

A New Quadratic Formulation to Ensure Maximum Profit of a Textile Industry and a Modified Harmonic Average Technique to Solve Multi Objective Quadratic Programming Problem (MOQPP)

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Abstract: This paper presents a survey of QPP in corporate and economic planning formulation. The research aims to find maximum profits diagnostics of a Textile industry by using QPP. Here a MOQPP has been formulated from real industrial data to change the problem into mathematical formula then solved it by AMPL software. Chandra sen's technique, Statistical (arithmetic, geometric, harmonic) Average technique and modified statistical average technique are used to solve multi objective quadratic programming problem (MOQPP) converting it into single objective QPP then solving by AMPL software. After comparing, the results show that the modified harmonic average technique is best than others. The conclusion will be focus on the maximum profit of the industry which ensures by using Modified harmonic average technique.

Keywords: Quadratic formulation, Multi objective quadratic programming problem, AMPL software, Statistical Average technique, Modified statistical average technique

1. Introduction

The fundamentals and applications of linear programming have been already explored in great detail. In real life problems, many different situations lead to nonlinear formulation of constraints and objective functions. The importance of nonlinear programming application is growing due to rapidly increasing sophistications of managers and operation researchers in implementing decision oriented mathematical models. Among all nonlinear programming problems, the quadratic formulation is the easiest to solve when one deals with a QP involving linear inequality constraints. In many situations, decision makers want to optimize several different objective functions at the same time under same constraints. This leads to Multi Objective concept. The MOQPP has been greatly received by user to solve different types of real life problems.

Converting multi objective LPP into single objective LPP, a study was done by Chandra Sen [1]. AMPL software is developed to solve optimization problems [2]. Sulaiman [3] described the computational aspects of single objective in definite QPP. Mohammad and Sulaiman [4] studied the MOF complementary program. Sulaiman and Sadiq [5] presented the multi objective function by solving MOLPP using mean and median, named as Sadiq's approach. Using arithmetic average value, Hamad -Amin [6] studied the multi objective function by solving MOLPP. Sulaiman & Abdul Rahim [7] used the optimal cutting plane procedure to solve MOQPP. Nahar and Alim [8] studied new statistical average method to solve MOLPP. Arithmetic average transformation technique is applied by Sulaiman & Abulrahim [9] to solve MOQPP. Using optimal geometric average technique Sulaiman et al. [10] solved extreme point

MOQPP. LPP and QPP also can be solved by using AMPL software [13].

In this paper, I have extended the work and a new Modified Harmonic Average technique is applied to solve MOQPP which gives better optimal solutions than others.

MOQPP has attracted the interest of many researchers due to its application in many important fields such as production planning, financial and corporate planning, portfolio selection, monopolist's profit maximization, inequality constrained least squares estimation, spatial equilibrium analysis, goal programming, optimal decision rules etc.

Jae K.Shim [11] studied a survey of Quadratic Programming applications to business and economics. A quadratic programming approach to the problem of the optimal use of milk was introduced by Louwes, Boot & Wage [12].

In our present study, we have formulated a new Multi Objective Quadratic Programming Formulation. This aims to find the maximum profit of a textile industry for each product by using MOQPP formulation. It depends on formulating technique to change the problem into Quadratic problem. The results may have some influences on future production decision and it illustrates an application of quadratic programming since the profit to be maximized may be written as quadratic function of product quantities and prices. A full description of MOQPP and the solution procedures of MOQPP such as AMPL algorithm, Chandra Sen's technique, Statistical average technique, Modified statistical average technique are discussed. We have used some numerical examples and presented the detail of practical problem that is taken from list of information of

case study (Textile industry). Finally the results and discussion are given that are followed by our conclusion.

2. Mathematical formulation of MOQPP

Multi-objective quadratic programming problem (MOQPP) involves a decision-making unit (DMU) with multiple objective functions. The objective functions of the DMU are quadratic in nature and the constraints are linear function. Mathematically the multi objective quadratic programming problem (MOQPP) can be stated as:

$$\begin{aligned}
 \text{Max } F_1 &= \frac{1}{2} x^T P_1 x + C_1^T x \\
 \text{Max } F_2 &= \frac{1}{2} x^T P_2 x + C_2^T x \\
 &\dots \\
 \text{Max } F_r &= \frac{1}{2} x^T P_r x + C_r^T x \\
 \text{Max } F_{r+1} &= \frac{1}{2} x^T P_{r+1} x + C_{r+1}^T x \\
 \text{Min } F_{r+2} &= \frac{1}{2} x^T P_{r+2} x + C_{r+2}^T x \\
 &\dots \\
 &\dots \\
 \text{Min } F_s &= \frac{1}{2} x^T P_s x + C_s^T x \\
 &\text{subject to} \\
 Ax &\begin{cases} \leq \\ > \end{cases} b \\
 x &\geq 0
 \end{aligned} \tag{1}$$

