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Some Fixed Point Theorems of Integral Function in Menger Spaces with Property (E.A.)

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Abstract: ***In this paper, we obtain some fixed point theorems using integral type inequality in Menger space employing the property (E.A). Our results improve and generalize several known fixed point theorems existing in the literature.***

Keywords: Menger space, Integral Function, Weakly compatible mappings, Property (E.A)

Mathematics Subject Classification (2010): Primary 47H10, Secondary 54H25.

1. Introduction

In the year 1942 Menger [21] introduced the notion of a probabilistic metric space (PM- space) which was, in fact, a generalization of metric space. The idea behind this is to associate a distribution function with a pair of points, say (p,q), denoted by Fp,q(t) where t > 0 and interpret this function as the probability that distance between p and q is less than t, whereas in the metric space, the distance function is a single positive number. Sehgal [37] initiated the study of fixed points in probabilistic metric spaces. The study of these spaces was expanded rapidly with the pioneering works of Schweizer and Sklar [7]. Jungck [13] introduced the notion of compatible mappings and utilized the same to improve commutativity conditions in common fixed point theorems. This concept has been frequently employed to prove existence theorems on common fixed points. However, the study of common fixed points of non-compatible mappings was initiated by Pant [29]. Recently, Aamri and Moutawakil [1] and Liu et al. [34] respectively defined the property (E.A) and the common property (E.A) and proved interesting common fixed point theorems in metric spaces. Most recently, Kubiaczyk and Sharma [15] adopted the property (E.A) in PM spaces and used it to prove results on common fixed points. Recently, Imdad et al. [26] adopted the common property (E.A) in PM spaces and proved some coincidence and common fixed point results in Menger spaces.

1) Preliminaries: Before going to our main result we require some more definitions and Lemma**,**

Definition 1.1 [8]**:** Let X be a non empty set and L denote the set of all distribution functions. A probabilistic metric space is an ordered pair (X,F) where F :X * X→L. we shall denote the distribution function by F (p, q) or F p ,q ; p, q ϵ X and F(p, q, x) will represent the value of F (p, q) at x ϵ R. the function F p, q is assumed to satisfy the following conditions :

1. F p, q (t) = 1, ∀ t > 0 if and if p = q
2. F p, q (0) = 0 for every p, q ϵ X
3. F p, q (t) = F q, p (t) for every p, q ϵ X
4. If F p, q (t) = 1 and F q, r (s) = 1 it follows that F q r (t +s) =1 ∀ p, q, r ϵ X and t ,s ≥ 0.

In metric space (X ,d) , the metric d induces a mapping F : X * X→L such that F p, q (t) = H(t-d(p, q)) for all p, q ϵ X and t ϵ R , where H is the distribution function defined as

$$H(x) = \begin{cases} 0, \text{if } x \le 0 \\ 1, \text{if } x > 0 \end{cases}$$

Definition 1.2 [8] **:**A mapping Δ: [0, 1] * [0,1] →[0,1] is called t- norm if the following conditions are satisfied
(1) Δ (a, 1) = a for all a ϵ [0, 1], Δ (0,0) = 0,
(2) Δ (a ,b) = Δ (b, a)
(3) Δ (c, d) ≤ Δ (a, b) for c ≥ a, d ≥ b, and
(4) Δ (Δ (c, d),c) = Δ (a,Δ (b, c)) for all a, b, c ϵ [0,1]

Example 1[8] The following are the four basic t-norms:
(i) The minimum t-norm: $T_M(a, b) = \min\{a,b\}$.
(ii) The product t-norm: $T_P(a,b) = a.b$
(iii) The Lukasiewicz t-norm: $T_L(a, b) = \max\{a + b - 1, 0\}$.
(iv) The weakest t-norm, the drastic product:

$$T_D(a, b) = \begin{cases} \min\{a, b\}, & if \ \max\{a, b\} = 1 \\ 0, & otherwise \end{cases}$$

In respect of above mentioned t-norms, we have the following ordering:
$T_D < T_L < T_P < T_M$.

