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On the Correctness of Error Approximation in Calibration of Instrument Transformer

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Abstract: ***International standards for instrument transformers in interpreting definitions of ratio error and phase displacement allows simplification in vector analysis. This paper is devoted to finding the limits of the correctness of applying approximations when estimating errors of instrument transformers. The dependence of the difference between the obtained results of the error determination is considered on the example of current transformers, both without the use of approximation and with its application. In particular, the case, where the proposed approximation leads to a significant discrepancy between the simulated measurement results, is outlined.***

Keywords: current transformer, ratio error, phase displacement, calibration, phasor

1. Introduction

The energy sector of the economy is an important component of the life of a technically developed state. Ukraine takes not the last place in many technically advanced countries in Europe and is one of almost 40 countries operating nuclear power plants. Developed infrastructure of enterprises with significant production capacity, densely populated territories of Ukraine and other factors have led to the widespread distribution of power supply networks. The huge number of substations and switchgears in cities and villages in the country concentrates hundreds of thousands of measuring instruments of alternating voltage and current, frequency and power consumption. Such devices must provide operating personnel with accurate information about the state of the grid, the amount of electricity consumed, and they are characterized metrologically by measurement range and accuracy. The verified test equipment and working standards with small measurement uncertainty are used to verify and confirm the metrological characteristics.

In Ukraine, about one hundred thousand pieces of current transformers (CT), which perform the function of large-scale reduction of current to a value that is convenient for measurement, are in operation. The CTs with accuracy class 0.5S (in some cases 0.2S) are most commonly used when accounting for electricity consumption [1]. The measuring systems consisting of a high AC source, a laboratory CT (working standard), a device for comparing two secondary currents, a loading device (burden) are used to check compliance with the specified accuracy classes. In the field of metrological support for measurements of electrical quantities, one has to perform periodically the task of metrological characterization (calibration) of a comparator of two alternating currents [2]. Such devices are characterized by the accuracy of measuring the errors of the TCs. Some measurement bridges, such as Tettex 2767 [3], allow comparing two alternating currents with a significant difference (up to two times) which makes it possible to use working standards with fewer primary currents. However, in the countries of the former USSR, in particular, Ukraine, Belarus, Russia, Kazakhstan, the comparators that compare almost identical alternating currents are widespread. A working standard must necessarily have identical primary currents to be able to calibrate CTs or verify its accuracy across the range of possible rated primary currents.

2. Applied Materials

Instrument transformers are characterized by amplitude error (ratio error) and phase error (phase displacement) [4]. Ratio error (RE) ε is the error that an instrument transformer introduces into the measurement and which arises from the fact that the actual transformation ratio is not equal to the rated transformation ratio. According to the definition, the formula for calculating the RE of currents is as follows [5]:

$$\varepsilon_I = 100 \cdot \left(K_T \cdot I_2 - I_1\right) / I_1 \qquad (1)$$

where K_T is the rated transformation ratio; I_1 is the actual primary current; I_2 is the actual secondary current when I_1 is flowing, under the conditions of measurement.

Phase displacement (PD) $\Delta\varphi$ is a difference in phase between the primary voltage or current and the secondary voltage or current phasors, the direction of the phasors being so chosen that the angle is zero for an ideal transformer [4]. According to the definition, the formula can be as follows:

$$\Delta\varphi = \varphi_2 - \varphi_1 \qquad (2)$$

where φ_1 is the actual initial phase of primary current; φ_2 is the actual initial phase of secondary current when I_1 is flowing, under the conditions of measurement.

As noted above, when determining CT errors, two transformers with the same rated primary and secondary currents are often used. As the same current flows through the primary windings of both the device under test (DUT) and the working standard connected in series, the secondary currents will differ depending on the REs and PDs of the two transformers [6]. The errors of the working standard are often equated to zero because its errors are usually much smaller than the errors of the DUT. In this case, the RE of the DUT will be equal to the difference of the amplitudes of the secondary current of the working standard and the secondary current of the DUT. Similarly, the PD of the DUT will be equal to the phase difference of the said secondary currents.