Where r is the number of objective function to be maximized, s is the number of objective function to be maximized and minimized and (s-r) is the number of objective function to be minimized. Here P is a (n × n) symmetric matrix of coefficients, x is an n-dimensional vector of decision variables, C is the n-dimensional vector of constants, b is m-dimensional vector of constants. A is (m × n) matrix of coefficients. All vectors are assumed to be column vectors unless transposed.

3. Solution Procedure for MOQPP

A Mathematical Programming Language (AMPL) is an algebraic modeling language to describe and solve high-complexity problems for large-scale mathematical computing such as large-scale optimization problems. AMPL can be used to solve QPP.

At first Chandra Sen presented the Multi objective programming (MOP) method to solve Multi objective optimization problems. Different types of statistical average (mean) techniques can give better result than Sen's approach. These all techniques are briefly described below.

3.1 AMPL- A Mathematical Programming Language

Practical large-scale mathematical programming involves more than just the application of an algorithm to minimize or maximize an objective function subject to constraints equations and inequalities. AMPL is a new language

designed to solve those problems easily and with less error-prone.

Algorithm for AMPL:

Step 1: Once the file is created and we have checked the file name, run the AMPL program by selecting it from appropriate start menu. After some text appears, we will see the AMPL prompt, which is

ampl:

Step 2: We have to select a solver, which is the program that AMPL calls to solve the math program. We will be using the solver called cplex, So type:

option solver cplex;

Step 3: Type in the following command, which tells AMPL to read in the file we have created:

model problem.mod;

Step 4: If we type in the model correctly, we will get no error messages. If we made a mistake, correct the model file and first reset AMPL by typing:

reset;

To reload the model, type in: **model problem.mod;**

Step 5: For data file do the same process and type:

data problem.dat;

Step 6: The files are successfully loaded, tell AMPL to run the model by typing:

solve;

Once AMPL has solved the model, it displays the results.

To convert MOQPP into Single objective QPP, Procedures are:

3.2 Chandra Sen's Technique

In Sen's technique, all objective functions need to be maximized or minimized firstly by Simplex method. Each objective function of equation (1) are solved and following equations are obtained:

$$\begin{aligned}
 \text{Max } Z_1 &= \varphi_1 \\
 \text{Max } Z_2 &= \varphi_2 \\
 &\dots \\
 &\dots \\
 \text{Max } Z_r &= \varphi_r \\
 \text{Max } Z_{r+1} &= \varphi_{r+1} \\
 \text{Max } Z_{r+2} &= \varphi_{r+2} \\
 &\dots \\
 &\dots \\
 \text{Max } Z_s &= \varphi_s
 \end{aligned} \tag{3}$$

Where $\varphi_1, \varphi_2, \dots, \varphi_s$ are optimal values of the objective functions.

By Chandra Sen's technique, these values are used to form a single objective function by adding (for maximization) and subtracting (for minimization). Mathematically,

$$\text{Max } Z = \sum_{i=1}^r \frac{z_i}{|\varphi_i|} - \sum_{i=r+1}^s \frac{z_i}{|\varphi_i|} \quad \text{where } |\varphi_i| \neq 0$$

Subject to constraints of equation (1) remains same. The optimum value φ_i may be positive or negative.

3.3 Statistical Average Techniques

All the objectives are optimized individually. The mean, median and optimal averages of optimal values for maximization and minimization objectives are estimated separately. The combined objective function is constructed

by weighting the objective functions by the inverse of the mean, median and optimal average as given below:

$$\text{Max } Z = \sum_{i=1}^r \frac{Z_i}{\text{mean, median, optimal average}} - \sum_{i=r+1}^s \frac{Z_i}{\text{mean, median, optimal average}}$$

3.3.1 Different kinds of mean technique:

**Arithmetic Mean technique:*

$$\text{Max } Z = \sum_{i=1}^r \frac{Z_i}{A.M_1} - \sum_{i=r+1}^s \frac{Z_i}{A.M_2}$$

Where $A.M_1 = A.M(|\varphi_1|, |\varphi_2|, \dots, |\varphi_r|)$ & $A.M_2 = A.M(|\varphi_{r+1}|, |\varphi_{r+2}|, \dots, |\varphi_s|)$ and A.M is arithmetic mean.