Definition 1.3 [21]**:** A Menger probabilistic space is a triplet (X, F, Δ) where (X, F) is a PM-space and Δ is a t- norm with the following condition
$F_{p,r}(t+s) \ge \Delta(F_{p,r}(t), F_{p,r}(s))$ for all p, q, r ϵ X and t, s ≥ 0.
The above inequality is called Menger's triangle inequality.

Definition 1.4 [28] **:** A sequence $\{x_n\}$ in (X, F, Δ) is said to be a convergent to a point x ϵ X if for every ε > 0 and λ > 0, there exists an integer N=N (ε ,λ) such that $F_{x_n, x}(\varepsilon) \to 1-\lambda$ ∀ n ≥ N (ε ,λ).

Definition 1.5 [28] **:** A sequence $\{x_n\}$ in (X, F, Δ) is said to be a Cauchy sequence if for every ε > 0 and λ >0 , there exists an integer N=N (ε ,λ) such that $F_{x_n, x_m}(\varepsilon) \to 1-\lambda$ ∀ n, m ≥ N (ε ,λ).

Definition 1.6 [28] **:** A Menger Space (X, F, Δ) with the continuous t- norm is said to be complete if every Cauchy sequence in X converges to a point in X.

Definition 1.7 [24]**:** Let (X, F, Δ) be a Menger PM Space. A pair (f, g) of self mapping on X is said to be weakly commuting if and only if $F_{fgx\,,\,gfx\,,}(t) \geq F_{fx\,,\,gx,}(t)$ for each x ϵ X and t > 0.

Definition 1.8 [31]**:** Let (X, F, Δ) be a Menger PM Space . A pair (f, g) of self mapping on X is said to be compatible if and only if $F_{fgx_n\,,\,gfx_n\,,}$ (t) →1 for all t > 0 whenever $\{x_n\}$ in X such that $fx_n, gx_n \to z$ for some z ϵ X as n → ∞.
Clearly, a weakly commuting pair is compatible but every compatible pair need not be weakly commuting.

Definition 1.10 [19]**:** Let (X, F, Δ) be a Menger PM Space . A pair (f, g) of self mapping on X is said to be non–compatible if and only if there exist at least one sequence $\{x_n\}$ in X such that
$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n =$ z, for some z ϵ X, implies that $\lim_{n\to\infty} F_{fgx_n\,,gfx_n}(t_0)$ (for some $t_0 > 0$) is either less than 1 or non-existent.

Definition 1.11 [15] **:** Let (X, F, Δ) be a Menger PM Space . A pair (f, g) of self mapping on X is said to satisfy the property (E.A) if there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n =$ z, for some z ϵ X.
Clearly, a pair of compatible mappings as well as non-Comatible mappings satisfies the property (E.A).
Inspired by Liu et al. [39], Imdad et al. [26] defined the following:

Definition 1.12 [34]: Two pairs (f, g) and (p, q) of self mappings of a Menger PM space (X,F, Δ) are said to satisfy the common property (E.A) if there exist two sequences $\{x_n\}$, $\{y_n\}$ in X and some t in X such that
$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = \lim_{n\to\infty} px_n = \lim_{n\to\infty} qx_n = z$

Definition1.13. [24] Two finite families of self mappings $\{A_i\}$ and $\{B_j\}$ are said to be pairwise commuting if:
(i) $A_iA_j = A_jA_i$, i, j ∊ {1, 2...m},
(ii) $B_iB_j = B_jB_i$, i, j ∊ {1, 2...n},
(iii) $A_iB_j = B_jA_i$, i ∊ {1, 2...m}, j ∊ {1, 2...n}.

2. Main Result

The following lemma is useful for the proof of succeeding theorems.

