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Chromatic Number and Weak Complement of L-Fuzzy Graphs

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Abstract: ***Fuzzy Graph colouring techniques are used to solve many complex real world problems. Fuzzy graph colouring can be extended to L-Fuzzy graph. In this paper we studied the chromatic number of L-Fuzzy graph and fuzzy chromatic number of L- Fuzzy graph. We have also tried to define weak complement of L Fuzzy graph and its properties.***

Keywords: L-Fuzzy graphs, chromatic number of L-Fuzzy graphs, Fuzzy chromatic number of L-Fuzzy graph, weak complement of L-Fuzzy graphs, weak complement properties

1. Introduction

Fuzzy graph theory has numerous applications in modern science and technology especially in the field of information theory, neutral networks, expert systems, cluster analysis, medical diagnosis and control theory. Several papers in related areas of fuzzy graph theory are available in literature. [1,2,3,4,5,6,7]

In the classical paper [8] Rosenfield introduced the concept of fuzzy graphs, as a means to model several real life situations. Ever since then, fuzzy graph theory has witnessed tremendous growth.

Graphs are models of relations between objects. The objects are represented by vertices and relations by edges. When the description of the objects, or relationships, or both happens to possessuncertainty, we design a fuzzy graph model. A Somasundaram and S Somasundaram [9] introduced the concept of strength reducing sets and t-connected fuzzy graphs.

Today fuzzy graph theory finds applications in areas as diverse as computer science, artificial intelligence, decision analysis, pattern recognition, medicine, geography, linguistics and even robotics. An application of fuzzy graph theory in the human cardiac function has been discussed in [12]

According to Klir and Yuan [14] an L-Fuzzy set is a fuzzy set in which the range [0,1] is replaced by a lattice.

The concept of chromatic number of fuzzy graph was introduced by Munoz et al [15]. The authors considered fuzzy graphs with crisp vertex set i.e. fuzzy graphs for which $\sigma(x)=1$ $x\epsilon$ V and edges with membership degree in [0,1]. The concept of fuzzy chromatic number was defined by Eslahehi and Onagh [16]

Pramada Ramachandran and K V Thomas introduce the concept of L-Fuzzy graphs.In this paper we introduce the chromatic number of L-Fuzzy graphs, fuzzy chromatic number of L-fuzzy graphs and weak complement of L-Fuzzy graph. Throughout this paper 'L' is a finite lattice (L, ∧, ∨, 0,1) and '≤' denotes the partial order of the lattice.

2. Preliminaries

Definition 2.1 [13]:- *A fuzzy relation on a set V is a map, $\mu : V \times V \rightarrow [0,1]$. A fuzzy graph $G=(V, \sigma, \mu)$ with the underlying Set V is a non-empty set together with a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ u, v in V*

Definition2.2 [15] :-*If $G : (v , \mu)$ is such a fuzzy graph where $v = \{1, 2, 3, ... n\}$ and μ is a fuzzy number on the set of all subsets of V x V. Assume $I=A \cup \{0\}$, where $A=\{\alpha_1<\alpha_2<....< \alpha_n\}$ is the fundamental set (levels set) of G. For each $\alpha \in I$, G_α denotes the crisp graph $G_\alpha =(V, E_\alpha)$ where $E_\alpha= \{ ij / 1 \leq i, j \leq n, \mu(i , j) \geq \alpha \}$ and $X_\alpha= X(G_\alpha)$ denote the chromatic number of crisp growth G_α. By this definition the chromatic number of fuzzy graph is the fuzzy number $X(G) = \{(i , v(i) / (i\epsilon x))\}$ where $v(i) = \max \{ \alpha\epsilon z\ I\ i\epsilon A_\alpha\}$ and $A_\alpha = \{1, 2, 3, ... X_2\}$*

Definition 2.3 [16]:-*A family $\Gamma = \{ \gamma_1,\gamma_2, ... \gamma_k\}$ of fuzzy sets on a set v is called a k-fuzzy colouring of $G = (v, \sigma, \mu)$; Γ*

(i) $\vee \Gamma = \sigma$

(ii) $\gamma_i \wedge \gamma_j = 0$

(iii) For every strong edge (x,y) (u, $\mu(x,y) > 0$) of G $\min \{ \gamma_i(x), \gamma_i(y) \} = 0$ $(1 \leq i \leq k)$

The minimum number k for which there exists a k-fuzzy colouring is called the fuzzy chromatic number of G, denoted by $X^f(G)$.