The arithmetic mean of a set of observed data is defined as being equal to the sum of the numerical values of each and every observation divided by the total number of observations.

**Geometric Mean technique:*

$$\text{Max } Z = \sum_{i=1}^r \frac{Z_i}{G.M_1} - \sum_{i=r+1}^s \frac{Z_i}{G.M_2}$$

Where $G.M_1 = G.M(|\varphi_1|, |\varphi_2|, \dots, |\varphi_r|)$ & $G.M_2 = G.M(|\varphi_{r+1}|, |\varphi_{r+2}|, \dots, |\varphi_s|)$ and G.M is geometric mean.

The geometric mean is defined as the *n*th root of the product of *n* numbers.

**Harmonic Mean technique:*

$$\text{Max } Z = \sum_{i=1}^r \frac{Z_i}{H.M_1} - \sum_{i=r+1}^s \frac{Z_i}{H.M_2}$$

Where $H.M_1 = H.M(|\varphi_1|, |\varphi_2|, \dots, |\varphi_r|)$ & $H.M_2 = H.M(|\varphi_{r+1}|, |\varphi_{r+2}|, \dots, |\varphi_s|)$ and H.M is harmonic mean. The harmonic mean can be expressed as the reciprocal of the arithmetic mean of the reciprocals of the given set of observations.

3.4 Modified Statistical Average Techniques

**Modified Arithmetic Mean technique:*

$$\text{Max } Z = \frac{\sum_{i=1}^r Z_i - \sum_{i=r+1}^s Z_i}{A.M.AV}$$

where $A.M.AV = (m_1 + m_2)/2$ and $m_1 = \min(|\varphi_1|, |\varphi_2|, \dots, |\varphi_r|)$ $m_2 = \min(|\varphi_{r+1}|, |\varphi_{r+2}|, \dots, |\varphi_s|)$

**Modified Geometric Mean technique:*

$$\text{Max } Z = \frac{\sum_{i=1}^r Z_i - \sum_{i=r+1}^s Z_i}{G.M.AV}$$

Where $G.M.AV = \sqrt{m_1 m_2}$ and $m_1 = \min(|\varphi_1|, |\varphi_2|, \dots, |\varphi_r|)$ $m_2 = \min(|\varphi_{r+1}|, |\varphi_{r+2}|, \dots, |\varphi_s|)$

**Modified Harmonic Mean technique:*

$$\text{Max } Z = \frac{\sum_{i=1}^r Z_i - \sum_{i=r+1}^s Z_i}{H.M.AV}$$

Where $H.M.AV = \frac{2}{\frac{1}{m_1} + \frac{1}{m_2}}$ and $m_1 = \min(|\varphi_1|, |\varphi_2|, \dots, |\varphi_r|)$ $m_2 = \min(|\varphi_{r+1}|, |\varphi_{r+2}|, \dots, |\varphi_s|)$

4. Numerical Examples

We use some numerical examples to solve MOQPP. Example (1): Solve the following MOQPP by using Chandra Sen's Technique

$$\begin{aligned} \text{Max } Z_1 &= 4x_1 + 2x_2 - x_1^2 - x_2^2 + 5 \\ \text{Max } Z_2 &= 2x_1 + x_2 - x_1^2 \\ \text{Min } Z_3 &= 6 - 6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2 \\ \text{Min } Z_4 &= 2x_1 + 3x_2 - 2x_1^2 \\ \text{Subject to } & \quad x_1 + 4x_2 \leq 9 \\ & \quad x_1 + x_2 \leq 3 \\ & \quad 3x_1 + 2x_2 \leq 8 \\ & \quad x_1, x_2 \geq 0 \end{aligned}$$

After finding the value of each of individual objective functions of example (1) by using AMPL software, the numerical results are given below in table 1:

Table 1

<i>i</i>	<i>Z_i</i>	<i>x_i</i>	<i>φ_i</i>	<i>OA_i</i>	<i>OL_i</i>
1	10	(2,1)	10	10	
2	3.0156	(.875, 2.0313)	3.0156	3.0156	
3	15	(0,2.25)	15		15
4	6.9453	(0.3125, 2.1719)	6.9453		6.9453