An alternating current, unlike a direct current, is a variable in time and is characterized by amplitude and initial phase. When calibrating CTs, two alternating currents are compared, differing in the initial phase in units or tens of angular minutes. The difference between the amplitude values of the compared currents is often thousandths, hundredths or tenths of a percent for the CTs used in power engineering.

It is often convenient to represent alternating current as a phasor to solve electrical engineering tasks. In this representation, two phasors differing by several hundredths or tenths of a percent are virtually superimposed, since the phase shift between them is measured by minutes. In this case, a graphical representation is convenient with the exclusion of a large portion of the length of the phasors as shown in **Figure 1**.

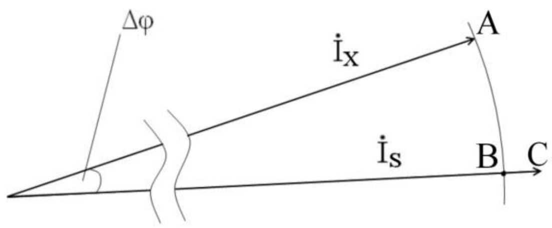

Figure 1: Phasor representation of two almost identical alternating currents

A visual representation of two current phasors having a small phase shift and a small difference in amplitudes can be seen in **Figure 1**. The phasors of both the secondary current of the DUT, the errors of which are needed to be determined, and the secondary current of the working standard are torn by two curves. This approach allows us to depict not the entire length of the phasors and to demonstrate the difference between positions of the end points of these phasors, greatly scaling the length and phase shift. Drawing the arc AB of the circle centered at the point of origin of phasors, we can determine the BC difference in the amplitudes of $\mathbf{I_X}$ and $\mathbf{I_S}$ phasors. At the same time, the phase shift $\Delta\varphi$ determines the difference in the initial phases of the compared alternating currents.

3. Settling Goal

An international standard IEC 61869-2 [1] has the annex A in which an explicative vector diagram is given to explain the determination of the RE. This annex illustrates the error triangle which distinguishes two components of composite error, namely: the in-phase component (ΔI) and the quadrature component (ΔI_q) of exciting current I_e which is a composite error. The foregoing may be presented in **Figure 2**, which is a graphical illustration of the difference between the secondary current and the reduced primary current of the CT.

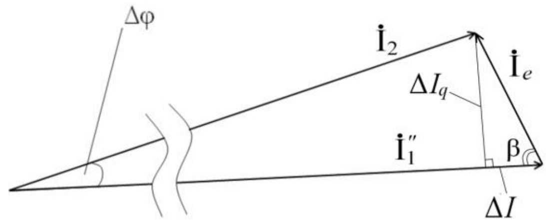

Figure 2: Graphic illustration of vector difference of currents of current transformer

The $\mathbf{I}_1''$ phasor of reduced current represents the primary current multiplied by transformation ratio. The $\mathbf{I}_1''$ and $\mathbf{I_2}$ phasors have lengths many times greater than the length of the $\mathbf{I}_e$ phasor. Therefore, the $\Delta\varphi$ phase shift at several angular minutes allows us to consider the phasors $\mathbf{I}_1''$ and $\mathbf{I_2}$ as parallel. In this case, the in-phase component ΔI will be the difference between the lengths of these phasors, that is, the current RE ε_I.

Assuming that the phase shift between the primary and secondary currents of the TC is too small, it is suggested to use the in-phase component to estimate the RE, and the quadrature component can be used to estimate the PD of the TC. Since the composite error is the hypotenuse of the rectangular error triangle, then the in-phase component is the projection of the phasor of exciting current onto an axis drawn through the reduced primary current phasor. In this case, the equation is correct:

$$\Delta I = I_e \cdot \cos\beta \qquad (3)$$

The quadrature component is a triangle's leg opposite to the phase shift angle between the phasors of the currents of the TC. To determine the quadrature component, the following expression is correct:

$$\Delta I_q = I_e \cdot \sin\beta \qquad (4)$$

Therefore, the PD of such TC can be determined by the expression:

$$\Delta\varphi = \operatorname{atan}\left[I_e \cdot \sin\beta \Big/ \left(I_1'' - \Delta I \right) \right] \qquad (5)$$

The idea of the error triangle is based on the assumption of the parallelism of the phasors of the primary and secondary currents of the TC. As noted above (when determining TC errors by comparing with working standard) it is necessary to compare two secondary currents. This means that the assumption of the parallelism of the phasors of the primary and secondary currents must be transferred to the two secondary currents being compared. The above statements will remain valid for two secondary currents when replacing phasors $\mathbf{I}_1''$ and $\mathbf{I_2}$ by phasors $\mathbf{I_S}$ and $\mathbf{I_X}$, respectively. The $\mathbf{I}_e$ phasor must be replaced by the phasor of the difference between two currents.