Lemma 2.1 [14]: Let (X, F, Δ) be a Menger space. If there exists some k ∈ (0, 1) such that for all p, q, X and all x > 0,

$$\int_0^{F_{p,q}(kt)} \emptyset(u)du \geq \int_0^{F_{p,q}(t)} \emptyset(u)du \quad \text{- - - (2.1.1)}$$

Where ϕ : [0, ∞) → [0, ∞) is a non-negative summable Lebesgue integrable function such that

$$\int_\varepsilon^1 \emptyset(u)du > 0 \text{ for each } \varepsilon \in [0,1) \text{ then } p = q.$$

Proof. From (2.1.1)

$$\int_0^{F_{p,q}(t)} \emptyset(u)du \geq \int_0^{F_{p,q}(k^{-1}t)} \emptyset(u)du$$

one can inductively write (for m ∈ N)

$$\int_0^{F_{p,q}(t)} \emptyset(u)du \geq \int_0^{F_{p,q}(k^{-1}t)} \emptyset(u)du \geq \text{- - -} \geq \int_0^{F_{p,q}(k^{-m}t)} \emptyset(u)du$$

$$\geq \text{- - -} \to \int_0^1 \emptyset(u)du \quad \text{as } m \to \infty.$$

Therefore

$$\int_0^{F_{p,q}(t)} \emptyset(u)du - \int_0^1 \emptyset(u)du \geq 0$$

And hence,

$$\int_0^{F_{p,q}(t)} \emptyset(u)du \left(\int_0^{F_{p,q}(t)} \emptyset(u)du - \int_0^1 \emptyset(u)du\right) \geq 0$$

Or,

$$\int_{F_{p,q}(t)}^1 \emptyset(u)du \leq 0.$$

which amounts to say that Fp,q(t) ≥ 1 for all t ≥ 0. Thus, we get p = q.

Remark : By setting φ(t) = 1 (for each t ≥ 0) in (2.1.1) of Lemma 2.1, we have

$$\int_0^{F_{p,q}(kt)} \emptyset(u)du = F_{p,q}(kt) \geq F_{p,q}(t) = \int_0^{F_{p,q}(t)} \emptyset(u)du,$$

which shows that Lemma 1 is a generalization of the Lemma 2 (contained in [34]).

In what follows, Δ is a continuous t-norm (in the product topology).

Lemma 2.2: Let (X.F, Δ) be a complete Menger Space and let f, g, p and q be self mapping of X satisfying the conditions:
(i) Pairs {p, f} and {q, g} satisfies the property E.A.
(ii) B(y_n) converges for every sequence $\{y_n\}$ in X whenever T(y_n) converges,
(iii) for any x,y ∊ X and for all t > 0,

$$\int_0^{F_{px,qy}(kt)} \emptyset(u)du \geq \int_0^{m(x,y)} \emptyset(u)du \quad \text{- - - (2.2.1)}$$

Where ϕ : [0, ∞) → [0, ∞) is a non-negative summable Lebesgue integral function such that
$\int_c^1 \emptyset(u)du > 0$ for each u ∊ [0,1), where 0 < k < 1 and

$$m(x,y) = \min\Big\{F_{fx,gy}(t), F_{fx,px}(t), F_{gy,qy}(t), F_{fx,qy}(t), F_{gy,px}(t), \frac{F_{fx,gy}(t).\ F_{gy,qy}(t)}{F_{fx,qy}(t)}, \frac{F_{fx,gy}(t).\ F_{fx,,px}(t)}{F_{gy,px}(t)}, \frac{F_{qy,gy}(t) + F_{fx,,qy}(t)}{2}, \frac{F_{fx,,gy}(t) + F_{px,gy}(t)}{2}$$

(iv) p(X) ⊂ g(X) (or q(X) ⊂ f(X)).
Then the pair (p,f) and (q,g) share the common property (E.A.).

Proof : Suppose that the pair (p,f) enjoys the property (E.A.),then there exist a sequence $\{x_n\}$in X such that

$$\lim_{n\to\infty} px_n = \lim_{n\to\infty} fx_n = u, \text{ for some } u \in X.$$

Since p(X) ⊂ g(X), for each x_n there exists y_n ∊ X.
Such that $px_n = gy_n$, and hence

$$\lim_{n\to\infty} gy_n = \lim_{n\to\infty} px_n = u$$

Thus in all, we have $px_n \to u$, $fx_n \to u$ and $gy_n \to u$. Now we assert that $qy_n \to u$.