By Chandra Sen's Approach,

$$\begin{aligned} \text{Max } Z &= \sum_{k=1}^r \frac{Z_k}{|\varphi_k|} - \sum_{k=r+1}^s \frac{Z_k}{|\varphi_k|} \\ \text{Max } Z &= \frac{Z_1}{\varphi_1} + \frac{Z_2}{\varphi_2} - \frac{Z_3}{\varphi_3} - \frac{Z_4}{\varphi_4} \\ &= \frac{4x_1 + 2x_2 - x_1^2 - x_2^2 + 5}{10} \\ &+ \frac{2x_1 + x_2 - x_1^2}{3.0156} \\ &- \frac{6 - 6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2}{15} \\ &- \frac{2x_1 + 3x_2 - 2x_1^2}{6.9453} \\ &= 1.1734x_1 + 0.0968x_2 - 0.2751x_1^2 - 0.2333x_2^2 \\ &+ 0.1333x_1x_2 + 0.1 \end{aligned}$$

Using Chandra Sen's Approach, the system becomes,

$$\text{Max } Z = 1.1734x_1 + 0.0968x_2 - 0.2751x_1^2 - 0.2333x_2^2 + 0.1333x_1x_2 + 0.1$$

$$\begin{aligned} \text{Subject to } & \quad x_1 + 4x_2 \leq 9 \\ & \quad x_1 + x_2 \leq 3 \\ & \quad 3x_1 + 2x_2 \leq 8 \\ & \quad x_1, x_2 \geq 0 \end{aligned}$$

Using AMPL software solving this, we get,

$$Z_{max} = 1.50957 \quad \text{with } x_1 = 2.18077 \text{ \& } x_2 = 0.728841$$

Using Arithmetic Average Approach,

$$\begin{aligned} A.M(10, 3.0157) &= 6.5, \quad A.M(15, 6.9) = 10.95 \\ \text{Now } \text{Max } Z &= \frac{1}{6.5}(Z_1 + Z_2) - \frac{1}{10.95}(Z_3 + Z_4) \\ &= 1.288x_1 + 0.1876x_2 - 0.3077x_1^2 - 0.6105x_2^2 + 0.1826x_1x_2 \\ &+ 2213 \end{aligned}$$

Using Arithmetic Average Approach, the system becomes,

$$\text{Max } Z = 1.288x_1 + 0.1876x_2 - 0.3077x_1^2 - 0.6105x_2^2 + 0.1826x_1x_2 + 2213$$

$$\text{Subject to } \begin{aligned} x_1 + 4x_2 &\leq 9 \\ x_1 + x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 8 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Using AMPL software solving this, we get,

$$Z_{\text{max}} = 1.70917 \text{ with } x_1 = 2.23852 \text{ \& } x_2 = 0.488414$$

Using Geometric Average Approach,

$$G.M(10,3.0156) = \sqrt{10 \times 3.0156} = 5.4914$$

$$G.M(15,6.9) = \sqrt{15 \times 6.9} = 10.1735$$

$$\text{Now Max } Z = \frac{1}{5.4914}(Z_1 + Z_2) - \frac{1}{10.1735}(Z_3 + Z_4)$$

$$= 1.4858x_1 + 0.2514x_2 - 0.3642x_1^2 - 0.6735x_2^2 + 0.1966x_1x_2 + 0.3207$$

Using Geometric Average Approach, the system becomes,

$$\text{Max } Z = 1.4858x_1 + 0.2514x_2 - 0.3642x_1^2 - 0.6735x_2^2 + 0.1966x_1x_2 + 0.3207$$

$$\text{Subject to } \begin{aligned} x_1 + 4x_2 &\leq 9 \\ x_1 + x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 8 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Solving this, we get,

$$Z_{\text{max}} = 1.99275 \text{ with } x_1 = 2.03927 \text{ \& } x_2 = 0.441095$$

Using Harmonic Average Approach,

$$H.M(10,3.0156) = \frac{2}{\frac{1}{10} + \frac{1}{3.0156}} = 4.6338$$

$$H.M(15,6.9) = \frac{2}{\frac{1}{15} + \frac{1}{6.9}} = 9.4521$$

$$\text{Max } Z = \frac{1}{4.6338}(Z_1 + Z_2) - \frac{1}{9.4521}(Z_3 + Z_4)$$

$$= 1.718x_1 + 0.33x_2 - 0.4316x_1^2 - 0.7448x_2^2 + 0.2116x_1x_2 + 0.4442$$

Using Harmonic Average Approach, the system becomes,

$$\text{Max } Z = 1.718x_1 + 0.33x_2 - 0.4316x_1^2 - 0.7448x_2^2 + 0.2116x_1x_2 + 0.4442$$

$$\text{Subject to } \begin{aligned} x_1 + 4x_2 &\leq 9 \\ x_1 + x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 8 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Solving this, we get,

$$Z_{\text{max}} = 2.35006 \text{ with } x_1 = 2.11834 \text{ \& } x_2 = 0.522449$$