Incorporating the features of above definitions 3.1 and 3.2 the chromatic number X(G) of fuzzy graph is modified by Anjaly and Sunitha [17] as follows:

Definition 2.4 [17] :-*For each $\alpha \in I$, G_α denotes the crisp graph $G_\alpha = (\sigma_{\alpha,} \mu_\alpha)$ and $X_\alpha= X(G_\alpha)$ denote the chromatic number of crisp graph G_α. The chromatic number of fuzzy graph G is the number $X(G) = \max \{ X(G_\alpha)\ I\ \alpha \in \lambda \}$*

Definition 2.5 [18] :- *An L-Fuzzy graph (LFG) $G^L = (V, \sigma, \mu)$ with the underlying set V is a non empty set together with a pair of functions, $\sigma : V \rightarrow L$ and $\mu : V \times V\rightarrow L$ such that $\mu (u, v) \leq \sigma(u) \wedge \sigma(v)$, ∀ u, v ϵ V*

Definition 2.6 [18]:- *Two vertices x and y in G are called adjacent if*
(½) min { σ(x), σ(y) } ≤ μ(xy). The edge xy of G is called strong if x and y are adjacent and it is called weak otherwise.

Remark 2.7 [17] :- If $\alpha_i \leq \alpha_j$ then $X(G(\alpha_i)) \geq X(G(\alpha_j))$

Definition 2.8[18] :- *An L-Fuzzy Graph* $G^L = (V, \sigma, \mu)$ *is said to be strong if it satisfies the condition* $\mu(u_i, u_j) = \sigma(u_i) \wedge \sigma(u_j)\ \forall\ (u_i, u_j) \epsilon\ \mu$

Definition 2.9[?]:-*Complement of an element (in lattice)*
a is called complement of b if
(i) $a \vee b = 1$
(ii) $a \wedge b = 0$

Definition 2.10[?]:- Complemented lattice: - A bounded lattice where every element has a complement

3. Chromatic number of L- Fuzzy Graph

Incorporating the features of the above definitions, Sreedevi V. S. And Bloomy Joseph has studied the chromatic number of L-Fuzzy Graph and fuzzy chromatic number of L- Fuzzy graph.

Definition 3.1: *Consider L-Fuzzy Graph* $G^L = (V, \sigma, \mu)$, *define* $G^\alpha = (\sigma_\alpha, \mu_\alpha)$
$\sigma_\alpha = \{ v \epsilon V\ I\ \sigma(v) \geq \alpha \}, \alpha \epsilon L$
$\mu_\alpha = \{ vw \epsilon V \times V\ I\ \mu(vw) \geq \alpha \}, \alpha \epsilon L$
$X_\alpha = X(G_\alpha)$ *(Chromatic number of crisp graph* G_α*)*

Then the chromatic number of L-Fuzzy Graph G^L is the number
$X^L(G) = \max \{ X(G)\ I\ \alpha \epsilon \lambda \}$
e.g. Consider the lattice fig(a.1) and L-Fuzzy graph fig (a)

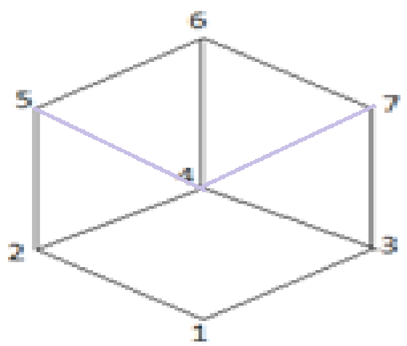


Figure (a.1)

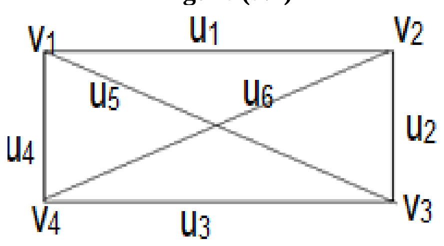


Figure (a)