To accomplish this, using (2.2.1) with x = x_n and y = y_n, we get

$$\int_0^{F_{px_n,qy_n}(kt)} \emptyset(u)du \geq \int_0^{m(x_n,y_n)} \emptyset(u)du$$

Where, $m(x_n, y_n) = \min\{F_{fx_n,gy_n}(t),$

$F_{fx_n,px_n}(t)$, $F_{gy_n,qy_n}(t)$, $F_{fx_n,qy_n}(t)$, $F_{gy_n,px_n}(t)$, $\frac{F_{fx_n,qy_n}(t).F_{gy_n,qy_n}(t)}{F_{fx_n,qy_n}(t)}$,

$\frac{F_{fx_n,gy_n}(t).F_{fx_n,px_n}(t)}{F_{gy_n,px_n}(t)}$, $\frac{F_{qy_n,gy_n}(t)+F_{fx_n,qy_n}(t)}{2}$, $\frac{F_{fx_n,gy_n}(t)+F_{px_n,gy_n}(t)}{2}\}$

Let, $\lim_{n\to\infty} q(y_n) = v$

Also, let t > 0 be such that $F_{u,v}(.)$ is continuous in t and kt.

Then, on making n → ∞ in the above inequality, we get

$$\int_0^{F_{u,v}(kt)} \emptyset(u)du \geq \int_0^{\min\{F_{u,u}(t), F_{fu,u}(t), F_{u,v}(t), F_{u,v}(t), F_{u,u}(t), \frac{F_{u,u}(t).F_{u,v}(t)}{F_{u,v}(t)}, \frac{F_{u,u}(t).F_{u,u}(t)}{F_{u,u}(t)}, \frac{F_{u,v}(t)+F_{u,v}(t)}{2}, \frac{F_{u,u}(t)+F_{u,u}(t)}{2}} \emptyset(u)du$$

Or, $\int_0^{F_{u,v}(kt)} \emptyset(u)du \geq \int_0^{F_{u,v}(t)} \emptyset(u)du$

This, implies that v = u (in view of Lemma 2.1) which shows that the pair (p,f) and (q,g) share the common property (E.A).

Theorem 2.3: Let f, g, p and q be self mappings of a Menger space (X, F, Δ) which satisfy the inequality (2.2.1) together with the conditions :

(i) the pairs (p, f) and (q, g) share the common property (E.A),

(ii) f(X) and g(X) are closed subsets of X.

Then the pairs (p, f) and (q, g) have a point of coincidence each. Moreover, f, g, p and q have a unique common fixed point provided both the pairs (p, f) and (q, g) are weakly compatible.

Proof. Since the pairs (p, f) and (q, g) share the common property (E.A), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} px_n = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} qy_n =$ u, for some u ∈ X.

Since f(X) is a closed subset of X, hence $\lim_{n\to\infty} fx_n$ = u ∈ f(X).

Therefore, there exists a point z ∈ X such that fz = u.

Now, we assert that pz = fz.

To prove this, on using (2.2.1) with x = z, y = y_n, we get

$$\int_0^{F_{pz,qy_n}(kt)} \emptyset(u)du \geq \int_0^{\min\{F_{fz,qy_n}(t), F_{fz,pz}(t), F_{gy_n,qy_n}(t), F_{fz,qy_n}(t), F_{gy_n,pz}(t), \frac{F_{fz,qy_n}(t).F_{gy_n,qy_n}(t)}{F_{fz,qy_n}(t)}, \frac{F_{fz,gy_n}(t).F_{fz,pz}(t)}{F_{gy_n,pz}(t)}, \frac{F_{qy_n,gy_n}(t)+F_{fz,qy_n}(t)}{2}, \frac{F_{fz,gy_n}(t)+F_{pz,gy_n}(t)}{2}} \emptyset(u)$$

On taking n ⟶ ∞, reduces to

$$\int_0^{F_{pz,u}(kt)} \emptyset(u)du \geq \int_0^{\min\{F_{u,u}(t), F_{u,pz}(t), F_{u,u}(t), F_{u,u}(t), F_{u,pz}(t), \frac{F_{u,u}(t).F_{u,u}(t)}{F_{u,u}(t)}, \frac{F_{u,u}(t).F_{u,pz}(t)}{F_{u,pz}(t)}, \frac{F_{u,u}(t)+F_{u,u}(t)}{2}, \frac{F_{u,u}(t)+F_{pz,u}(t)}{2}\}} \emptyset(u)du$$

$$\int_0^{F_{pz,u}(kt)} \emptyset(u)du \geq \int_0^{F_{pz,u}(t)} \emptyset(u)du$$

Now on appealing Lemma 2.1, we get pz = u and hence pz = fz. Therefore, z is a coincidence point of the pair (p, f).

