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Polarized Gluon Distribution Functions in Leading Order and Next-to-Leading Order

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1. Introduction

Quark and gluon contributions to the nucleon spin are described by polarized parton distribution functions (polarized PDF) and their first moments. It became clear that only a small fraction of nucleon spin is carried by quarks and antiquarks. Therefore, a large gluon polarization or effect of orbital angular momenta should be possible sources for explaining the origin of the nucleon spin. Polarized PDFs have been investigated by global analyses of data on polarized lepton - nucleon DIS and proton - proton collisions [1, 2, 3, 4, 5-12]. Polarized quark distributions are determined relatively well, however the polarized gluon distribution is not accurately determined. The gluon distribution contributes to the structure function g_1 as a higher order effect in the expansion by the running coupling constant α_s of QCD.

The polarized gluon distribution functions have been obtained by solving DGLAP evolution equations in LO and NLO at the small-x limit. Here we have used a Taylor's series expansion and then the method of characteristics to solve the evolution equations. We have also calculated t and x-evolutions of gluon distribution function and the results are compared with the graph obtained by B Ziaja with the help of numerical method [13]. Here the detailed phenomenological study is not possible due to shortage of experimental data of polarized gluon distribution function.

2. Theory

The DGLAP evolution equations in standard forms for polarized gluon distribution functions $\Delta G(x,t)$, in LO and NLO [14, 15, 16, 17] are

$$\frac{\partial \Delta G(x,t)}{\partial t}-\frac{\alpha_S}{2\pi}J_1^G(x,t)=0 \quad (1)$$

$$\frac{\partial \Delta G(x,t)}{\partial t}-\frac{\alpha_S}{2\pi}J_1^G(x,t)-\left(\frac{\alpha_S}{2\pi}\right)^2 J_2^G(x,t)=0, \quad (2)$$

where integrals $J_1^G(x,t)$, $J_2^G(x,t)$ are defined in Appendix G.

Let us introduce the variable u = 1 ω and using Taylor's expansion series we can rewrite

$$\Delta G\left(\frac{x}{\omega},t\right)\approx \Delta G(x,t)+\frac{xu}{1-u}\frac{\partial \Delta G(x,t)}{\partial x} \quad (3)$$

$$g_1^S\left(\frac{x}{\omega},t\right)=g_1^S(x,t)+\frac{xu}{1-u}\frac{\partial g_1^S(x,t)}{\partial x} \quad (4)$$

Using equations (3) and (4) and performing u-integrations we get equation (1) as

$$\frac{\partial \Delta G(x,t)}{\partial t}-\frac{A_f}{t}\left[\begin{array}{l}A_1'^g(x)\Delta G(x,t)+A_2'^g(x)\dfrac{\partial \Delta G(x,t)}{\partial x}\\ +A_3'^g(x)g_1^S(x,t)+A_4'^g(x)\dfrac{\partial g_1^S(x,t)}{\partial x}\end{array}\right]=0. \quad (5)$$

Equation (4) is a partial differential equation of two variables and two functions. To convert it to one function, we have to establish a relation between them. At high-Q^2 and small-x, we can assume that sea quarks and gluons have no clear-cut distinction. Thus, we can assume that $g_1^S(x,t)=R'(x)\Delta G(x,t)$, where R'(x) is a suitable function of x or may be a constant. Now equation (5) gives

$$-t\frac{\partial \Delta G(x,t)}{\partial t}+A_f L_1'^g(x)\frac{\partial \Delta G(x,t)}{\partial x}+A_f M_1'^g(x)\Delta G(x,t)=0 \quad (6)$$

where

$$L_1'^g(x)=A_2'^g(x)+R'(x)A_4'^g(x) \quad (7)$$

$$M_1'^g(x)=A_1'^g(x)+R'(x)A_3'^g(x)+A_4'^g(x)\frac{\partial R'(x)}{\partial x} \quad (8)$$

To introduce the method of characteristics, let us consider two new variables S and τ instead of x and t, such that

$$\frac{dt}{dS}=-t, \quad (9)$$

$$\frac{dx}{dS}=A_f L_1'^g(x). \quad (10)$$

Putting these in equation (5), we get

$$\frac{d\Delta G(x,t)}{dt}+M_1'^g(S,\tau)\Delta G(S,\tau)=0, \quad (11)$$

where $M_1'^g(S,\tau)=A_f M_1'^g(x)$.

