_2$ is for finite number of terms (n). X_n and Y_n are the last terms of two geometric progressions, S_1 and S_2. r_1 and r_2 are the common ratios of S_1 and S_2.***

Keywords: G.P, Common ratio (r), Last term (X_n or Y_n) and etc.

1. Theorem

If there are two different GP's [first one (S_1) having even numbers of terms] having same first term (a) and term occurring at (n-1)thplace then, the ratio of last term of S_1 (x_n) to the last term of S_2 (y_n) is always equal to the ratio of common ratio of S_1 (r_1) to the common ratio of S_2(r_2).

Mathematical Form: - $X_n/Y_n = r_1/r_2$.

Note:-

1) (S_1) should consists of even number of terms
2) Number of terms in (S_1) may be 4,6,8,10,12,.....and so on
3) First term of both S1 and S2 should be equal and term occurring at (n-1) th place of both S1 and S2 should be equal.
4) S1 and S2 should form some pattern, such as
   a) Third term of S_1 is equal to the second term of S_2.
   b) Fifth term of S_1 is equal to third term of S_2.
   c) Seventh term of S_1 is equal to the fourth term of S_2.
   And so on.
   (It depends on the number of terms (n) of the GP.)
5) S1 and S2 must be in the form of
   a) S_1= a,ar,ar^2,ar^3
      S_2 = a,ar^2,ar^4
   b) S_1 = a,ar,ar^2,ar^3,ar^4,ar^5,ar^6,ar^7
      S_2 = a,ar^2,ar^4,ar^6,ar^8
   c) S_1 = a,ar,ar^2,ar^3,ar^4,ar^5
      s_2= a,ar^2,ar^4,ar^6

Clearly, the first term and term occurring at (n-1)thplace of both S_1 and S_2 in case (a) , (b), (c)are same. Number of terms (n) in S_1 is even

If all above conditions (note) are satisfied then we can apply the following relation
$X_n/Y_n = r_1/r_2$.

Here, X_n and Y_n are the last terms of S_1 and S_2. r_1 and r_2 are the common ratios of S_1 and S_2.

In Mathematical Form

S_1=a, ar, ar^2,ar^3.
S_2= a ,ar^2,ar^4.
X_n=ar^3,y_n=ar^4

$$x_n/y_n=ar^3/ar^4=1/r \quad \text{---------- (1)}$$

r_1=ar/a=r
r_2=ar^2/a=r^2

$$r_1/r_2=r/r^2=1/r \quad \text{---------- (2)}$$

Therefore, from (1) and (2)
$X_n/Y_n = r_1/r_2$

Some important examples:

1) S_1= 1/3, 1/6, 1/12, 1/24
S_2 = 1/3, 1/12, 1/48
X_n= 1/24, Y_n= 1/48
X_n/Y_n= 48/24 = 2
r_1= 1/2, r_2= 1/4
r_1/r_2= 2
Hence, $X_n/Y_n = r_1/r_2$

2) S_1= 2, 1/4, 1/32, 1/256
S_2 = 2, 1/32, 1/2048
X_n = 1/256, Y_n = 1/2048
X_n/Y_n = 8
r_1= 1/8, r_2= 1/64
r_1/r_2= 8
Hence, $X_n/Y_n = r_1/r_2$

3) S_1= 256, 128, 64, 32, 16, 8, 4, 2
S_2 = 256, 64, 16, 4, 1
X_n = 2 , Y_n = 1
X_n/Y_n = 2
r_1= 1/2, r_2= 1/4
r_1/r_2= 2
Hence, $X_n/Y_n = r_1/r_2$

4) S_1= 3, 6, 12, 24, 48, 96, 192, 384
S_2 = 3, 12, 48, 192, 768
X_n = 384, Y_n = 768
X_n/Y_n = 1/2
r_1= 2, r_2= 4
r_1 / r_2= 1/2
Hence, $X_n / Y_n = r_1/r_2$

5) S_1= 1/9, -1/27, 1/81, -1/243
S_2 = 1/9, 1/81, 1/729
X_n = -1/243, Yn= 1/729
X_n/Y_n = -3
r_1= -1/3, r_2= 1/9
r_1/r_2= -3

Hence, $X_n/Y_n = r_1/r_2$

Proof:-
According to my Previousarticle

HCF X LCM / Common ratio (r)= first term (a) X [term occuring at the place (n-1)] .

The above relation is for odd number of terms. But it is correct for even number of terms also.

HCF of a G.P is always its first term. ThereforeHCF = a and LCM of a G.P is always its last term.

Therefore , LCM= x_n or y_n
a_1 X x_n /r_1 =a X[term occuring at (n-1) place]--------(1)
[for even number of terms] (S_1)

a_2 X y_n /r_2 = a [term occuring at (n-1) place] ----------(2)
(S_2)
from (1) and (2) , we get
a_1 X x_n / r_1 = a_2 X y_n / r_2
since , $a_1 = a_2$
therefore, x_n / y_n = r_1 / r_2
Hence , the relation is proved .

2. Benefits

It should encourage student to think and discover more about this topic. It is a new relation which creates some new questions related to this topic.

3. Acknowledgement

It's my individual work. This relation is made by me only. I thank to my parents and teachers of SICES Degree College for support and motivation. My father's constant motivation has been responsible for completion of my work.

References

[1] 11th Standard CBSE Mathematics Textbook.