Since g(X) is a closed subset of X, therefore $\lim_{n\to\infty} gy_n$ = u ∈ g(X) and hence we can find a point w∈ X such that gw = u

Now we show that qw = gw.

To accomplish this, on using (2.2.1) with x = x_n, y = w, we have

$$\int_0^{F_{px_n,qw}(kt)} \emptyset(u)du \geq \int_0^{m(x_n,w)} \emptyset(u)du$$

Where, $m(x_n,w) = \min\{F_{fx_n,qw}(t)$, $F_{fx_n,px_n}(t)$, $F_{gw,qw}(t)$, $F_{fx_n,qw}(t)$, $F_{gw,px_n}(t)$, $\frac{F_{fx_n,qw}(t).F_{gw,qw}(t)}{F_{fx_n,qw}(t)}$,

$\frac{F_{fx_n,gw}(t).F_{fx_n,px_n}(t)}{F_{gw,px_n}(t)}$, $\frac{F_{qw,gw}(t)+F_{fx_n,qw}(t)}{2}$, $\frac{F_{fx_n,gw}(t)+F_{px_n,gw}(t)}{2}\}$

Which on making n ⟶ ∞, reduces to

$$\int_0^{F_{u,qw}(kt)} \emptyset(u)du \geq \int_0^{\min\{F_{u,u}(t), F_{u,u}(t), F_{u,qw}(t), F_{u,qw}(t), F_{u,u}(t), \frac{F_{u,u}(t).F_{u,qw}(t)}{F_{u,qw}(t)}, \frac{F_{u,u}(t).F_{u,u}(t)}{F_{gw,u}(t)}, \frac{F_{u,gw}(t)+F_{u,u}(t)}{2}, \frac{F_{u,gw}(t)+F_{u,gw}(t)}{2}} \emptyset(u)du$$

$$\int_0^{F_{u,qw}(kt)} \emptyset(u)du \geq \int_0^{F_{u,qw}(t)} \emptyset(u)du$$

on employing Lemma 2.1, we get qw = u and gw = qw

Therefore, w is a coincidence point of the pair (q, g).

Since the pair (p, f) is weakly compatible and pz = fz, therefore pu = pfz = fpz = fu.

Again, on using (2.2.1) with x = u, y = w, we have

$$\int_0^{F_{pu,qw}(kt)} \emptyset(u)du \geq \int_0^{m(u,w)} \emptyset(u)du$$

Where, $m(u,w) = \min\{F_{fu,qw}(t)$, $F_{fu,pu}(t)$, $F_{gw,qw}(t)$, $F_{fu,qw}(t)$, $F_{gw,pu}(t)$, $\frac{F_{fu,gw}(t).F_{gw,qw}(t)}{F_{fu,qw}(t)}$,

$\frac{F_{fu,gw}(t).F_{fu,pu}(t)}{F_{gw,pu}(t)}$, $\frac{F_{qw,gw}(t)+F_{fu,qw}(t)}{2}$, $\frac{F_{fu,gw}(t)+F_{pu,gw}(t)}{2}\}$

Or,

$$\int_0^{F_{pu,u}(kt)} \emptyset(u)du \geq$$

$$\int_0^{\min\{F_{fu,u}(t),\ F_{pu,pu}(t), F_{u,,u}(t), F_{pu,u}(t), F_{u,pu}(t),\ \frac{F_{pu,u}(t).F_{u,,u}(t)}{F_{pu,u}(t)},\ \frac{F_{pu,u}(t).F_{pu,,pu}(t)}{F_{u,,pu}(t)},\ \frac{F_{u,u}(t)+F_{pu,,u}(t)}{2}, \frac{F_{pu,u}(t)+F_{pu,u}(t)}{2}} \emptyset(u)du$$

Or

,

$$\int_0^{F_{pu,u}(kt)} \emptyset(u)du \geq \int_0^{F_{pu,u}(t)} \emptyset(u)du$$

On employing Lemma 2.1, we have pu = fu = u, which shows that u is a common fixed point of the pair (p, f).