Using Modified Arithmetic Average Approach,

$$\text{Let } m_1 = 3.0156 \text{ \& } m_2 = 6.9$$

$$\text{Then } A.AV = \frac{m_1+m_2}{2} = 4.9578$$

$$\text{Max } Z = \frac{1}{A.AV}(Z_1 + Z_2 - Z_3 - Z_4)$$

$$= 2.017x_1 - 0.4034x_1^2 - 1.2102x_2^2 + 0.4034x_1x_2 - 0.2017$$

Thus the QPP becomes,

$$\text{Max } Z = 2.017x_1 - 0.4034x_1^2 - 1.2102x_2^2 + 0.4034x_1x_2 - 0.2017$$

$$\text{Subject to } \begin{aligned} x_1 + 4x_2 &\leq 9 \\ x_1 + x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 8 \end{aligned}$$

$$\text{Solving this, we get,}$$

$$Z_{\text{max}} = 2.51307 \text{ with } x_1 = 2.43243 \text{ \& } x_2 = 0.351351$$

Using Modified Geometric Average Approach,

$$G.AV = \sqrt{m_1m_2} = \sqrt{3.0156 \times 6.9} = 4.5615$$

$$\text{Max } Z = \frac{1}{G.AV}(Z_1 + Z_2 - Z_3 - Z_4)$$

$$= 2.1923x_1 - 0.4385x_1^2 - 1.3154x_2^2 + 0.4358x_1x_2 - 0.2192$$

Thus the QPP becomes,

$$\text{Max } Z = 2.1923x_1 - 0.4385x_1^2 - 1.3154x_2^2 + 0.4358x_1x_2 - 0.2192$$

$$\text{Subject to } \begin{aligned} x_1 + 4x_2 &\leq 9 \\ x_1 + x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 8 \end{aligned}$$

$$\text{Solving, we get,}$$

$$Z_{\text{max}} = 2.73131 \text{ with } x_1 = 2.43239 \text{ \& } x_2 = 0.351408$$

Using Modified Harmonic Average Approach,

$$H.AV = \frac{2}{\frac{1}{m_1} + \frac{1}{m_2}} = \frac{2}{\frac{1}{3.0156} + \frac{1}{6.9}} = 4.1969$$

$$\text{Max } Z = \frac{1}{H.AV}(Z_1 + Z_2 - Z_3 - Z_4)$$

$$= 2.3827x_1 - 0.4765x_1^2 - 1.4296x_2^2 + 0.4765x_1x_2 - 0.2383$$

Thus the QPP becomes,

$$\text{Max } Z = 2.3827x_1 - 0.4765x_1^2 - 1.4296x_2^2 + 0.4765x_1x_2 - 0.2383$$

$$\text{Subject to } \begin{aligned} x_1 + 4x_2 &\leq 9 \\ x_1 + x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 8 \end{aligned}$$

$$\text{Solving this, we get,}$$

$$Z_{\text{max}} = 2.85616 \text{ with } x_1 = 2.16219 \text{ \& } x_2 = 0.25671$$

Table 2 summarizes the solutions of the MOQPP using different approaches:

Comparison Table

Chandra Sen's Approach	Using Arithmetic Average Technique	Using Geometric Average Technique	Using Harmonic Average Technique	Using Modified Arithmetic Average Technique	Using Modified Geometric Average Technique	Using Modified Harmonic Average Technique
Z = 1.5	Z = 1.7	Z = 1.99	Z = 2.35	Z = 2.51	Z = 2.73	Z = 2.85
x ₁ = 2.18	x ₁ = 2.24	x ₁ = 2.04	x ₁ = 2.11	x ₁ = 2.43	x ₁ = 2.43	x ₁ = 2.16
x ₂ = 0.73	x ₂ = 0.49	x ₂ = 0.44	x ₂ = 0.52	x ₂ = 0.35	x ₂ = 0.35	x ₂ = 0.26

It shows that the solution of the objective functions improved when we used the proposed modified techniques. Modified Harmonic average technique gives better solutions than others.

5. Practical Aspects (Formulations)

Example: A textile company produces different types of products. Such as- Tshirt, Ladies tops, Jeans etc. They use several raw materials like cotton, silk, wool, polyester, viscous etc.

In Bangladesh, there are many textile industries. Consider data from one of the well known textile industry DBL group. DBL group has several buyers to purchase their different kinds of products. One of their buyer purchases only Polo Tshirt and Ladies tops. To provide that two products DBL group needs raw materials, mainly cotton, polyester and viscous.