For two parallel vectors lying on one line, the phase shift can be zero, the angle β will also be zero, and the cosine β will be 1. In this case, the in-phase component ΔI will be equal to the length of the phasor $\mathbf{I_e}$. But if the phase shift $\Delta\varphi$ reaches values that affect the result of estimating the difference of the lengths of the phasors considered, it is no longer correct to consider these phasors as parallel.

In a previous work [7], [8] it was proposed a model that allows calculating the RE taking into account the true PD between the $\mathbf{I_S}$ and $\mathbf{I_X}$ phasors when calibrating the comparator of two alternating currents. When developing a virtual instrument for testing current and voltage transformers [9], the assumption of parallelism of the $\mathbf{V_{2S}}$ and $\mathbf{V_{2X}}$ phasors was used in analyzing systematic errors (the analysis was performed for two almost identical secondary voltages). To analyze the sources of uncertainty in the calibration of instrument transformers at the German Metrology Institute, the question of the correctness of the use of ΔI and ΔI_q values was also considered [10]. In particular, it was confirmed that these values are almost equal to the error values ε_I and $\Delta\varphi$.

The purpose of this work is to establish the limitation for the correctness of the application of the simplified analysis of the interrelation between the phasors of secondary currents of the DUT and working standard, which is based on the assumption of parallelism of these two phasors. Since two nearly identical secondary voltage phasors are also compared when determining the errors of the voltage transformers, the results of the analysis can also be extended to these measuring instruments.

4. Search Method

The standard Microsoft Excel software environment was used to find the limitation of the application of the aforementioned assumption. A certain number of the PD values with a small constant increment of this argument were set using computer simulation. The RE values were calculated both for the case of known values of the amplitudes of both secondary currents (see **Figure 1**) and for the approximation of this characteristic by the projection of the exciting current phasor onto an axis drawn through the phasor of the reduced primary current according to expression (3).

Figure 2 shows that the projection of the phasor of exciting current on the axis through the phasor of the reduced primary current is equal to the difference between the length of the last phasor and the projection of the secondary current phasor on the same axis, i.e.:

$$\Delta I = I_1'' - I_2 \cdot \cos(\Delta\varphi) = I_e \cdot \cos\beta \tag{6}$$

From the obtained equation (6), it can be seen that the change in the PD $\Delta\varphi$ leads to a corresponding change in the in-phase component related to the angle β.

The difference between the results obtained was evaluated for its significance relative to the obtained ε_I value of RE.
Concerning the PD, **Figure 2** shows that the projection of the phasor of exciting current on an axis perpendicular to the $\mathbf{I_1''}$ phasor, i.e. the quadrature component ΔI_q, changes according to the dependence:

$$\Delta I_q = I_2 \cdot \sin(\Delta\varphi) = I_e \cdot \sin\beta \tag{7}$$

Since PD can be determined by the tangent according to the expression (5), it is possible to simulate several PD and RE relations of the TC by varying the length of the phasor $\mathbf{I_2}$ and the PD value. The PD can also be determined as follows:

$$\Delta\varphi = \mathrm{atan}\left[\Delta I_q / \left(I_1'' - \Delta I\right)\right] \approx \Delta I_q / \left(I_1'' - \Delta I\right) \tag{8}$$

The last approximate equality in expression (8) is appropriate for small angles. Such approximation (simplification) also raises the question of the limitation of the admissibility of its application in practical activities, in particular when calibrating the TC. To solve this question, a number of the PD values were set for fixed interrelations between the $\mathbf{I_1''}$ and $\mathbf{I_2}$ phasors (see **Figure 2**). An approximation of this characteristic was also performed by projecting the phasor of exciting current onto an axis perpendicular to the phasor of the reduced primary current according to the expression (8).