$\sigma(v_1) = 5\ \ \sigma(v_2) = 6\ \ \sigma(v_3) = 7\ \ \sigma(v_4) = 4,\ \mu(u_1) = 2\ \ \mu(u_2) = 3$
$\mu(u_3) = 3\ \ \mu(u_4) = 2\ \mu(u_5) = 1\ \ \mu(u_6) = 1$

Here level set $\lambda = \{ 1, 2, 3, 4, 5, 6, 7 \}$

Chromatic number of L-Fuzzy Graph $G^L = \max \{2,4\} = 4$

Definition 3.2:- *A family* $\Gamma = \{ \gamma_1, \gamma_2, \ldots \gamma_k \}$ *of L Fuzzy set on X is called k-L Fuzzy colouring of* $G = (X, \sigma, \mu)$ *if*
(a) $\vee\ \Gamma = \sigma$
(b) $\gamma_i \wedge \gamma_j = 0\ (1 \leq i,j \leq k)$
(c) For every strong edge of xy of G, $\min \{ \gamma_i(x), \gamma_i(y) \} = 0$ $(1 \leq i \leq k)$

The least value of k for which G has a k-L Fuzzy colouring, denoted by $X^{f_L}(G)$ is called the L- Fuzzy chromatic number of G.

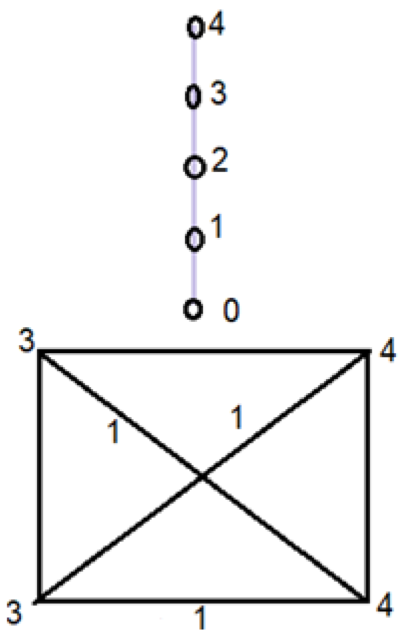
Remark 3.3 :- Consider L-Fuzzy Graph $G^L = (V, \sigma, \mu)$ Let $G^\alpha = (\sigma_\alpha, \mu_\alpha)$ $\alpha \epsilon \lambda$, if $\alpha_i \leq \alpha_j$ then $X(G(\alpha_i)) \geq X(G(\alpha_j))$
Illustration

From fig(a) put $\alpha_1=1$, $\alpha_2=2$ $X(G_1)=4$ $X(G_2)=2$ $X(G_1) \geq X(G_2)$
$\alpha_2=2$, $\alpha_3=3$ $X(G_2)=4$ $X(G_3)=2$ $X(G_2) \geq X(G_3)$

Definition 3.4 *[18] :- An L-Fuzzy Graph* $G^L = (V, \sigma, \mu)$ *is said to be complete if it satisfies the condition* $\mu(u_i, u_j) = \sigma(u_i) \wedge \sigma(u_j)\ \forall\ (u_i, u_j) \epsilon\ V \times V$
From the above definition we derive the following result

Result 3.5 :- Let $G^L = (V, \sigma, \mu)$ be L-Fuzzy Graph with any $\alpha_i \epsilon \lambda$ being comparable
then

If G_L if complete then $X(G^L) = I\ V\ I$
Illustration



Consider the lattice and L-Fuzzy graph

The chromatic number of given L-Fuzzy Graph is 4.