According to their monthly order from that one buyer they need 50 tons cotton, 7 ton polyester and 10 tons viscous as raw material. These materials are first transformed to yarn and then after processing these are turned into finished fabric. After all these steps: 50 ton cotton has converted into 16 ton finished fabric, 7 ton polyester into 5 ton finished fabric and 10 ton viscous into 7 ton finished fabric.

According to their respective 3 months data:

Month 1

<u>Polo Tshirt</u>	<u>Ladies Tops</u>
Quantity- 70,000 pcs	Quantity- 50,000 pcs
Profit per piece- 1/-	Profit per piece- 2/-

Month 2

<u>Polo Tshirt</u>	<u>Ladies Tops</u>
Quantity- 68,000 pcs	Quantity- 55,000 pcs
Profit per piece- 1/-	Profit per piece- 4/-

Month 3

<u>Polo Tshirt</u>	<u>Ladies Tops</u>
Quantity- 71,000 pcs	Quantity- 53,000 pcs
Profit per piece- 2.5/-	Profit per piece- 3/-

The amount of raw materials to produce per piece Polo Tshirt and Ladies tops are given below:

	Cotton (in ton) (multiplied by 1000)	Polyester (in ton) (multiplied by 1000)	Viscous (in ton) (multiplied by 1000)
Polo Tshirt (per pc)	0.29	0.29	0.22
Ladies tops(per pc)	0.2	0.2	0.23

Formulate the quadratic programming model to find the maximum profit for each product and also find the number of products.

Formulation:

Let the quantity of Polo Tshirt be x_1
 And the quantity of Polo Ladies tops be x_2 .
 Also, let for maximum profit;
 The respective profit per piece of Polo Tshirt = P_1
 The respective profit per piece of Ladies tops = P_2

The demand functions are based on the three sets of data.

- (i) The actual quantities of product, say \bar{x}_1, \bar{x}_2 .
- (ii) Their average profit per piece say \bar{P}_1, \bar{P}_2 .
- (iii) Price elasticity say ϵ .

These values have been based on existing studies of consumer's behavior. The functions are assumed to be linear. For large positive deviation from the equilibrium prices, this will probably imply an underestimation of demand and hence revenue.

$$\frac{x_1 - \bar{x}_1}{\bar{x}_1} = -\epsilon \frac{P_1 - \bar{P}_1}{\bar{P}_1}$$

$$\frac{x_2 - \bar{x}_2}{\bar{x}_2} = -\epsilon \frac{P_2 - \bar{P}_2}{\bar{P}_2} \quad (5.1)$$

For the price elasticity, let the value is specified as follows:
 $\epsilon = 0.3$

For month 1:

Given $\bar{x}_1 = 70000, \bar{P}_1 = 1$ and $\bar{x}_2 = 50000, \bar{P}_2 = 2$
 Consider as, $\bar{x}_1 = 70, \bar{P}_1 = .001$ and $\bar{x}_2 = 50, \bar{P}_2 = .002$

Using (5.1)

$$\frac{x_1 - 70}{70} = -3 \times 10^{-4} \times \frac{P_1 - .001}{.001}$$

$$\Rightarrow P_1 = 3.5 - 0.5 x_1 \quad (5.2)$$

Also, $\frac{x_2 - 50}{50} = -3 \times 10^{-4} \times \frac{P_2 - .002}{.002}$

$$\Rightarrow P_2 = 6.5 - 0.13 x_2 \quad (5.3)$$

Now the function to be maximized is $\sum P_i x_i$.

Thus the objective function for this case is,

$$\text{Max } Z_1 = P_1 x_1 + P_2 x_2$$

$$= -0.5x_1^2 - 0.13x_2^2 + 3.5x_1 + 6.5x_2 \quad (5.4)$$

For month 2:

Given $\bar{x}_1 = 68000, \bar{P}_1 = 2$ and $\bar{x}_2 = 55000, \bar{P}_2 = 4$
 Consider as, $\bar{x}_1 = 68, \bar{P}_1 = .002$ and $\bar{x}_2 = 55, \bar{P}_2 = .004$

Using (5.1)

$$\frac{x_1 - 68}{68} = -3 \times 10^{-4} \times \frac{P_1 - .002}{.002}$$

$$\Rightarrow P_1 = 6.12 - 0.09 x_1 \quad (5.5)$$

Also, $\frac{x_2 - 55}{55} = -3 \times 10^{-4} \times \frac{P_2 - .004}{.004}$

$$\Rightarrow P_2 = 13.204 - 0.24 x_2 \quad (5.6)$$

Now the function to be maximized is $\sum P_i x_i$.