The difference in one unit of the second significant digit between the two results was chosen as a criterion on the search for the limitation of assumption admissibility. The values measured are usually rounded to the second significant digit in determining TC errors. This means that the second digit remains unchanged when rounding if the measurement result has the third significant digit less than 5. If the third digit exceeds 5, then the second digit is rounded to the next higher digit. Consequently, the measurement results of slightly more and slightly less than 5 in the third digit will differ by 1 in the second digit.

According to GUM 1995 [11], it is possible to evaluate the uncertainty of measurements for the case when the interval is known and the distribution law of a random variable is unknown. In this case, we must assume a uniform distribution law for the interval where a random variable is located and, then, the standard uncertainty u_X will be determined by the formula:

$$u_X = \frac{X}{2\cdot\sqrt{3}} \tag{9}$$

where X is the interval in which the random variable is located.

A standard uncertainty of about 0.3 can be obtained when substituting an interval of 1 into the formula (9). Thus, the standard uncertainty of applying such a criterion would be about 0.3 of the second significant digit. In the transition to the expanded uncertainty, the use of a coverage factor of 2 is common in the practice of calibration laboratories. In this case, the contribution to the expanded uncertainty estimate can be evaluated at 0.6 of the second significant digit. The authors decided to apply somewhat more stringent requirements, reducing the tolerance in the second digit to half for convenience.

It should be noted that such a criterion contains some irregularity due to its dependence on the measurement result. That is, the half of the second digit relative to the minimum modeled RE (e.g. 0.1%), the relative difference will be 5%, and for the maximum RE (i.e. 0.99%) such parameter will be 0.5%. To eliminate this imbalance of assessment, the criterion for exceeding the relative difference of 1% was also applied.

Thus, the final criterion was the difference in the results in the third significant digit by 5 or exceeding the relative difference of 1%.

5. Simulation Results

5.1 Ratio Error Modeling

According to the method described above, several values of the relative difference of the simulated measurement results were obtained. The values of RE were set at 0.01, 0.02, 0.05, 0.1, 0.2, 0.5 and 1%. For each of these values, a number of the PD values were set in such a way that the $\Delta\varphi/\varepsilon_I$ error ratio varied from 5 to about 170. The probability of the error ratio of TC corresponding to the given extreme values is not high. However, to find the limitations to the correctness of the simplifications used in determining TC errors, it is worth expanding the search range to unlikely error ratios. **Table 1** summarizes the main data analysis of simulated RE measurement results.

Table 1: Simulation of RE measurement results

Characteristic	*The characteristic value depending on the $\Delta\varphi/\varepsilon$ error ratio*				
	5	40	80	122	172
ε=0.01 %					
PD, min.	0.05	0.40	0.80	1.20	1.70
Difference in results, %	0.0001	0.0068	0.0274	0.0616	0.1237
ε=0.02 %					
PD, min.	0.10	0.80	1.60	2.40	3.40
Difference in results, %	0.0002	0.0137	0.0547	0.1232	0.2476
ε=0.05 %					
PD, min.	0.25	2.00	4.00	6.00	8.50
Difference in results, %	-0.0005	-0.0342	-0.1366	-0.3067	-0.6137
ε=0.1 %					
PD, min.	0.50	4.00	8.00	12.0	17.0
Difference in results, %	0.001	0.069	0.276	0.624	1.260
ε=0.2 %					
PD, min.	1.00	8.00	16.0	24.0	34.0
Difference in results, %	-0.002	-0.138	-0.548	-1.225	-2.429
ε=0.5 %					
PD, min.	2.50	20.0	40.0	60.0	85.0
Difference in results, %	0.005	0.346	1.400	3.207	6.650
ε=1.0 %					
PD, min.	5.0	40	80	120	170
Difference in results, %	0.01	0.70	2.84	6.63	14.25

Table 1 shows that for high-precision TCs (when ε < 0.1%), the relative difference in results will never be 1%. However, with an increase in RE up to 1%, the effect of approximation becomes significant exceeding the 1% threshold in the vicinity of 50 angular minutes. Regarding the second part of the criterion, it should be said that the difference in half of the second significant digit appears at the PD of 40 angular minutes for simulated RE of 1%. For simulated RE of 0.2%, the difference in half of the second significant digit appears at the PD of about 34 angular minutes.

