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Relation between the Highest Common Factor (HCF), Least Common Multiple (LCM) and Common Ratio of Geometric Progression (G.P) [For Finite Number of Terms]

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Abstract: ***There are two different theorems both are for finite number of terms (n). First theorem is for even number of terms (n) and second is for odd number of terms. Highest Common factor (HCF) of a geometric progression (GP) is always its first term and least Common Multiple (LCM) of a Geometric progression is always its last term.***

Keywords: HCF, LCM, GP, etc

Relation for even number of terms (n)

HCF x LCM/common ratio (r) = [Term occurring at $(n/2)^{th}$ place]^2

Relation for odd number of terms (n)

HCF X LCM/ Common ratio (r) = a x [Term occurring at $(n–1)^{th}$ place]

Here, a is the first term of GP

Relation between the Highest Common Factor (HCF), Least Common Multiple (LCM) and Common Ratio of Geometric Progression (G.P)

[For Finite Number of Terms]

1] For even number of terms theorem

For a geometric progression having even number of terms and terms being either natural numbers (increasing order / decreasing order) or fractions, the product of highest common factor (HCF) and least common multiple (LCM) of the terms divided by the common ratio (r) is equal to the square of the term occurring at the $(n/2)^{th}$ place, where n is the number of terms.

HCF x LCM/common ratio (r) =[term occurring at $(n/2)^{th}$ place]^2

In Mathematical Form

a, ar, ar^2, ar^3
(Here number of terms (n) is 4, which is even)
HCF of a GP is always its first term and LCM of GP is always its last term

Therefore,
HCF = a, LCM = ar^3
HCF X LCM / r = a x ar^3/ r = a^2 r^2 = $(ar)^2$
$(ar)^2$ = [term occurring at the $(n/2)^{th}$ place]^2
Hence, HCF x LCM /r =[Term occurring at $(n/2)^{th}$ place]^2

2] For Odd Number of Terms

Theorem:
For a geometric progression (G.P) having odd number of terms and terms being either natural numbers (increasing order/ decreasing order) or fractions, the product of highest common factor (HCF) and least common multiple (LCM) of the terms divided by the common ratio is equal to the product of first term and term occurring at $(n-1)^{th}$ places.

Relation

HCF x LCM / COMMON RATIO (r)= a x [term occurring at $(n-1)^{th}$ place]
Here, a is the first term of geometric progression

In Mathematical Form:

a, ar, ar^2 (Here, number of terms (n) is 3, which is odd)
HCF= a, LCM= ar^2
HCF x LCM/r = a x ar^2/r = a^2r
Here, first term is a and term occurring at $(n-1)^{th}$ place is ar.
First term x [term occurring at $(n-1)^{th}$ place]=a^2r
Hence, HCF x LCM /r =first term (a) x [Term occurring at $(n-1)^{th}$ place].
Here, r is the common ratio of a geometric progression.

NOTE:

1) In first case (for even number of terms) number of terms (n) may be 4, 6, 8, 10, 12, ……..and so on.
   a) Is also even, but if number of terms (n) is 2, then We cannot say that the series is in geometric progression, for defining a geometric progression (G.P) we must have number of terms (n)
   b) And first case is for even number of terms, Therefore, number of terms (n) may be 4, 6, 8, 10, 12, ……. and so on.
2) In second case (for odd number of terms) number of terms may be 3, 5, 7, 9, 11, ……and so on.

Examples

For Even Number of Terms:
2, 4, 8, 16 (Here, number of terms is 4, which is even)

HCF = 2 (first term), LCM = 16 (last term) and common ratio, r = 2
HCF x LCM / Common ratio (r) = 16
And the term occurring at the place (n/2) = 4
Therefore its square will be = $(4)^2 = 16$
Hence, HCF x LCM / Common ratio = [Term occurring at the place (n/2)]^2

For Odd Number of Terms
6, 12, 24 (Here number of terms is 3, which is odd)
HCF=6 (First term), LCM = 24 (Last term) and common ratio, r = 2
HCF x LCM / Common ratio = 72
First term, a = 6 and term occurring at (n-1)th place = 12
So, first term x [term occurring at (n-1)th place] = 6 x 12= 72
Hence, HCF x LCM / Common ratio = (first term) x [term occurring at (n-1)th place]

Benefits

It should encourage students to discover more about this topic. It is a new relation which creates some new questions related to the HCF and LCM of a GP.

Acknowledgement

It's my individual work. These relations are made by me only. I thank to my brother and my parents for their support and motivation. My Father's constant motivations have been responsible for the completion of my work.

References

[1] Source of information: CBSE Class 10th And 11th Mathematics Textbook.