Thus the objective function for this case is,

$$\text{Max } Z_2 = P_1 x_1 + P_2 x_2$$

$$= -0.09x_1^2 - 0.24x_2^2 + 6.12x_1 + 13.204x_2 \quad (5.7)$$

For month 3:

Given $\bar{x}_1 = 71000, \bar{P}_1 = 2.5$ and $\bar{x}_2 = 53000, \bar{P}_2 = 3$
 Consider as, $\bar{x}_1 = 71, \bar{P}_1 = .0025$ and $\bar{x}_2 = 53, \bar{P}_2 = .003$

Using (5.1)

$$\frac{x_1 - 71}{71} = -3 \times 10^{-4} \times \frac{P_1 - .0025}{.0025}$$

$$\Rightarrow P_1 = 8.5 - 0.12 x_1 \quad (5.8)$$

Also, $\frac{x_2 - 53}{53} = -3 \times 10^{-4} \times \frac{P_2 - .003}{.003}$

$$\Rightarrow P_2 = 10.073 - 0.19 x_2 \quad (5.9)$$

Now the function to be maximized is $\sum P_i x_i$.

Thus the objective function for this case is,

$$\begin{aligned} \text{Max } Z_3 &= P_1 x_1 + P_2 x_2 \\ &= -0.12x_1^2 - 0.19x_2^2 + 8.5x_1 + 10.73x_2 \quad (5.10) \end{aligned}$$

Finally using (5.4), (5.7) and (5.10) we get MOQPP with some constraints:

$$\begin{aligned} \text{Max } Z_1 &= -0.5x_1^2 - 0.13x_2^2 + 3.5x_1 + 6.5x_2 \\ \text{Max } Z_2 &= -0.09x_1^2 - 0.24x_2^2 + 6.12x_1 + 13.204x_2 \quad (5.11) \\ \text{Max } Z_3 &= -0.12x_1^2 - 0.19x_2^2 + 8.5x_1 + 10.73x_2 \end{aligned}$$

$$\begin{aligned} \text{Subject to } &.29x_1 + 0.2x_2 \leq 50000 \\ &.29x_1 + 0.2x_2 \leq 7000 \\ &.22x_1 + 0.23x_2 \leq 10000 \\ &x_1, x_2 \geq 0 \end{aligned}$$

After solving by AMPL software, we obtain the result as:

i	x_i	Z_i	φ_i
1	(4, 25)	87.38	87.38
2	(34, 28)	285.54	285.54
3	(35, 28.2)	302.01	302.01

Using Chandra Sen's Technique, to convert the multi objective function to a single objective function is,

$$\begin{aligned} \text{Max } Z &= \frac{Z_1}{\varphi_1} + \frac{Z_2}{\varphi_2} + \frac{Z_3}{\varphi_3} \\ &= \frac{1}{87.38} [-0.5x_1^2 - 0.13x_2^2 + 3.5x_1 + 6.5x_2] + \\ &\frac{1}{285.54} [-0.09x_1^2 - 0.24x_2^2 + 6.12x_1 + 13.204x_2] + \\ &\frac{1}{302.01} [-0.12x_1^2 - 0.19x_2^2 + 8.5x_1 + 10.73x_2] \\ &= -0.00643 x_1^2 - 0.00296 x_2^2 + 0.089 x_1 + 0.156 x_2 \end{aligned}$$

The system becomes,

$$\text{Max } Z = -0.00643x_1^2 - 0.00296 x_2^2 + 0.089 x_1 + 0.156 x_2$$

$$\begin{aligned} \text{Subject to } &.29x_1 + 0.2x_2 \leq 50000 \\ &.29x_1 + 0.2x_2 \leq 7000 \\ &.22x_1 + 0.23x_2 \leq 10000 \\ &x_1, x_2 \geq 0 \end{aligned}$$

After solving we get,

$$Z = 2.36 \text{ with } x_1 = 6.9, x_2 = 26.3$$

Using modified harmonic average technique,

$$H.AV = \frac{2}{\frac{1}{m_1} + \frac{1}{m_2}}$$

Here, $m_1 = 302.01$ and $m_2 = 87.38$

Then $H.AV = 135.54$

$$\begin{aligned} \text{Thus, Max } Z &= \frac{1}{H.AV} [Z_1 + Z_2 + Z_3] \\ &= -0.00524 x_1^2 - 0.00413 x_2^2 + 0.134 x_1 + 0.225 x_2 \end{aligned}$$