For better visual perception, the figure below shows an extract from the results obtained for the simulated RE of approximately 0.5%.

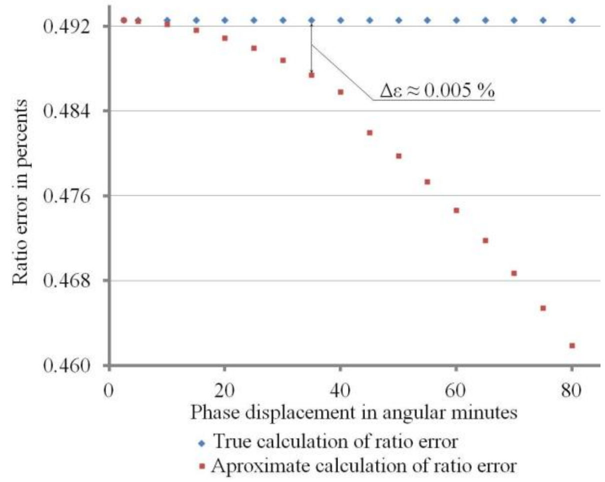

Figure 3: The difference between the simulated REs of 0.5% depending on the PD

In **Figure 3**, it can be seen that at a rated current of the TC with an accuracy class of 0.5S, a threshold of 1% is reached at the PD value of about 35 angular minutes. For simulated RE of 0.5%, the difference in half of the second significant digit also appears at the PD of 35 angular minutes.

It is worth noting that for the TCs that meet the requirements of the IEC standard for accuracy, the impact of the approximation can be considered to be negligible that is confirmed by the simulation results.

In the context of the study, it is also pertinent to mention that the TC errors are normalized in the range of the secondary winding load. The secondary load of the TC may be in the range from 1 to several tens of VA and a power factor should be in the range from 0.8 to 1. In practice, the secondary winding load may be less than 1 VA and slightly more than 0 VA. The TCs are designed in such a way that the RE is negative at maximum load and it is positive at 0 VA. In such a case, the PD will most often have the same sign. The hypothetical example would be the following error relations:

- ε = -0.16%, Δφ = 10' at a secondary load of 10 VA;
- ε = 0.03%, Δφ = 9' at a secondary load of 10 VA at a secondary load of 2.5 VA;
- ε = 0.10%, Δφ = 8.4' at a secondary load of 0 VA.

For metrology, the degree of equivalence of the results of measuring the TC errors at these ratios is of interest, especially when the RE is equal to 0. This case was modeled and the changing nature of the relative difference in the RE measurement results for the above example was investigated. The simulation result is presented graphically below.

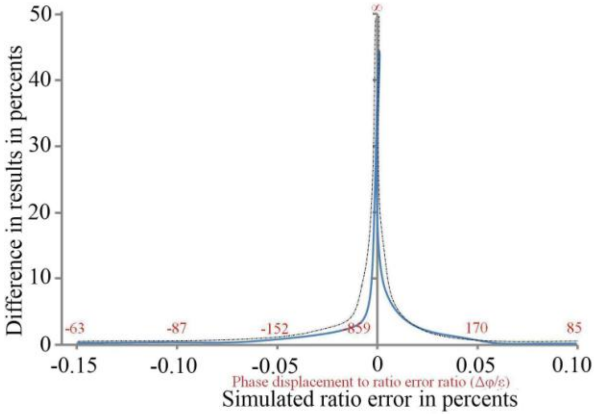

Figure 4: Dependence of the relative difference of the RE on both the value of the measured RE and the ratio of the PD to the RE ($\Delta\varphi/\varepsilon$)

Figure 4 shows that with the simulated RE approaching zero, the difference in the results calculated in two ways increases significantly. Since the PD does not change significantly, but the RE passes through zero, at this point the $\Delta\varphi/\varepsilon$ ratio grows to the infinity. In **Figure 4**, two dashed lines show an increase in the difference in the results to infinity, and the perpendicular to the abscissa axis through the coordinate $\varepsilon = 0$ is an asymptote.