Also the pair (q, g) is weakly compatible and qw = gw, hence

qu = qgw = gqw = gu.

Next, we show that u is a common fixed point of the pair (q, g). In order to accomplish this, using (2.2.1) with x = z, y = u, we get

$$\int_0^{F_{pz,qu}(kt)} \emptyset(u)du \geq$$

$$\int_0^{\min\{F_{fz,gu}(t),\ F_{fz,pz}(t), F_{gu,qu}(t), F_{fz,qu}(t), F_{gu,pz}(t),\ \frac{F_{fz,gu}(t).F_{gu,qu}(t)}{F_{fz,qu}(t)},\ \frac{F_{fz,gu}(t).F_{fz,,pz}(t)}{F_{gu,pz}(t)},\ \frac{F_{qu,gu}(t)+F_{fz,,qz}(t)}{2}, \frac{F_{fz,,gu}(t)+F_{pz,qu}(t)}{2}} \emptyset(u)du$$

Or,

$$\int_0^{F_{u,qu}(kt)} \emptyset(u)du \geq$$

$$\int_0^{\min\{F_{u,gu}(t),\ F_{u,,u}(t), F_{qu,qu}(t), F_{u,,qu}(t), F_{qu,u}(t),\ \frac{F_{u,qu}(t).F_{qu,qu}(t)}{F_{u,qu}(t)},\ \frac{F_{u,qu}(t).F_{u,,u}(t)}{F_{qu,u}(t)},\ \frac{F_{qu,qu}(t)+F_{u,,u}(t)}{2}, \frac{F_{u,,qu}(t)+F_{u,gu}(t)}{2}} \emptyset(u)du$$

Or,

$$\int_0^{F_{u,qu}(kt)} \emptyset(u)du \geq \int_0^{F_{u,qu}(t)} \emptyset(u)du$$

Using Lemma 2.1, we have qu = u which shows that u is a common fixed point of the pair (q, g). Hence u is a common fixed point of both the pairs (p, f) and (q, g). Uniqueness of common fixed point is an easy consequence of the inequality (2.2.1). This completes the proof.

References

[1] A. Aamri and D. El Moutawakil, Some new common fixed point theorems under strict contractive conditions, J.Math.Anal.Appl.270(2002),181-188.

[2] A. Aliouche, A common fixed point theorem for weakly compatible mappings in symmetric spaces satisfying a contractive condition of integral type, J. Math. Anal. Appl., 322(2)(2006), 796-802.

[3] A. Branciari, A fixed point theorem for mappings satisfying a general contractive condition of integral type, Int. J. Math. Math. Sci., 29(9)(2002), 531-536.

[4] A. Djoudi and A. Aliouche, Common fixed point theorems of Gregus type for weakly compatible Mappings satisfying contractive conditions of integral type, J. Math. Anal. Appl., 329 (1) (2007), 31-45.

[5] A. Razani and M. Shirdaryazdi, A common fixed point theorem of compatible maps in Menger space,Chaos, Solitions and Fractals, 32 (2007), 26-34.

[6] B. E. Rhoades, A comparison of various definitions of contractive mappings, Trans. Amer. Math. Soc., 226 (1977), 257-290.

[7] B. E. Rhoades, Two fixed point theorems for mapping satisfyng a general contractive condition of integral type, Int. J. Math. Math. Sci., 63 (2003), 4007-4013.

[8] B. Schweizer and A. Sklar, Probabilistic metric spaces, Elsevier, North Holand, New York, 1983.

[9] B. Singh and S. Jain, A fixed point theorem in Menger spaces through weak compatibility, J. Math. Anal. Appl., 301 (2005), 439-448.

[10] D. Mihet, A generalization of a contraction principle in probabilistic metric spaces (II), Int. J. Math. Math. Sci 5 (2005), 729-736.