The system becomes,

$$\text{Max } Z = -0.00524x_1^2 - 0.00413 x_2^2 + 0.134x_1 + 0.225 x_2$$

$$\begin{aligned} \text{Subject to } &.29x_1 + 0.2x_2 \leq 50000 \\ &.29x_1 + 0.2x_2 \leq 7000 \\ &.22x_1 + 0.23x_2 \leq 10000 \end{aligned}$$

$$x_1, x_2 \geq 0$$

After solving we get,

$$Z = 3.9 \text{ with } x_1 = 12.78, x_2 = 27.24$$

Table 3 summarizes the solutions of the MOQPP:

(Since x_i presents quantities, consider integer values)

Chandra Sen's Approach	Using Modified Harmonic Average Technique
$Z = 2.36$	$Z = 3.9$
$x_1 = 7, x_2 = 26$	$x_1 = 13, x_2 = 27$

Which shows that the maximum profit can be ensured by using modified harmonic average technique.

6. Result and Discussion

A survey has been presented on the use of quadratic programming in solving business planning problem. It has been seen that the optimization result using modified harmonic average technique is better than that of other techniques. The quadratic programming formulations are more realistic than is linear programming. The optimization model discussed here is more consistent with the decision maker's preference postulates. For maximum profit (per piece product), the ratio of quantities of products can be fixed with the help of modified harmonic average technique.

7. Conclusion

In this paper we have defined and discussed a number of techniques which we have used in order to get optimal solution of the MOQPP. The comparisons of these techniques are based on the values of the objective functions. It is observed that the statistical average techniques result better optimization than Chandra Sen's approach. To solve MOQPP we have proposed a modified statistical average technique. From the analysis, it has been found that MOQPP are best optimized using modified harmonic average technique. Also we have formulated a new quadratic formulation that can be added a new dimension for industrial aspects.

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References

- [1] C. Sen, "A new approach for multi objective rural development planning", *The Indian Economic Journal*, Vol. 30, pp. 91-96, 1983.
- [2] AMPL- A modeling language for mathematical programming; Author: Robert Fourer, David M. Gay, Brian W.Kernighan.
- [3] Sulaiman, N. A., (1989) "Extreme Point Quadratic Programming Problem Techniques", M.Sc. Thesis, University of Salahaddin, Hawler/Iraq.
- [4] Mohammad, M. F. and Sulaiman, N. A., (1992) "Multi-Objective Fractional Complimentary Program", Zanco Journal, University of Salahaddin, Hawler / Iraq, Vol. 4, No. 4.
- [5] Sulaiman, N. A. and Sadiq, G. W., (2006) "Solving the Linear Multi-Objective Programming Problems; Using Mean and Median Value", *Al-Rafiden Journal of computer sciences and mathematics*, University of Mosul, Vol. 3, No. 1, PP. 69-83.

- [6] Hamad-Amin A. O., (2008) “ An Adaptive Arithmetic Average Transformation Technique for Solving MOOPP”, M.Sc. Thesis, University of Koya, koya/Iraq .
- [7] Sulaiman, N. A., and Abdul-Rahim B., K. (2011)“Optimal Cutting Plane Procedure for Multi-Objective Quadratic Programming Problem”, Journal of Koya University, ISSN. 2073-0713,No.20, PP. 119-130.
- [8] S. Nahar and Abdul Alim,(2017) ‘A new statistical average method to solve Multi Objective Linear Programming Problem”, International Journal of Science and Research, volume:6, Issue: 8, ISSN:2319-7064.
- [9] N.A. Sulaiman, Abulrahim,(2013), “ Arithmetic Average Transformation Technique to solve Multi-Objective Quadratic Programming Problem”, Journal of Zankoy Sulaimani –part A,15(1)
- [10] N.A.Sulaiman, R.M. Abdullah, Snur O.Abdull,(2016), “ Using Optimal Geometric Average Technique to solve Extreme Point Multi-Objective Quadratic Programming Problem”, Journal of Zankoy Sulaimani –part A,18(3)
- [11] Jae K.Shim,(2007), “A Survey of Quadratic Programming applications to business and economics”, International Journal of System Science, <http://dx.doi.org/10.1080/00207728308926445>
- [12] S.L.Louwes, J.C.G. Boot and S.Wage,(2015) “A Quadratic Programming Approach to the problem of the optimal use of milk in the Netherlands”, Oxford Journal, <http://ajae.oxfordjournal.org/> at UB Giessen.