5.2 Phase Displacement Modeling

Table 2 summarizes the main data analysis of simulated PD measurement results. When modeling the PD measurement results, the RE values of 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, and 1% were set.

Table 2: Simulation of PD measurement results

Characteristic	*The characteristic value depending on the RE*				
	0.01 %	0.05 %	0.10 %	0.50 %	1.00 %
$\Delta\varphi$=0.05 angular minutes					
Error ratio $\Delta\varphi/\varepsilon$	5.050	1.010	0.509	0.102	0.051
Difference in results, %	$7.1{\cdot}10^{-9}$	$7.1{\cdot}10^{-9}$	$7.1{\cdot}10^{-9}$	$7.1{\cdot}10^{-9}$	$7.1{\cdot}10^{-9}$
$\Delta\varphi$=30 angular minutes					
Error ratio $\Delta\varphi/\varepsilon$	3030	6063	305	61	30
Difference in results, %	$2.5{\cdot}10^{-3}$	$2.5{\cdot}10^{-3}$	$2.5{\cdot}10^{-3}$	$2.5{\cdot}10^{-3}$	$2.5{\cdot}10^{-3}$
$\Delta\varphi$=100 angular minutes					
Error ratio $\Delta\varphi/\varepsilon$	10100	2021	1017	203	101
Difference in results, %	$2.8{\cdot}10^{-2}$	$2.8{\cdot}10^{-2}$	$2.8{\cdot}10^{-2}$	$2.8{\cdot}10^{-2}$	$2.8{\cdot}10^{-2}$
$\Delta\varphi$=300 angular minutes					
Error ratio $\Delta\varphi/\varepsilon$	30300	6063	3051	609	303
Difference in results, %	0.25	0.25	0.25	0.25	0.25

Several PD values were determined for each of the above values to vary the error ratio from 0.05 to about 30,000.

Table 2 shows that for the same PD, the difference in results is negligibly small and does not depend on the ratio between the PD and RE. It should be noted that the relative difference in the results increases with the increase of the PD, but does not reach up to 1% as well as does not reach half of the second significant digit, even at the PD of 300 angular minutes.

For a better visual perception, the following drawing is an extract from the obtained results of the simulated PD for the RE of about 0.5%. Moreover, the maximum value of the PD has been chosen such that it is determined by the IEC standard for the corresponding accuracy class of the TC for the rated current.

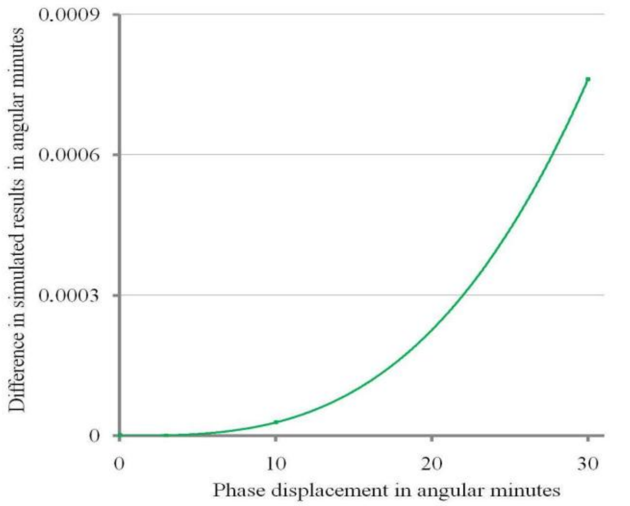

Figure 5: The difference between the simulated PDs depending on the PD value

Figure 5 shows that the dependence of the difference in results on the measured PD is polynomial of the third degree. When measuring 20 angular minutes, the difference in the results is $2.3{\cdot}10^{-4}$ angular minutes (i.e. 0.0011%), and when measuring 30 angular minutes, this parameter gains $7.6{\cdot}10^{-4}$ angular minutes (i.e. 0.0025%). Thus, the approximation in the measurement of the PD does not distort the results obtained for the TC with errors corresponding to the accuracy class of 0.5.

