Savage Minimax Regret Criterion Method to Find Optimal Decision

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Abstract: *In decision under uncertainty, decision makers have to choose one of a set number of alternatives with complete information about their outcomes but in the absence of any information or data about the probabilities of the various state of nature, we focus on avoiding regrets that may result from making a non-optimal decision. Although regret is a subjective emotional state, the assumption is made that it is quantifiable in direct (linear) relation to the rewards of the payoff. The classical approach of Savage Minimax regret criterion can be used to find an optimal decision and avoid regret. The savage minimax regret criterion examines the regret, opportunity cost or loss resulting when a particular situation occurs and the payoff of the selected alternative is smaller than the payoff that could have been attained with that particular situation. This approach is independent of the probabilities of the various outcomes: thus if regret can be accurately computed, one can reliably use minimax regret criterion*

1. Introduction

Typically, personal and professional decisions can be made with some difficulty. Either the best course of action is clear or the varieties of the decision are not significant enough to require a great amount of attention. Occasionally, decisions arise where the path is not clear and it is necessary to take substantial time and effort in devising a systematic method of analyzing the various courses of action. With decisions under uncertainty, the decision maker should:
1) Take an inventory of all viable options available for gathering information, for experimentation and for action
2) List all events that may occur
3) Arrange all pertinent information and choices/assumptions made
4) Rank the consequences resulting from the various courses of action
5) Determine the probability of an uncertain event occurring.

Upon systematically describing the problem and recording all necessary data, judgments, and preferences, the decision maker should synthesize the information set before using the most appropriate decision rules. Decision rules prescribe how an individual faced with a decision under uncertainty should go about choosing a course of action consistent with the individual's basic judgments and preferences.

When a decision maker should choose one possible actions, the ultimate consequences of some, if not all of these actions will generally depend on uncertain events and future actions extending indefinitely far into the future. The minimax regret approach is to minimize the worst-case regret.'Regret' in this context is defined as the opportunity loss through having made the wrong decision. It is useful for a risk-neutral decision maker. Essentially, this technique is for a 'sore loser' who does not wish to make a wrong decision

2. Related Work

In the decisions under uncertainty individual decision makers have to choose one of presumed business alternatives with the extended information about their profitability, outcomes, costs, financial results, but in the absence of any information about the probabilities of the various states of nature.

The classical criterion of Wald's, Hurwicz's, Maximax, Savage's and Laplace's are assessed and compared .The assessment is made on the basis of financial results for individual business alternative evaluation.

Wald's Maximin Criterion suggests that the decision maker examines only the minimum payoffs of alternatives and chooses the alternative whose outcome is the least bad. This criterion appeals to the cautious decision maker who seeks insurance that in the event of an unfavorable outcome, there is at least a known minimum payoff.

The Hurwicz criterion attempts to find a middle ground between the extremes posed by the optimist and pessimist criteria. Instead of assuming total optimism or pessimism, Hurwicz incorporates a measure of both by assigning a certain percentage weight to optimism and the balance to pessimism. However, this approach attempts to strike a balance between the maximax and maximin criteria.

Maximax Criterion suggests that the decision maker examine the maximum payoffs of alternatives and choose the alternatives whose outcome is the best. This criterion appeals to the adventurous decision maker who is attracted by high payoffs.

Minimax regret approach is to minimize the worst-case regret. The aim of this is to perform as closely as possible to the optimal course. Since the minimax criterion applied here is to the regret (difference or ratio of the payoffs) rather than to the payoff itself, it is not as pessimistic as the ordinary minimax approach. Similar approaches have been used in a variety of areas such as:
- Hypothesis testing
- Prediction
- Economics

One benefit of minimax (as opposed to expected regret) is that it is independent of the probabilities of the various outcomes: thus if regret can be accurately computed, one can

reliably use minimax regret. However, probabilities of outcomes are hard to estimate.

This differs from the standard minimax approach in that it uses differences or ratios between outcomes, and thus requires interval or ratio measurements, as well as ordinal measurements (ranking), as in standard minimax.

3. The Expected Opportunity Loss Criterion

The savage minimax regret criterion examines the regret, opportunity cost or loss resulting when a particular situation occurs and the payoff of the selected alternative is smaller than the payoff that could have been attained with that particular situation.