[11] D. Mihet, Fixed point theorems in fuzzy metric spaces using property E.A., Nonlinear Anal., 73 (2010), 2184-2188.

[12] Dr. kamal Wadhwa, dr. ramakant bhardwaj and Jyoti panthi, Common fixed point theorems of integral type in Menger PM Spaces, Network and Complex Systems, vol.3, No.6(2013), 10-16.

[13] G. Jungck, Common fixed points for non continuous nonself maps on nonmetric spaces. Far East J. Math. Sci., 4 (2) (1996), 199-215.

[14] I. Altun, M. Tanveer and M.Imdad, Common fixed point theorems of integral type in Menger PM spaces, Journal of Nonlinear Analysis and Optimization, vol.(3) No.1(2012),55-56.

[15] I. Kubiaczyk and S. Sharma, Some common fixed point theorems in Menger space under strict Contractive conditions, Southeast Asian Bull. Math., 32 (2008), 117-124.

[16] I. N. Beg, Shazad and M. Iqbal, Fixed points theorems and best approximation in convex metric spaces, J. Approx. Theory Appl. 8(4)(1992), 97-105.

[17] J. Ali, M. Imdad and D. Bahuguna, Common fixed point theorems in Menger spaces with common property (E.A), Comp. Math. Appl., 60(2010), 3152-3159.

[18] J. Ali, M. Imdad, D. Mihe¸t, and M. Tanveer, Common fixed points of strict contractions in Menger spaces, Acta Math. Hung., 132(4)(2011), 367-386.

[19] J.X. Fang and Y. Gao, Common fixed point theorems under strict contractive conditions in Menger spaces, Nonlinear Analysis, 70 (2009). 184-193.

[20] K. Menger, Probabilistic geometry, Proc. Nat. Acad. Sci. USA, 37 (1951), 226-229.

[21] K. Menger, Statistical metrics, Proc. Nat. Acad. Sci. USA, 28 (1942), 535-537.

[22] Lj. B. Ciri´c, A generalization of Banach's contraction principle, Proc. Amer. Math. Soc., 45 (1974), 267-273.

[23] M. Imdad and J. Ali, Jungck's common fixed point theorem and E.A property, Acta Math. Cinica, 2(2008), 87-94.

[24] M. Imdad, Javid Ali and M. Tanveer, Coincidence and common fixed point theorems for nonlinear contractions in Menger PM spaces, Chaos, Solitons & Fractals, 42 (2009), 3121-3129.

[25] M. Imdad, M. Tanveer and M. Hasan, Erratum to "Some common fixed point theorems in Menger PM Spaces", Fixed Point Theory Appl., 2011, 2011:28.

[26] M. Imdad, M. Tanveer and M. Hasan, Some commn fixed point theorems in Menger PM spaces, Fixed Point Theory Appl., Vol. 2010, 14 pages.

[27] R. Chugh and S. Rathi, Weakly compatible maps in probabilistic metric spaces, J. Indian Math. Soc., 72 (2005), 131-140.

[28] O. Hadzic and E. Pap, Fixed point theory in probabilistic metric spaces, Kluwer Academic Publishers, Dordrecht, 2001.

[29] R.P. Pant, Common fixed points of noncommuting mappings, J. Math. Anal. Appl., 188 (1994), 436-440.

[30] S. A. Naimpally, K.L Singh and J.H.M. Whitefield, Fixed points and non-expansive retracts in locally convex spaces, Fund. Math., 120 (1984), 63-75.

[31] S.N. Mishra, Common fixed points of compatible mappings in PMspaces, Math. Japon., 36 (1991), 283-289.

[32] T. Suzuki, Meir Keeler contractions of integral type are still MeirKeeler contractions, Internat. J. Math. Math. Sci., (2007), Article ID 39281, 6 pages.

[33] V.M. Sehgal and A.T. Bharucha-Reid, Fixed point of contraction mappings on probabilistic metric spaces, Math. Systems Theory, 6 (1972), 97-102.

[34] Y. Liu, Jun Wu and Z. Li, Common fixed points of single-valued and multivalued maps, Int. J. Math. Math. Sci., 19 (2005), 3045-3055.