The absence of the effect of the RE change may be logically explained when considering the drawing below.

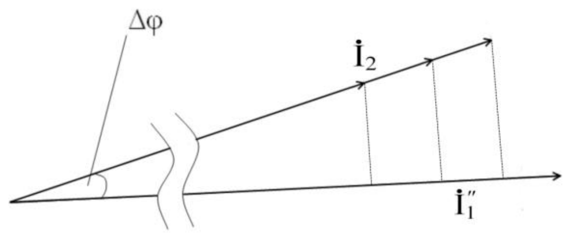

Figure 6: Graphical interpretation of changes in the ratio of $\mathbf{I}_1^{''}$ and $\mathbf{I}_2$ phasors

Figure 6 shows that when the length of the $\mathbf{I}_2$ phasor changes, the leg opposite the angle $\Delta\varphi$ changes proportionally by expression (7), as does the part of the $\mathbf{I}_1^{''}$ phasor that is an adjacent leg of the corresponding right triangle. Thus, at constant PD, the ratio of the quadrature component ΔI_q to the corresponding part of the $\mathbf{I}_1''$ phasor module is constant.

6. Conclusion

In assessing the correctness of the application of the approximation in determining the errors of the current transformer, it has been fixed the absence of a noticeable influence of the assumption of the equality of both the true ratio error and the value of the projection of the exciting current phasor on the axis drawn through the primary current phasor. This statement is correct provided that the errors of the transformer are found to satisfy the accuracy requirements of the international IEC standard. The application of the approximation in determining the ratio error will not give a correct estimate if the value of this characteristic is close to zero.

The absence of a noticeable influence of the assumption of the equality of both the true value of the phase displacement and the value of the projection of the phasor of exciting current on the axis perpendicular to the phasor of the primary current was noted as well. This statement is correct in a wide range of measured phase displacement from 0 to 300 angular minutes and even more.

References

[1] Instrument Transformers – Part 2: Additional Requirements for Current Transformers, IEC Standard 61869-2, 2012.
[2] LLC "Oltest," AMAK.411439.001 РЭ. CA507 Comparator. Manual. Part 1, oltest.com.ua. [Online]. Available: http://www.oltest.com.ua/file/04_Instr_CA507.pdf. [Accessed: Feb. 03, 2020].
[3] Haefely, 2763 / 2767 / 2769, Automatic Instrument Transformer Test Set, haefely.com. [Online]. Available: http://www.haefely.com/specsheet/2767_2763_2769_DS_v18.05_LW.pdf. [Accessed: Feb. 03, 2020].
[4] Instrument Transformers – Part 1: General Requirements, IEC Standard 61869-1, 2007.
[5] V. V. Afanasiev et al., Current Transformers, Energoatomizdat, Leningrad, 1989.
[6] V. Isaiev, "Finding Reference Values to Calibrate Two Alternating Currents Comparator," In Proceedings of the International Scientific and Practical Conference World Science, 1 (2), pp. 57-61, 2018.
[7] V. Isaiev, "The Problem of Defining Comparation Accuracy of AC Current Amplitude Values," Science and Education a New Dimension. Natural and Thechnical Sciences, V (121), pp. 57-60, 2017.
[8] V. Isaiev, "Method of Reference Values Defining for Calibration of Two Alternating Currents Comparator with Using Oscilloscope," World Science, 2 (32), pp. 42-49, 2018.
[9] N. Madhu Mohan, B. George, V. Jagadeesh Kumar, "Virtual Instrument for Testing of Current and Voltage Transformers," In Proceedings of the Instrumentation and Measurement Technology Conference (IMTC 2006), pp. 1163-1166, 2006.
[10] E. Mohns, P. Räther, "Test Equipment And its Effect on the Calibration of Instrument Transformers," Journal of Sensors and Sensor Systems, 7 (1), pp. 339-347, 2018.
[11] Evaluation of Measurement Data – Guide to the Expression of Uncertainty in Measurement, JCGM 100: 2008 (GUM 1995 with Minor Corrections), 2008.