The minimax criterion suggests that the decision maker look at the maximum regret of each strategy and select the one with the smallest value. This approach appeals to cautious decision makers who want to ensure that the selected alternative does well when compared to other alternatives regardless of what situation arises. It is particularly attractive to a decision maker who knows that several competitors face identical or similar circumstances and who is aware that the decision maker's performance will be evaluated in relation to the competitors. This criterion is applied to the same decision situation and transforms the payoff matrix into a regret matrix.

Minimax Regret is a better decision criterion than Maximax or Maximin and, arguably, Hurwicz as well. Although it employs the far-from-robust minimax logic, the values over which it operates (opportunity losses) contain more problem information (actual monetary losses plus unrealized potential profits), leading to a more informed decision than was possible with any of the three previous models. Nevertheless, it still fails to employ all of the available problem information and is therefore not a rationally acceptable criterion.

Minimax Regret is a conservative criterion, as is Maximin/Minimax. However, it is not as extreme in its pessimism as the latter. There is no guarantee this will always be so, but it does show that minimaxing regrets is not as conservative an approach as maximining positive-flow payoffs.

A. System Design
The expected opportunity loss (EOL) of action j is the loss, L_{ij}, for each combination of event i and action j multiplied by P_i, the probability of occurrence of the event i, summed over all events.

$$\text{EOL (j)} = \sum_{i=1}^{N} L_{ij} P_i$$

Where,
L_{ij}=opportunity loss that occurs when course of action is selected and event i occurs.
P_i=probability of occurrence of event i.

B. System Architecture
Purpose: To find the choice corresponding to minimum regret.
Input: The payoff matrix of the given problem.

Output: The regret matrix and the choice corresponding to the minimum regret.
STEP 1: From the given pay off matrix, develop an opportunity-loss (or regret) matrix.
1. Find the best pay off corresponding to each state of nature(maximum for profit and minimum for cost.
2.for i=0 to m-1
For j=0 to n-1
ith regret = (Maximum pay off-ith pay off) for the jth event
if the pay offs represents profit
end for
end for
STEP 2: Determine the maximum regret amount for each alternative.
STEP 3: Choose that alternative which corresponds to the minimum regrets.

The Minimax Regret criterion focuses on avoiding regrets that may result from making a non-optimal decision. The assumption is made that it is quantifiable in direct (linear) relation to the rewards of the payoff matrix. Regret is defined as the opportunity loss $r(a_i,\theta_j)$ to the decision maker if action alternative a_i is chosen and state of nature θj happens to occur. Opportunity loss is the payoff difference between the best possible outcome under θj and the actual outcome resulting from choosing a_i. Formally:
• If $v(a_i,\theta_j)$ corresponds to positive-flow payoff then :
$r(a_i,\theta_j) = \max\{v(a_k,\theta_j)\} - v(a_i,\theta_j)$ (1)
• If $v(a_i,\theta j)$ corresponds to negative-flow payoff then :
$r(a_i,\theta_j) = v(a_i,\theta_j) - \min\{v(a_k,\theta j)\}$ (2)

Initially we have to calculate the Opportunity Loss table using the formula with the help of given payoff matrix table. We have to calculate the maximum opportunity loss for each alternative.At the end we choose the alternative with the minimum maximum loss.

4. Results and Discussion

The maximax and maximin rules and the Hurwicz criterion can be criticized because they focus only on extreme payoffs and exclude the other payoffs. An approach that does take all payoffs into account is the Savage minimax regret criterion. This rule represents a pessimistic approach used for an opportunity loss table.

Minimax Regret is a conservative criterion, as is Maximin/Minimax. However, it is not as extreme in its pessimism as the latter. It is not necessary that equal differences in profit would always correspond to equal amounts of regret.A small advantage in one scenario may lead to the loss of larger advantages in other scenarios. Nevertheless it still fails to employ all of the available problem information and is therefore not a rationally acceptable criterion

The minimax regret criterion may have certain improvements in order to get better solutions.

5. Conclusion

Regret is the negative emotion experienced when learning that an alternative course of action would have resulted in a

more favorable outcome. The theory of regret aversion or anticipated regret proposes that when facing a decision, individuals may anticipate the possibility of feeling regret after the uncertainty is resolved and thus incorporate in their choice their desire to eliminate or reduce this possibility. One of the methods is Savage Minimax Regret Criterion. It minimizes the worst-case regret. It is an optimistic approach used in varieties of fields like Hypothesis testing, Prediction, Economics.

References

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