Equation (11) can be solved as

$$\Delta G(S,\tau)=\Delta G(0,\tau)\exp\left[-\int_0^S M_1'^g(S,\tau)\,dS\right] \quad (12)$$

For t-evolution, polarized gluon distribution functions vary with t, remaining x constant. Hence equation (9) can be used to solve the equation (11). Now we have to replace the co-ordinate system (S, τ) to (x, t), considering when S = 0, t = t_0 and the input function as $\Delta G(\tau)=\Delta G(x,t_0)$. So the t-evolution of polarized gluon distribution function in LO is given by

$$\Delta G(x,t)=\Delta G(x,t_0)\left(\frac{t}{t_0}\right)^{A_f M_1'^{g}(x)}. \quad (14)$$

Using equation (10) and replacing the co-ordinate system (S, τ) to (x, t), with consideration when τ = 0, $x = x_0$ the input function is $\Delta G(S)=\Delta G(x_0,t)$, we get the x-evolution of polarized gluon distribution function in LO as

$$\Delta G(x,t)=\Delta G(x_0,t)\exp\left(-\int_{x_0}^{x}\frac{M_1'^{g}(x)}{L_1'^{g}(x)}dx\right) \quad (15)$$

Considering the same procedure as in unpolarized cases, the t and x-evolution of polarized gluon distribution functions in NLO are given by

$$\Delta G(x,t)=\Delta G(x_0,t)\left(\frac{t}{t_0}\right)^{A_f\left\{M_1'^{g}(x)+T_0 M_2'^{g}(x)\right\}} \quad (16)$$

$$\Delta G(x,t)=\Delta G(x_0,t)\exp\left(-\int_{x_0}^{x}\frac{M_1'^{g}(x)+T_0M_2'^{g}(x)}{L_1'^{g}(x)+T_0L_2'^{g}(x)}dx\right), \quad (17)$$

where

$$L_2'^{g}(x)=B_2'^{g}(x)+R'(x)B_4'^{g}(x),$$

$$M_2'^{g}(x)=B_1'^{g}(x)+R'(x)B_3'^{g}(x)+B_4'^{g}(x)\frac{\partial R'(x)}{\partial x}$$

and $\left(\frac{\alpha_S(t)}{2\pi}\right)^2=T_0\left(\frac{\alpha_S(t)}{2\pi}\right)$.

Here T_0 is a **n**umerical parameter, which is not arbitrary chosen but obtained by phenomenological analysis [18, 19].

Here $\Delta G(x_0,t)$ and $\Delta G(x,t_0)$ are input functions. For phenomenological analysis we use equations (14) and (15) to study polarized gluon distribution functions in LO and equations (16) and (17) to study polarized gluon distribution functions in NLO.

3. Results and Discussions

Here we compare our result of x evolution of polarized gluon distribution function $\Delta G(x,t)$ in LO and NLO with the graphs obtained by numerical method of B Ziaja [13]. Each graph of our result is the best fit graph of our works with the numerical method. Here values at x = 0.001 is taken as input to test the x-evolution equations of our results. We compared our results for R'(x) = R', ax^b and ce^{dx}, where R', a, b, c and d are constants. In all figures, we have plotted computed values of polarized gluon distribution function $\Delta G(x,t)$ against the x values for a fixed Q^2 qualitatively. Here we have plotted the graphs for Q^2 =10 GeV^2 in the range of $0.001 \le x \le 0.00001$. In all graphs, solid lines represent our NLO results, dash lines represent our LO results of best fitted graphs and numerical methods are represented by the dotted lines. Since experimental data as well as parameterization results on t-evolution of polarized gluon distribution function is not found elsewhere, so we could not compare our t-evolution results here.

In figure 1, we have plotted our results for LO and NLO considering R'(x) = R', a constant. It is found that best fit results are for R' = 0.8 in both LO and NLO.