Theorem 3.6:-If G_1^L is isomorphic to G_2^L then chromatic number of G_1^L is equal to chromatic number of G_2^L

Proof:- Assume G_1^L is isomorphic to G_2^L

Then, $X(G_1^L) = \max\{X(G_1)/\alpha \text{ in } \lambda\}$
$= X(G1\alpha)$ for some α
$= X(G2\alpha)$ [there exist corresponding G_2 in G_2^L
$= X(G_2^L)$

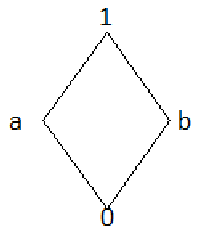
4. Weak Complement of L-Fuzzy graph

Anjaly Kishore and Sunitha [17] tried to compare Chromatic number of fuzzy graph and Chromatic number of complement of fuzzy graph. Here we try to study the chromatic number of weak complement of L-Fuzzy graph.
We define the weak complement of L-Fuzzy Graph as follows,

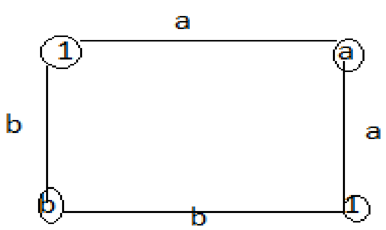
Definition 4.1:- *Let L is a complemented lattice. The weak complement of L-Fuzzy graph* $G^L = (V, \sigma, \mu)$ *is* $\overline{G}:(V, \overline{\sigma}, \overline{\mu})$ *with* $\overline{\sigma}(u) = \sigma(u)$ *and* $\overline{\mu}(u,v)$ *is the complement of* $\mu(u,v)$.

Result 4.2:-Weak Complement of an L-Fuzzy Graph need not be an L-Fuzzy Graph
Counter example

Consider lattice

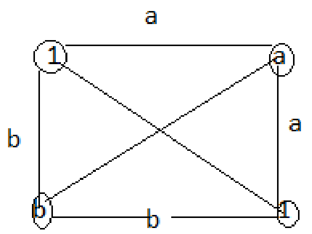


and L-Fuzzy Graph



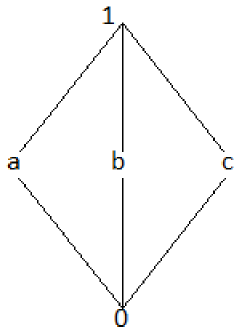
G^L

Weak Complement of $G^L(V, \sigma, \mu)$ is $\overline{G}^L:(V, \overline{\sigma}, \overline{\mu})$

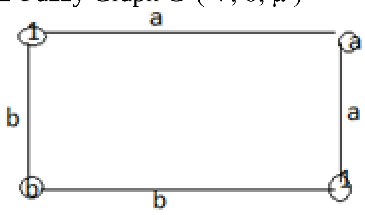


$\overline{G}^L$

Result 4.4: Weak Complement of an L-Fuzzy Graph need not be unique
e.g. Consider lattice

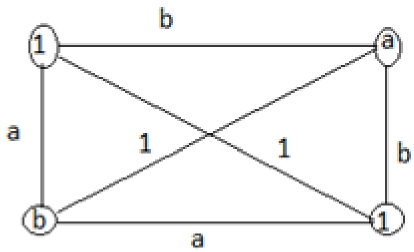


and consider L-Fuzzy Graph $G^L(V, \sigma, \mu)$

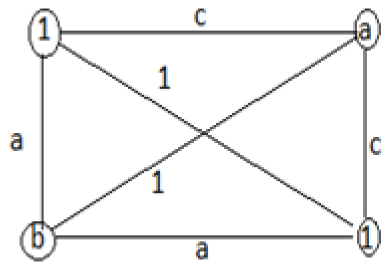


G^L

Weak Complements of $G^L(V, \sigma, \mu)$ are $G_1^L(V, \sigma, \mu)$ and $G_2^L(V, \sigma, \mu)$



G_2^L



G_1^L

5. Conclusion

The concept of chromatic number of L-Fuzzy Graph was studied . Further, the weak complement of L-Fuzzy Graph was defined and it was proved that the weak complement of an L-Fuzzy Graph need not be an L-Fuzzy Graph. It was also found out that the weak complement of L-Fuzzy Graph need not be unique. We find that there is a scope to modify the definition of weak complement of L-Fuzzy graph so that it satisfies closure property and uniqueness.

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