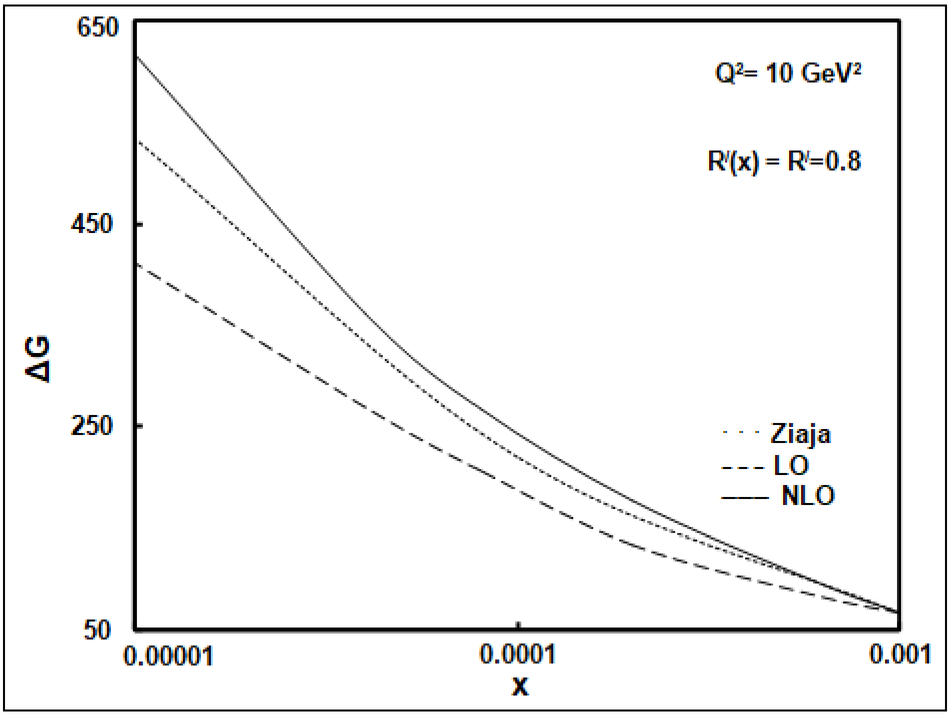

Figure 1: x-evolution of polarized gluon distribution functions in LO and NLO with R'(x) = R', compared with the graph obtained by numerical method

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In figure 2, we have plotted our results for $R'(x) = ax^b$, a power function of x. It is found that best fit results are for a = 0.01 and b = 0.002 in both LO and NLO.

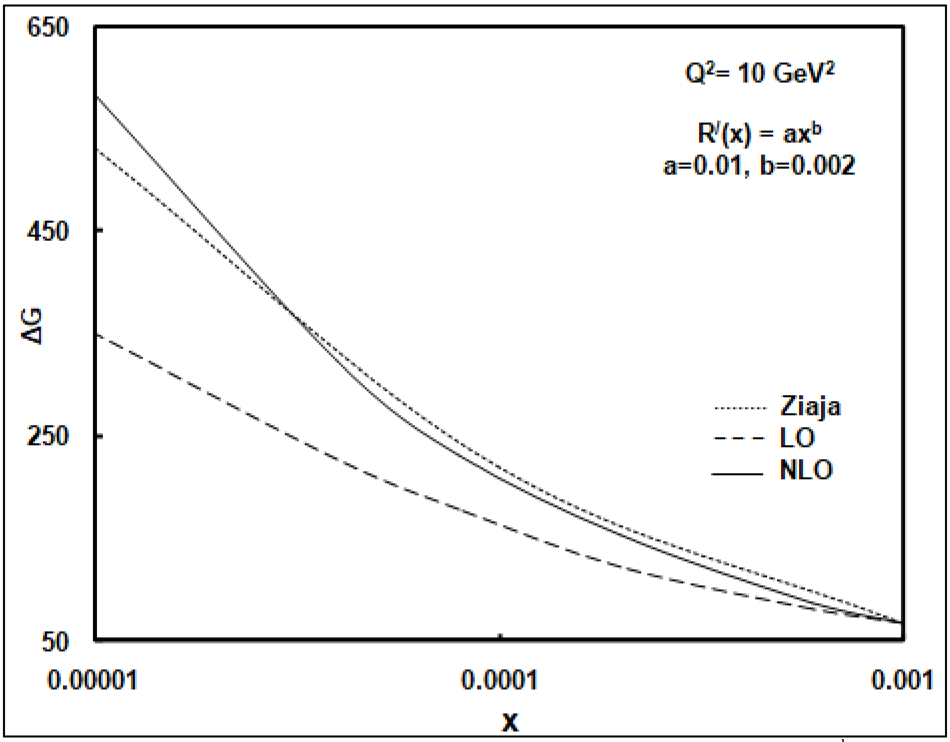

Figure 2: x-evolution of polarized gluon distribution functions in LO and NLO with $R'(x) = ax^b$, compared with the graph obtained by numerical method

In figure 3, we have plotted our results for $R^{/}(x) = ce^{\square\square dx}$, an exponential function of x. It is found that our best fit results are for c = 0.2 and d = 0.03 in both LO and NLO.

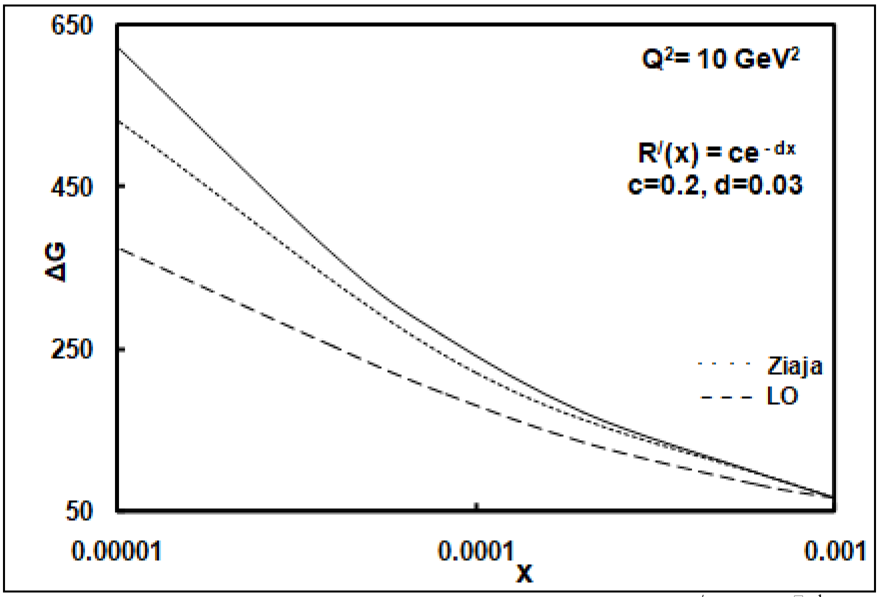

Figure 3: x-evolution of polarized gluon distribution functions in LO and NLO with $R^{/}(x) = ce^{\square\, dx}$, compared with the graph obtained by numerical method

4. Conclusion

Here we have solved DGLAP evolution equations for polarized gluon distribution function in LO and NLO using method of characteristics. Experimental data on t-evolution of polarized gluon distribution function is not found elsewhere. Thus, we could not compare our t-evolution results of polarized gluon distribution function. Our x-evolution graphs for polarized gluon distribution functions in both LO and NLO are compared are in good consistency with the results obtained by solving unified evolution equation by numerical method especially at small-x and high-Q^2 region. The mean percentage error of our LO and NLO results are 11.47% and 5.81% with data obtained from numerical method. Thus, the NLO shows significantly better fitting to the data sets obtained by numerical method than that of in LO and hence the NLO corrections have significant effect and we cannot ignore the contribution of NLO terms in high-Q^2 and small-x region. The polarized gluon contribution, which is largely responsible for scaling violations, appears to be positive, although quite poorly determined at this time. A major motivation for future high-energy polarized scattering experiments is to obtain more information on the polarized gluon contributions to the nucleon spin.

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